Acoustic Model of a Helmholtz Resonator under Grazing Turbulent Flow

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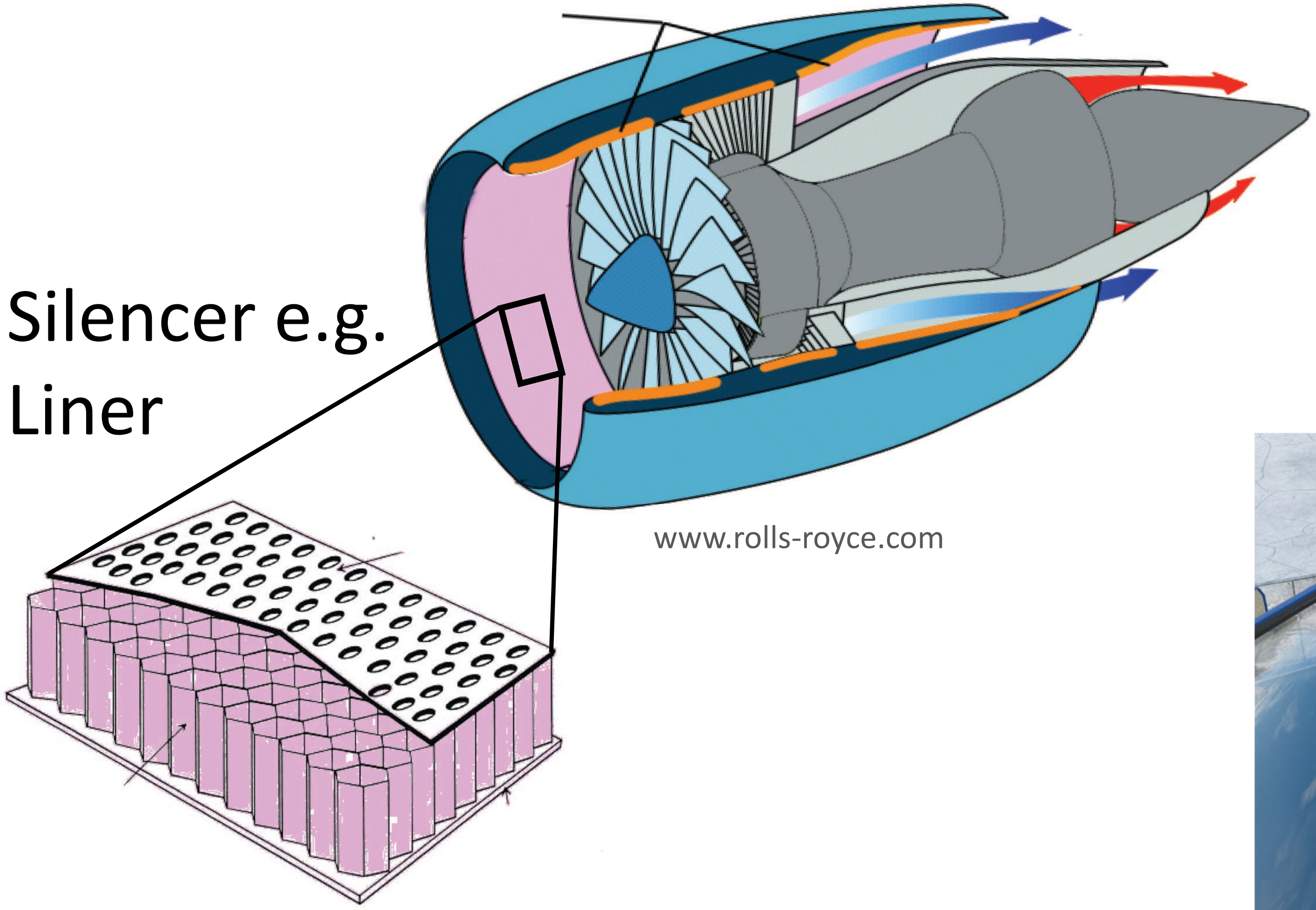
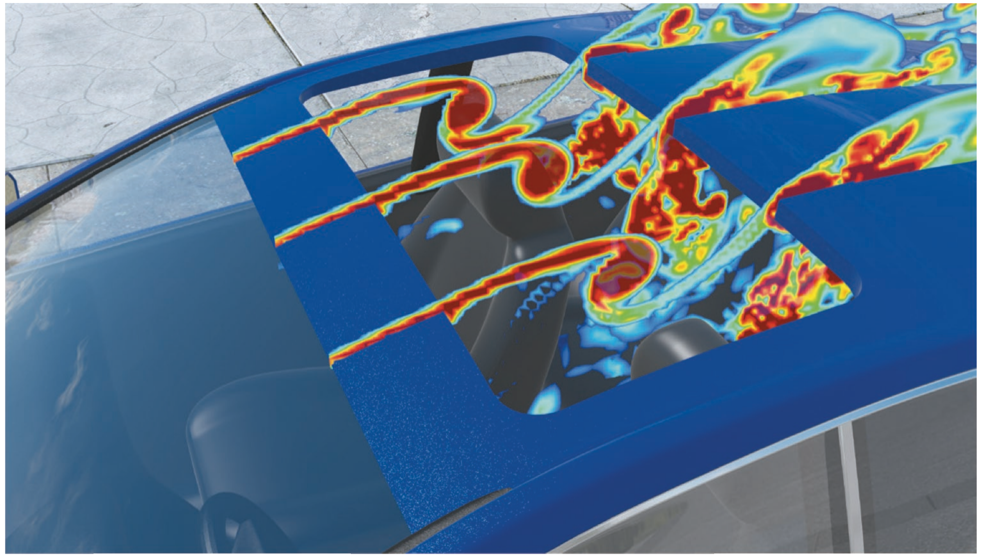
Goal: Model / Control Tonal Acoustics in Turbulent Flow past a Cavity

Silencer e.g. Liner



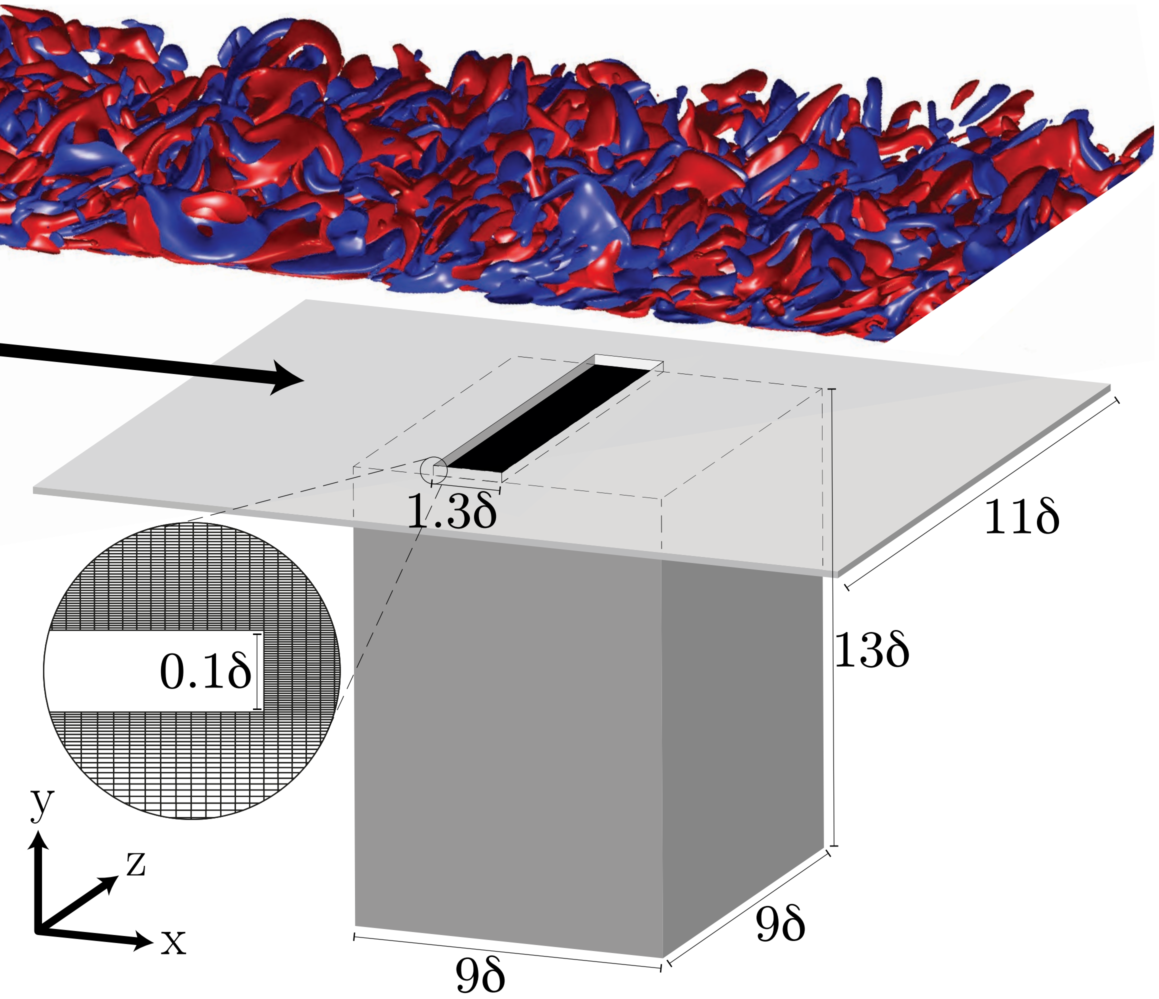
www.rolls-royce.com

Cavity Noise e.g. Sunroof Buffeting



www.exa.com

Case: Turbulent Boundary Layer flowing over a Cavity



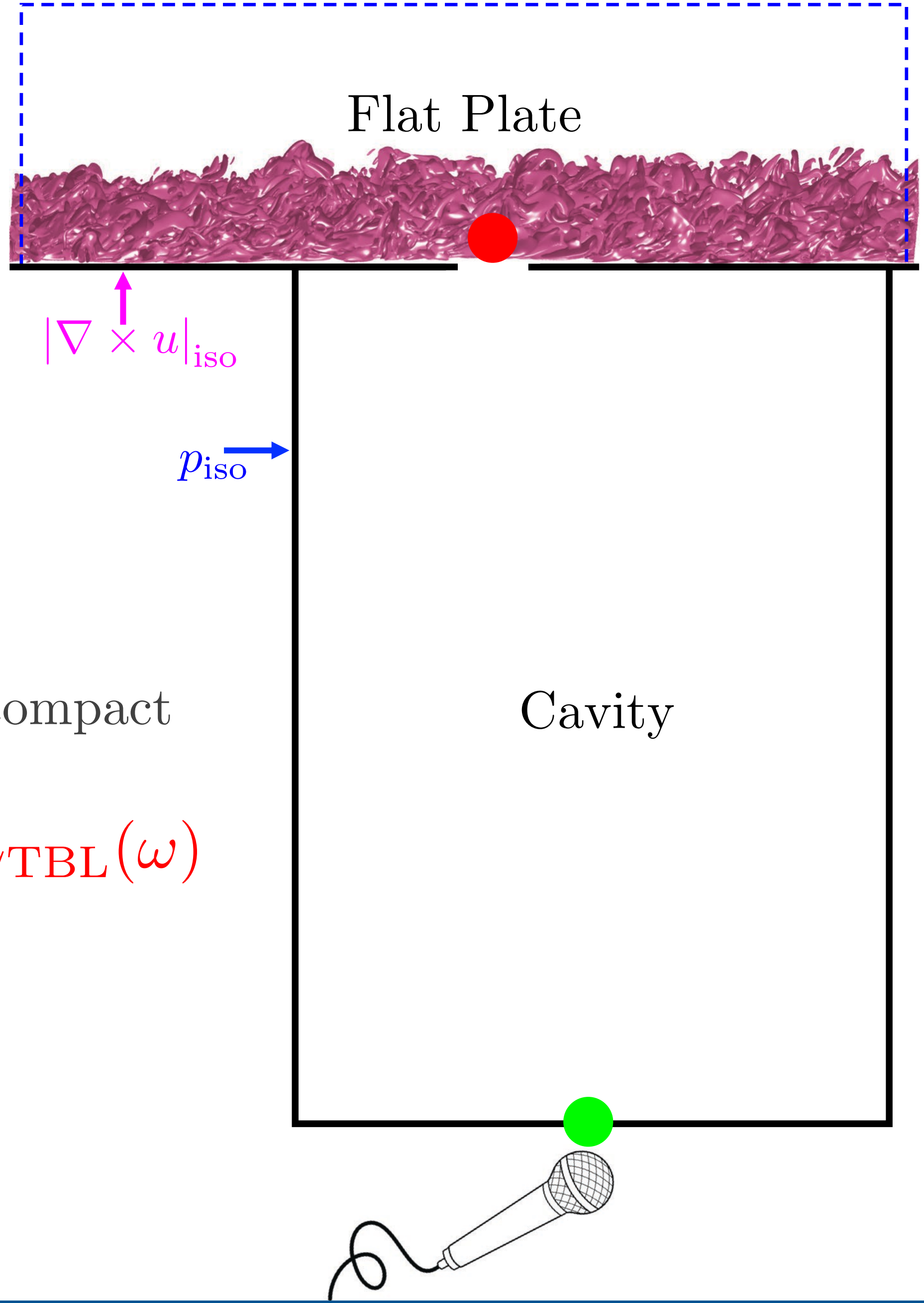
$$Re_{\delta_{99,neck}} \cong 4200$$
$$Re_{\tau_{neck}} \cong 170$$
$$M \cong 0.1$$
$$u_{ref} \cong 40\ m/s$$
$$\delta_{99,neck} \cong 1\ cm$$

1st Direct Numerical Simulation

grid points $\cong 1.2 \times 10^9$

computing time $\cong 3.6 \times 10^6$ CPU-h

Wanted Model



- To Understand Mechanisms
- Reveal Governing Parameters
- Easy, Low-cost Cavity Layout
- Operating Point: turbulent, low Mach, acoustically compact

$$SPL_{\text{MIC}}(\omega) = T(\text{Geometry}, Ma, Re, ?, ?)\ SPL_{\text{TBL}}(\omega)$$

Desired Transfer Function

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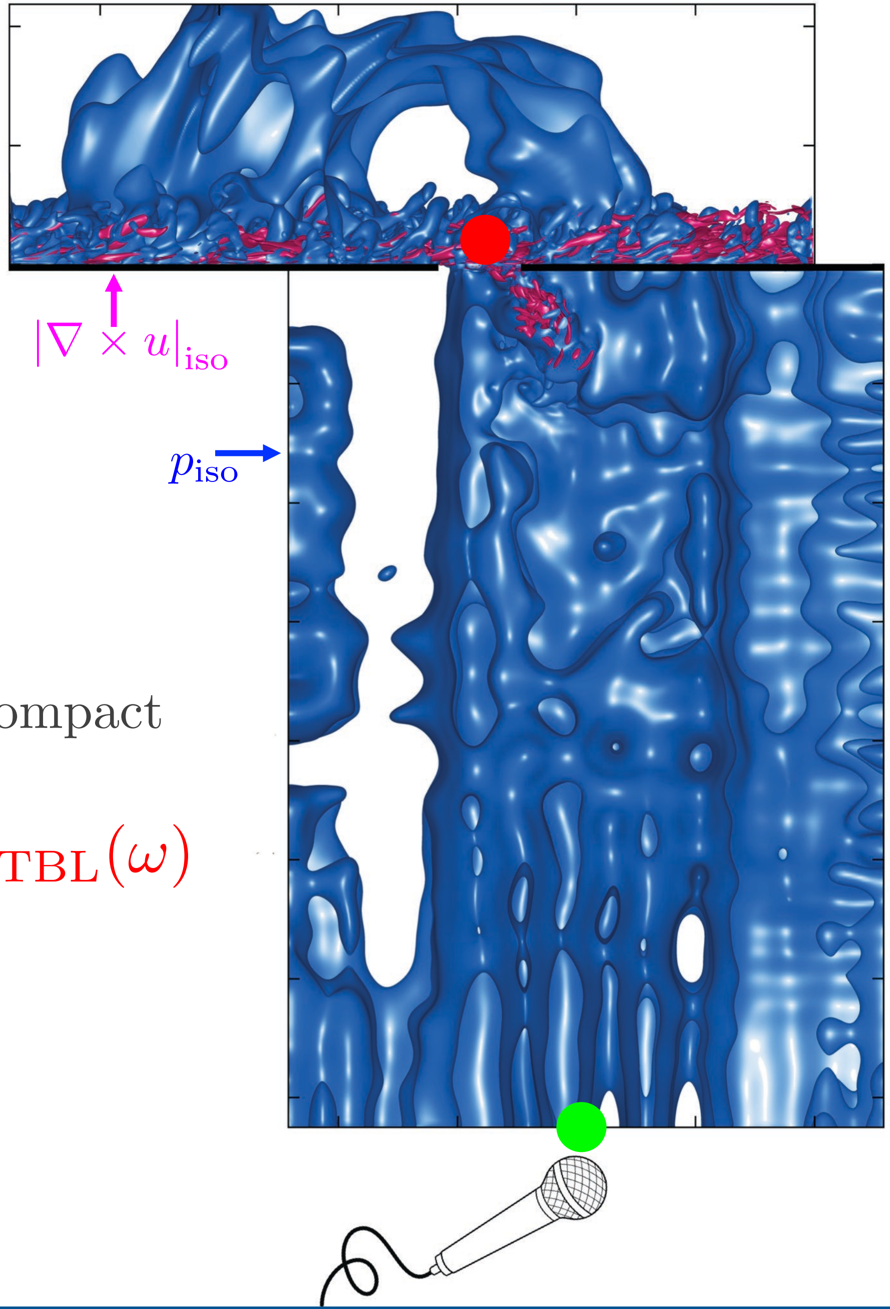
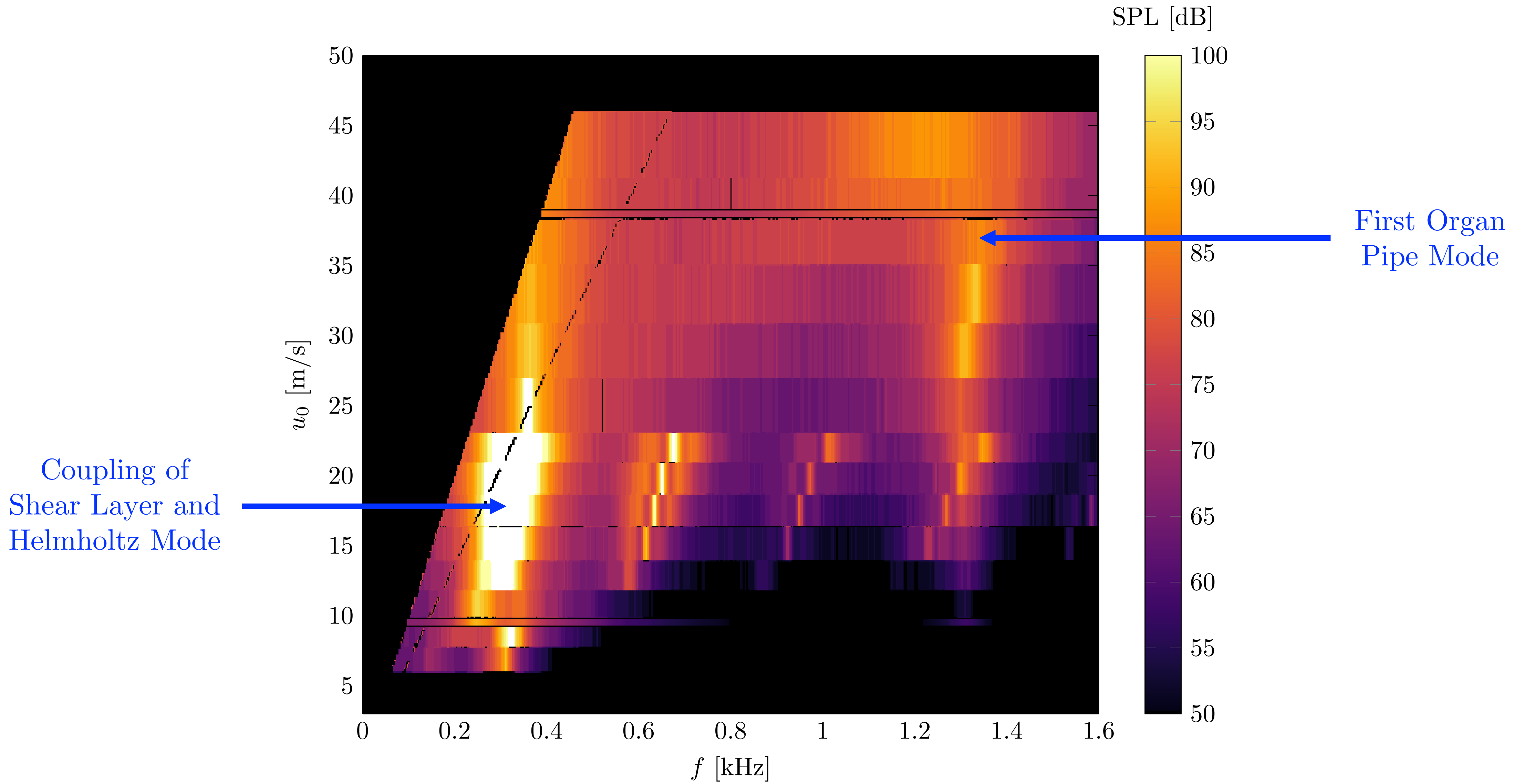


$SPL_{\mathrm{MIC}}(\omega)$ [J. Golliard, 2002]



Model Components

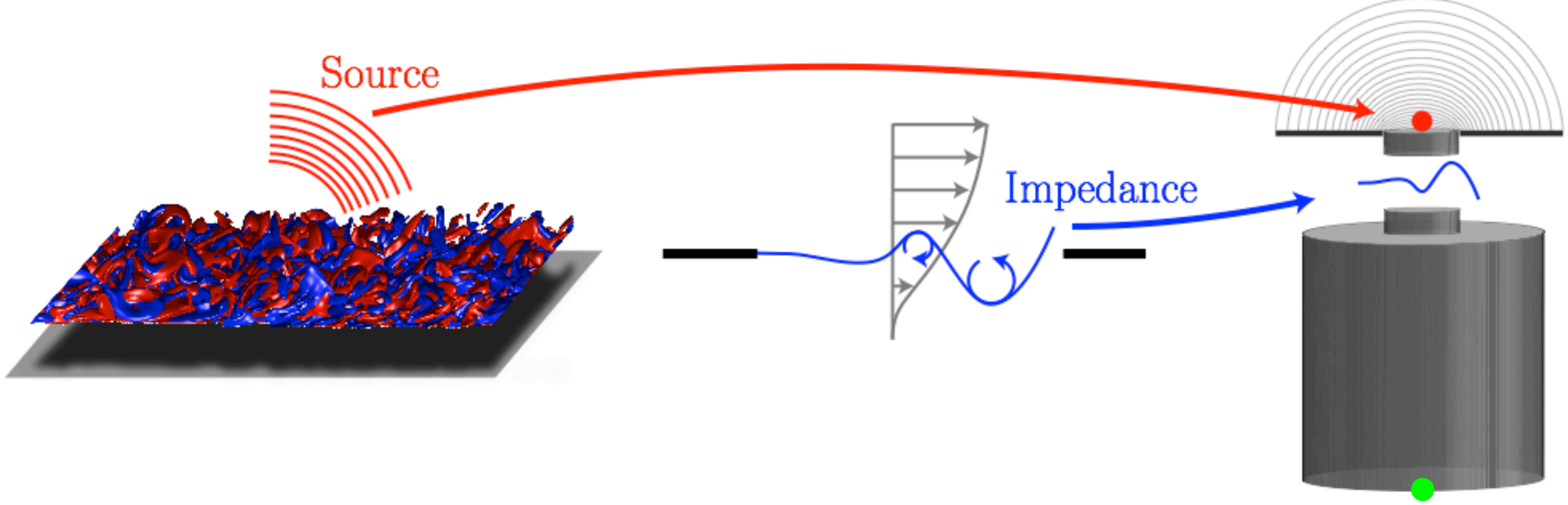
$SPL_{\mathrm{TBL}}(\omega)$

Goody's
Turbulent Flat Plate Flow
[Y. Hwang et al., 2009]

$T(\mathrm{Geometry}, Ma, Re, ?, ?)$

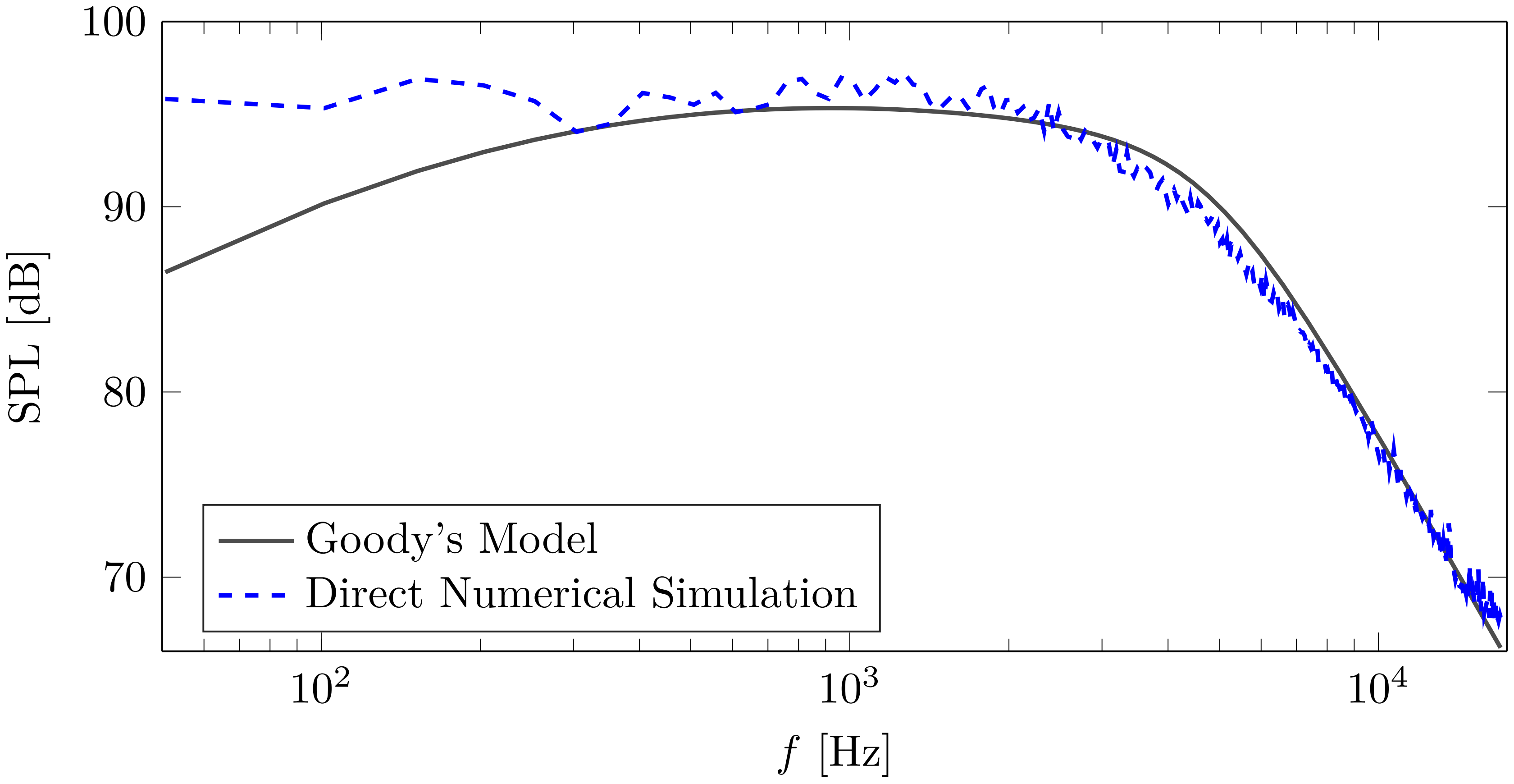
Rayleigh Conductivity
of an aperture
[M. S. Howe, 1998]

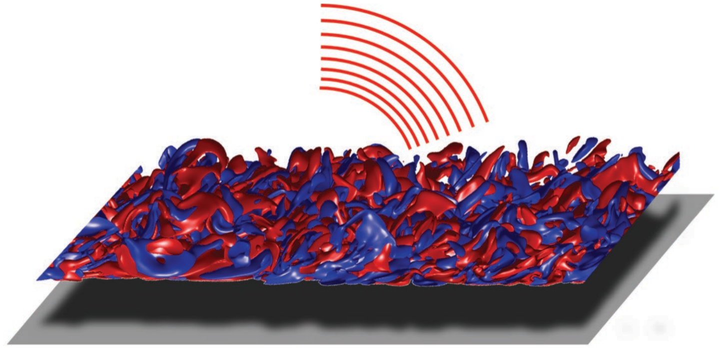
Lumped Acoustic Elements
(no flow)
[J. Golliard, 2002]



Goody's Model of a Turbulent Flat Plate Flow

$$\Phi_{pp}(Sr) = \mathrm{PressurePowerSpectrum}_{\mathrm{Goody}} = \rho_w^2 \tau_w^4 \frac{\delta_{99}}{u_0} \frac{C\ Sr^2}{\left\{Sr^{0.75} + 1/2\right\}^{3.7} + \left\{1.1\, Sr \left(Re_\tau \frac{u_\tau}{u_0}\right)^{-0.57}\right\}^7}$$





Golliard's 1D Lumped Element Method

Restrictions: no flow, linear

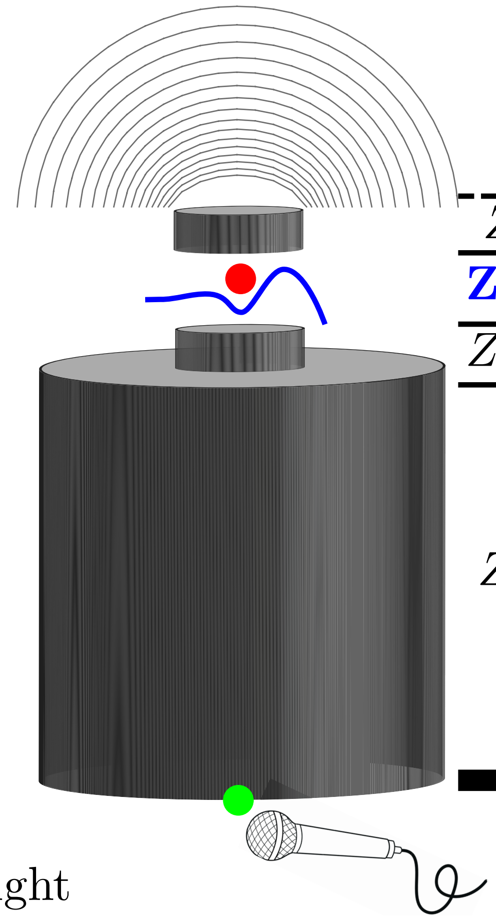
$$p_{\mathrm{MIC}}(\omega) = t(\omega, Z_{rad/howe/jump/pipe})\; p_{\mathrm{TBL}}(\omega)$$

$$T(\omega) = \left| \frac{p_{\mathrm{MIC}}}{p_{\mathrm{TBL}}} \right|^2 \qquad Z = \frac{\rho c}{S}\,(r + i\,k\,\delta)$$

$$T^{-1} = \sin^2(kH_{pipe})\,(\Lambda_\delta^2 + \Delta_r^2)$$

$$\Lambda_\delta = \cot(kH_{pipe}) - S_{ratio}(\delta_{jump} + \delta_{howe} + \delta_{rad}) \qquad \text{effective height}$$

$$\Delta_r = S_{ratio}(r_{howe} + r_{rad}) \qquad \text{energy transfer}$$



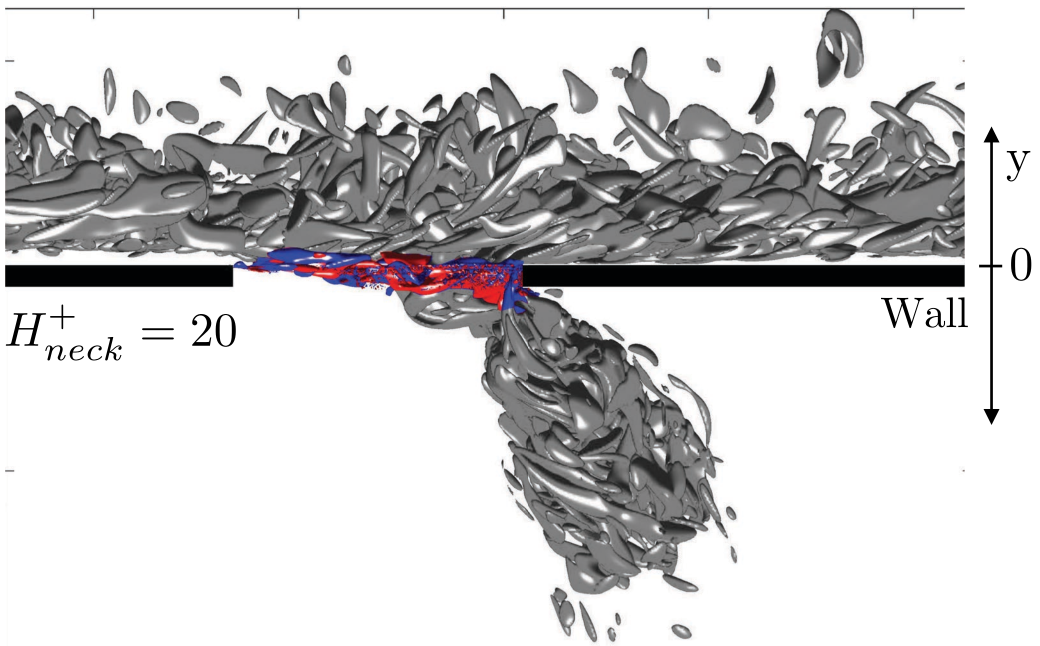
Elements:

Z_{rad}	infinite flange
$\mathbf{Z_{howe}}$	shear layer
Z_{jump}	area jump
Z_{pipe}	pipe

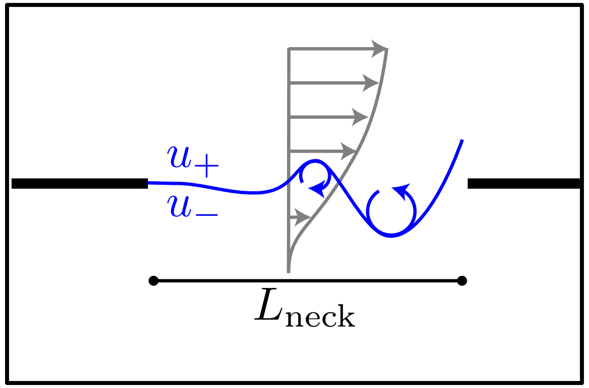
Howe's Opening Impedance

$$Z_{howe} = \frac{\rho c}{S_{neck}} \left(r_{howe} + i\, k\, \delta_{howe} \right)$$

DNS $|\nabla \times \mathbf{u}| = 3000\,\mathrm{Hz}$



Infinite Thin Vortex Sheet with Kelvin-Helmholtz Waves: $e^{i\sigma_\pm x}$



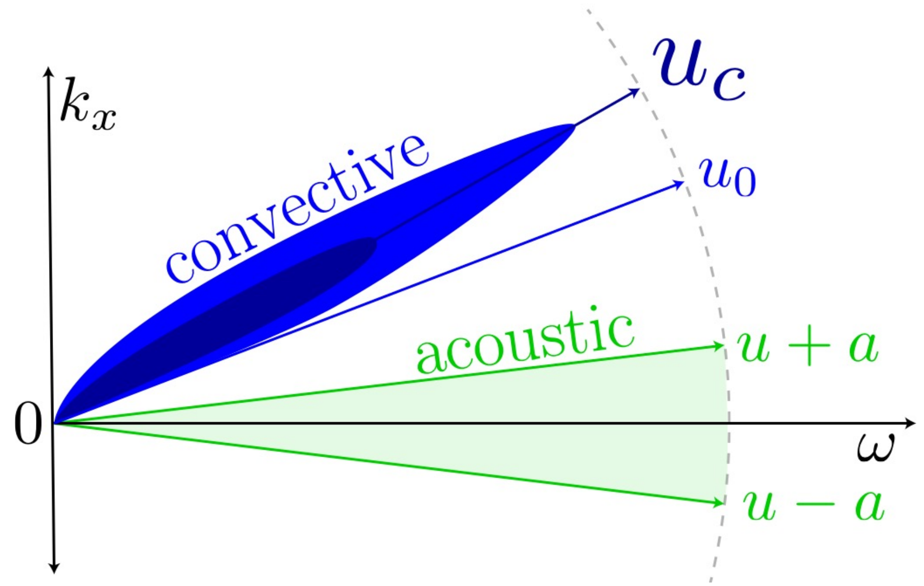
$$Z_{howe}(\sigma_\pm)$$

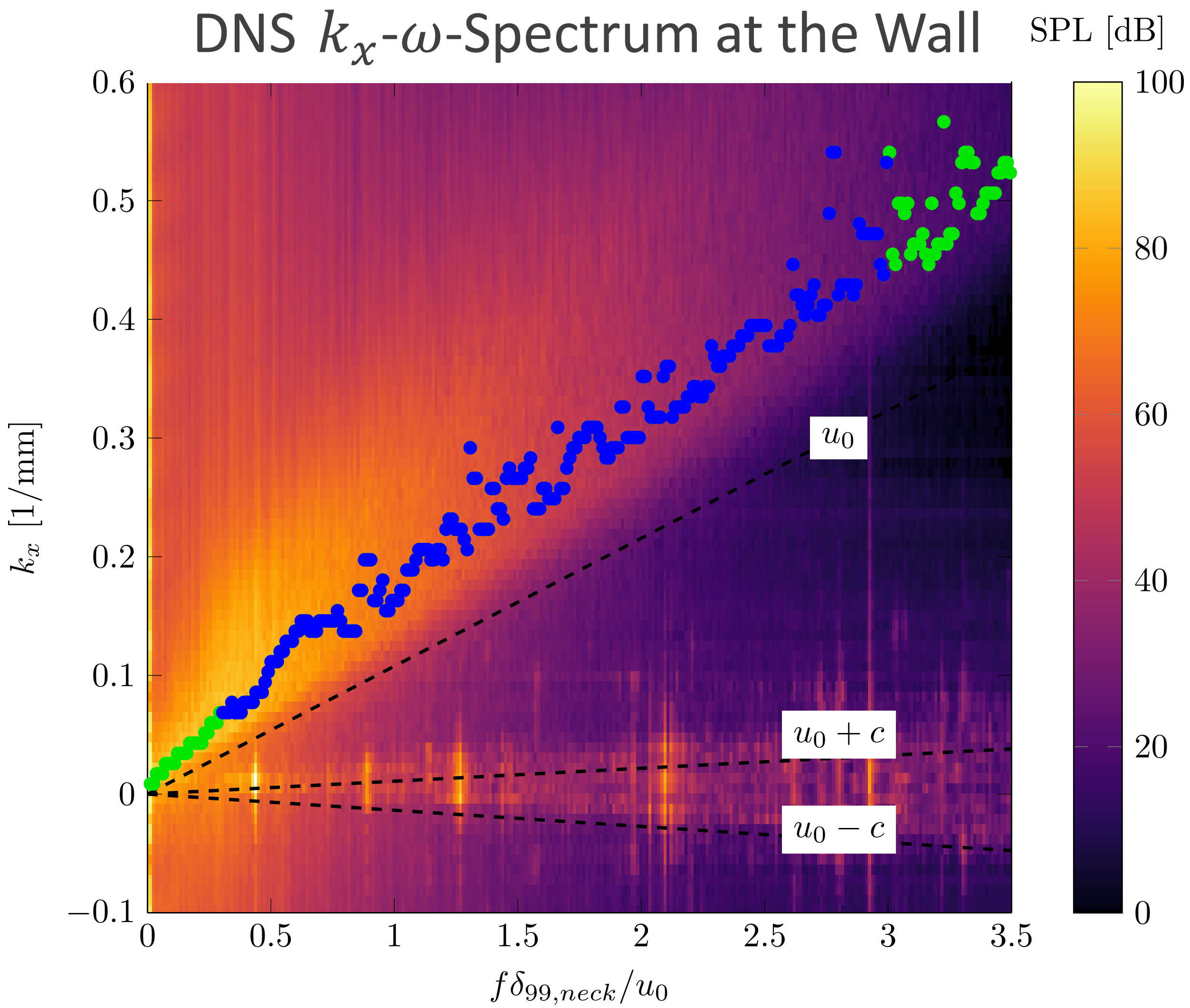
$$\sigma_\pm = \frac{\omega L_{\mathrm{neck}}}{2\, u_+} \frac{1 \pm i}{1 \pm i\, \frac{u_-}{u_+}}$$

$u_\pm$ unknown

General Convection Velocity Definition

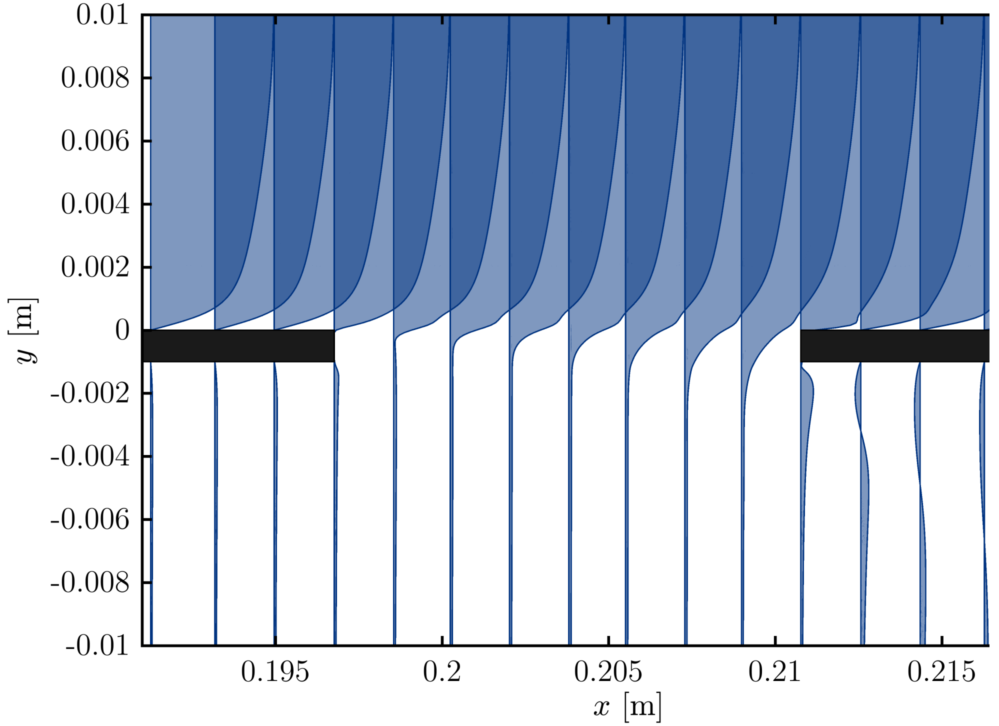
$$u_c(y,\omega) = \omega / k_{max}(y,\omega)$$
$$0 = \frac{\partial}{\partial k_x} \, \mathrm{SPL}(k_x, y, \omega)\Big|_{k_x = k_{max}}$$

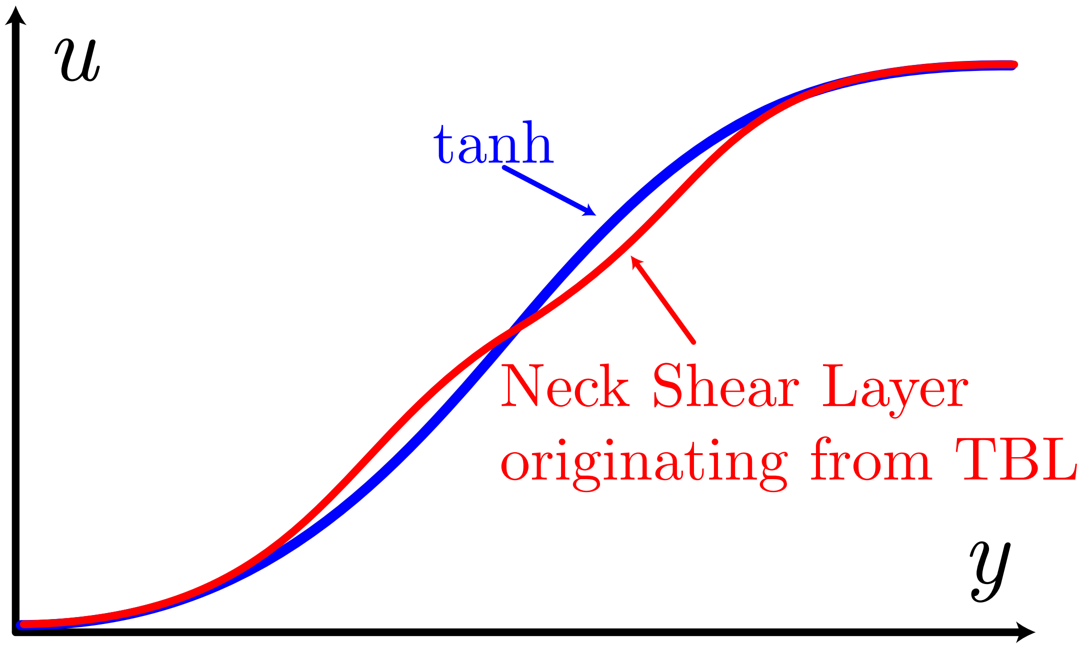




Constant Height of the Inflection Points

Velocity Profiles $u(x,y)$





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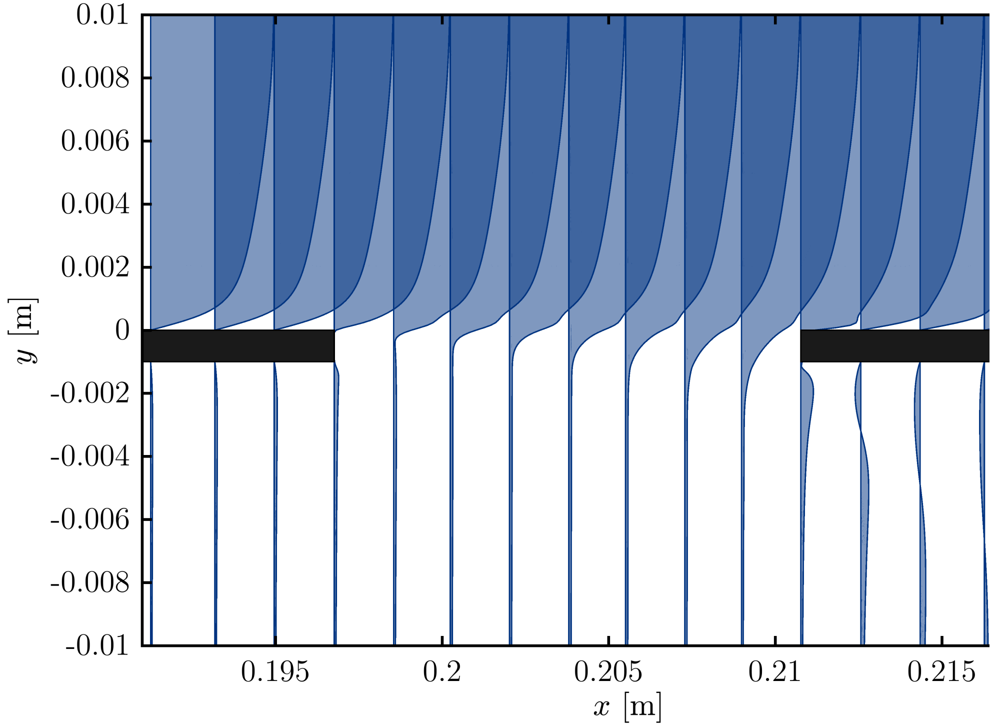
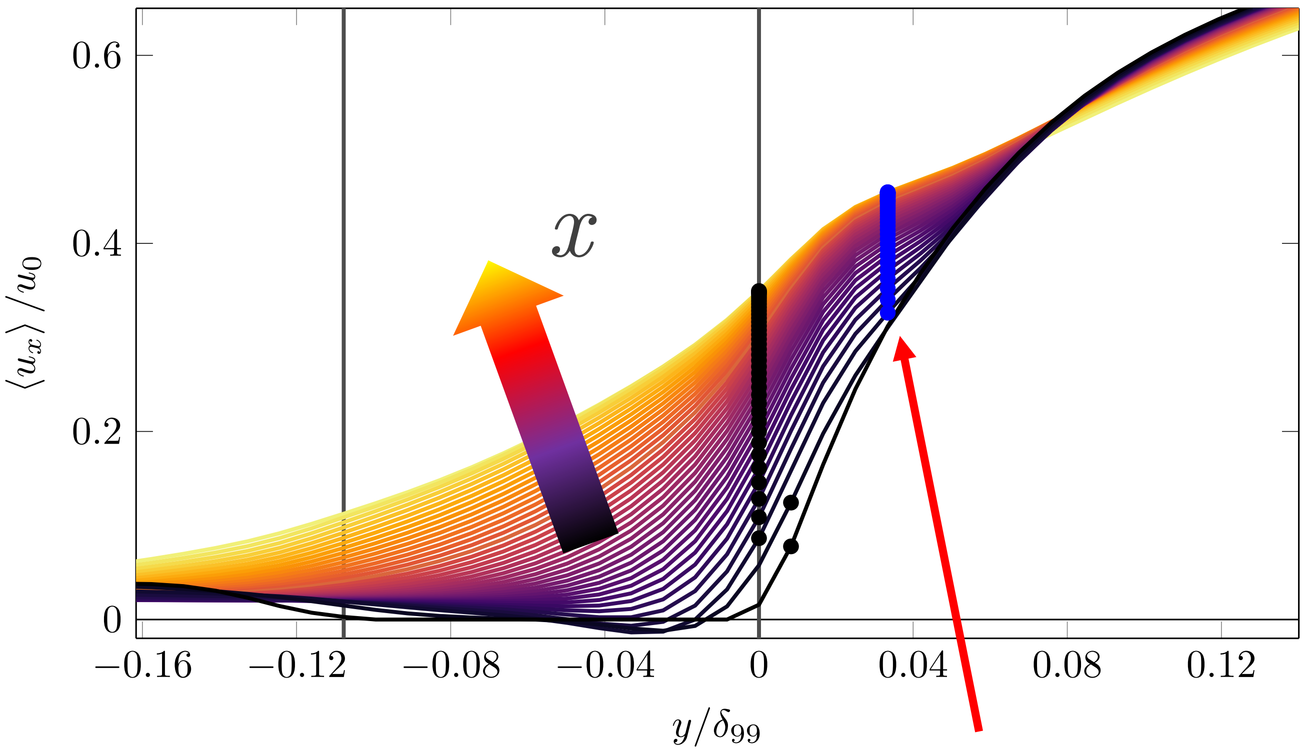
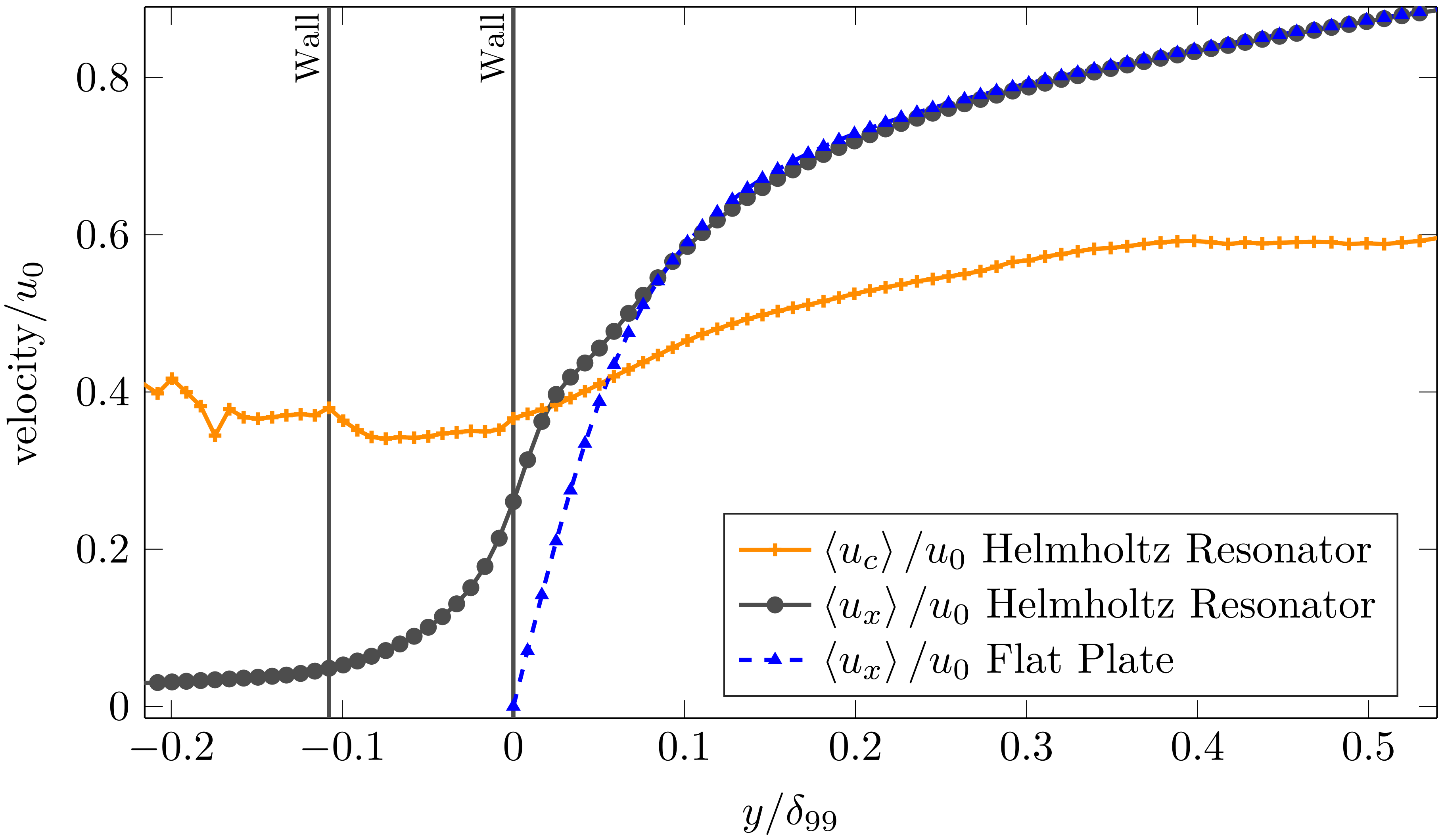


Velocity Profiles $u(y)$



New u_+ Definition



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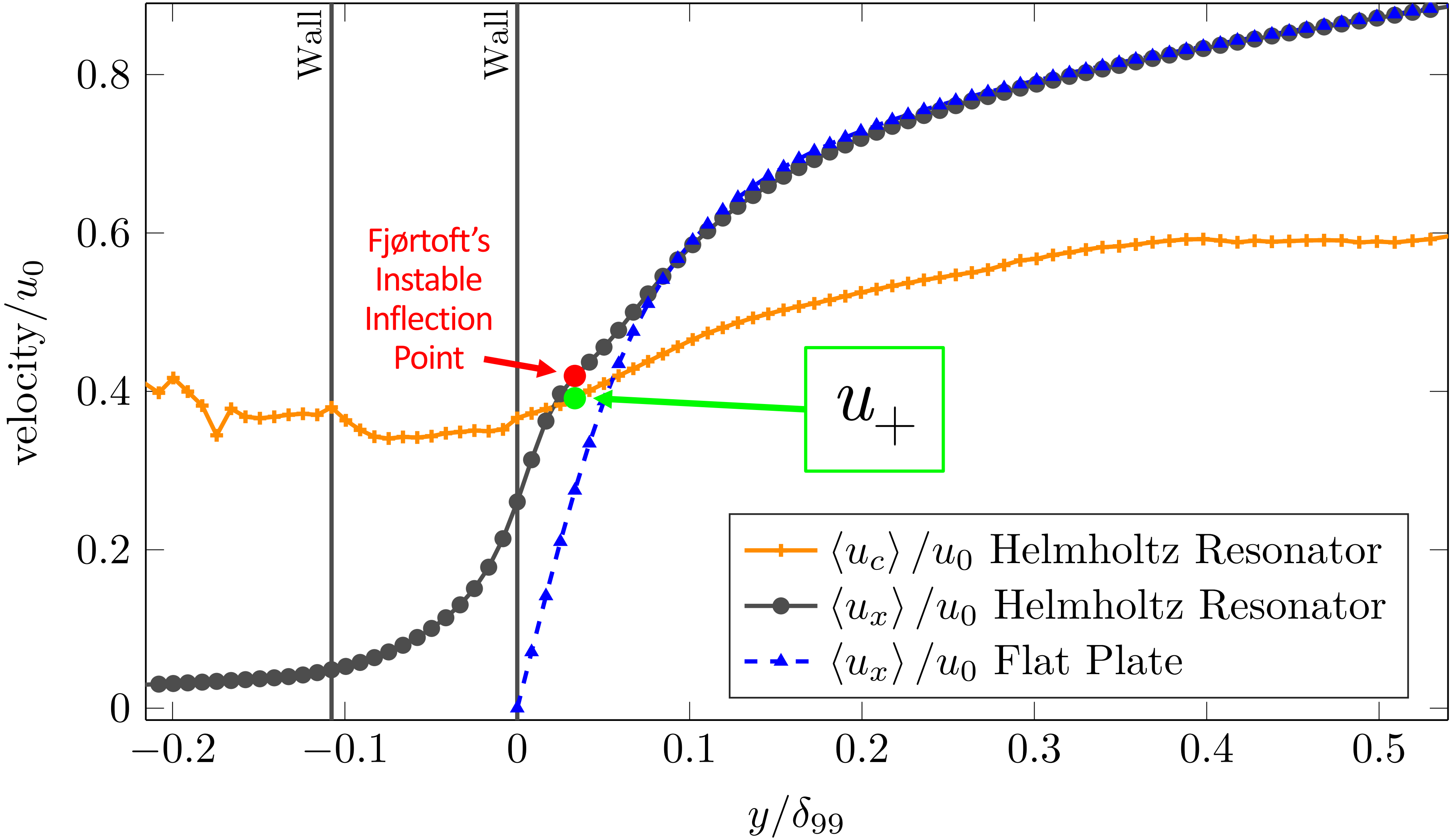
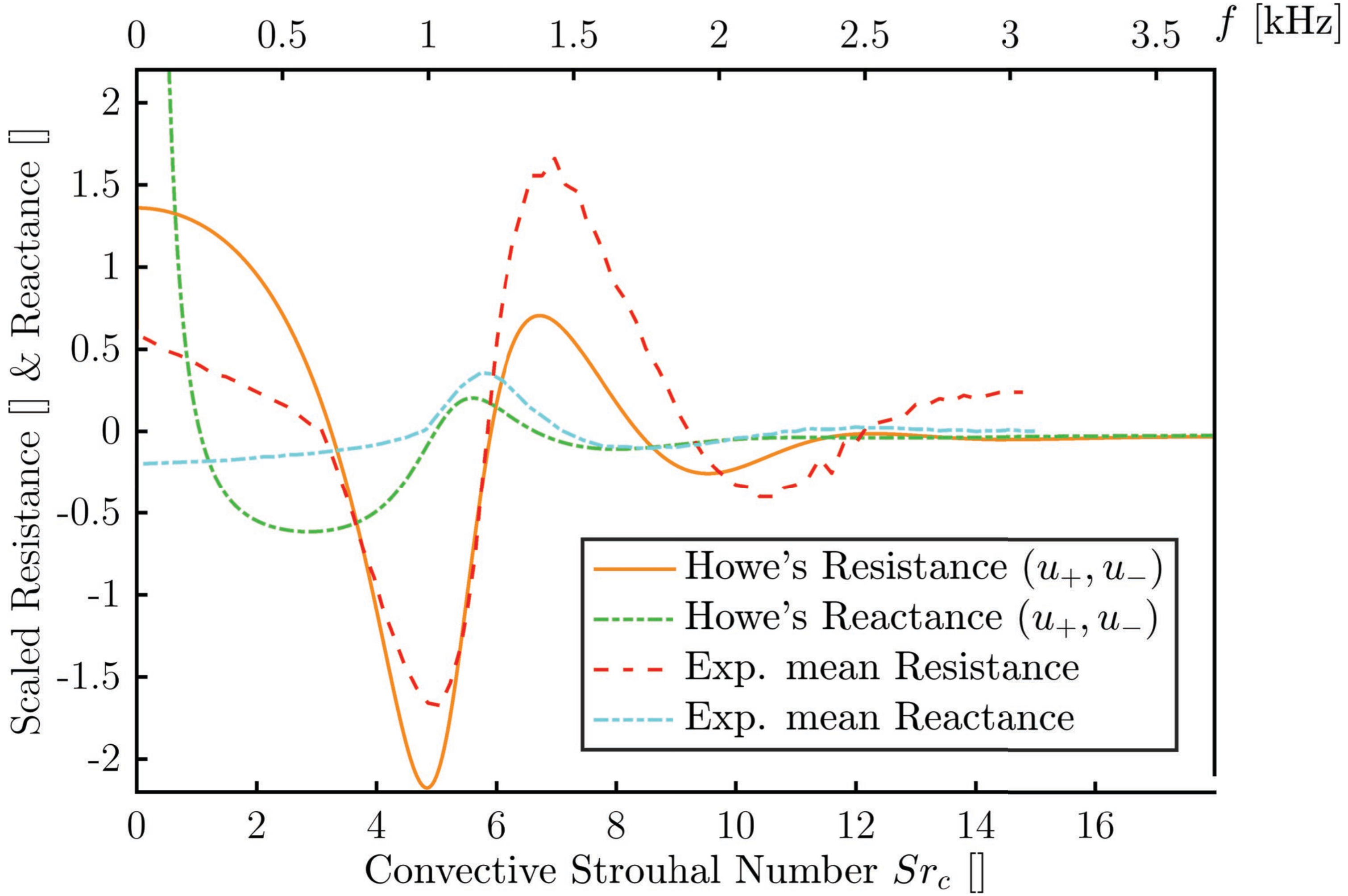


Experimental Verification of $Z_{howe}(u_+)$



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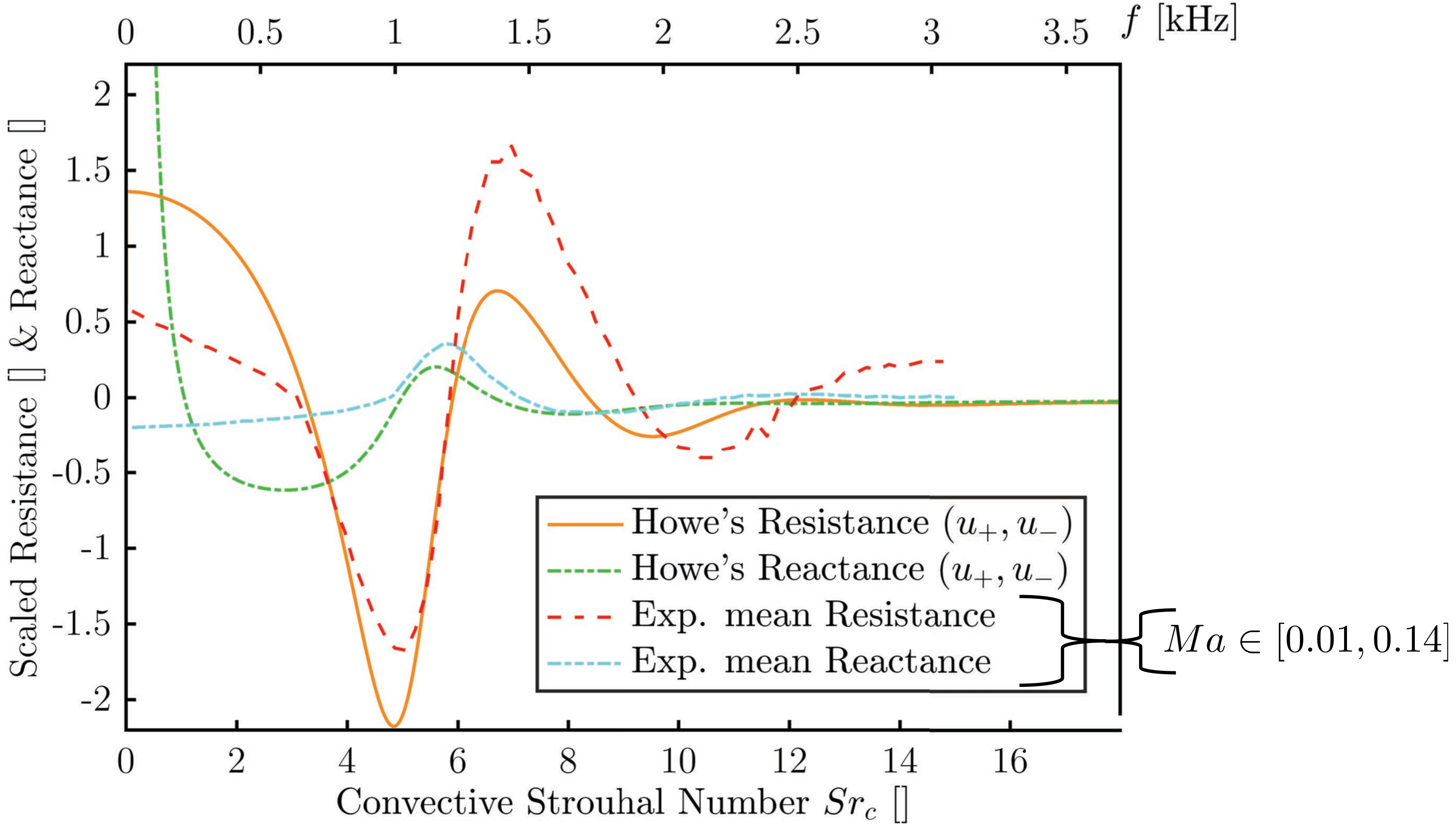

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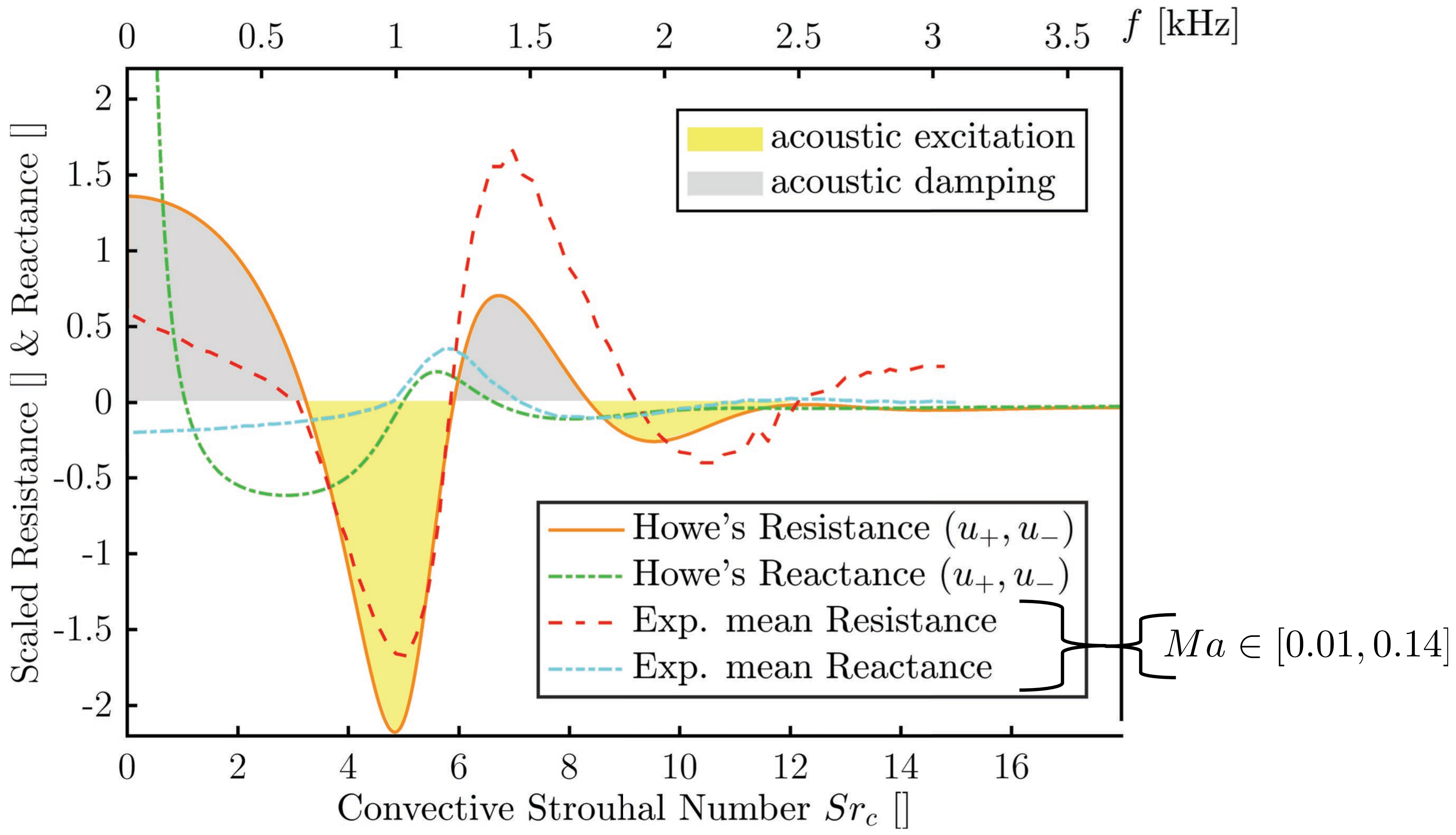
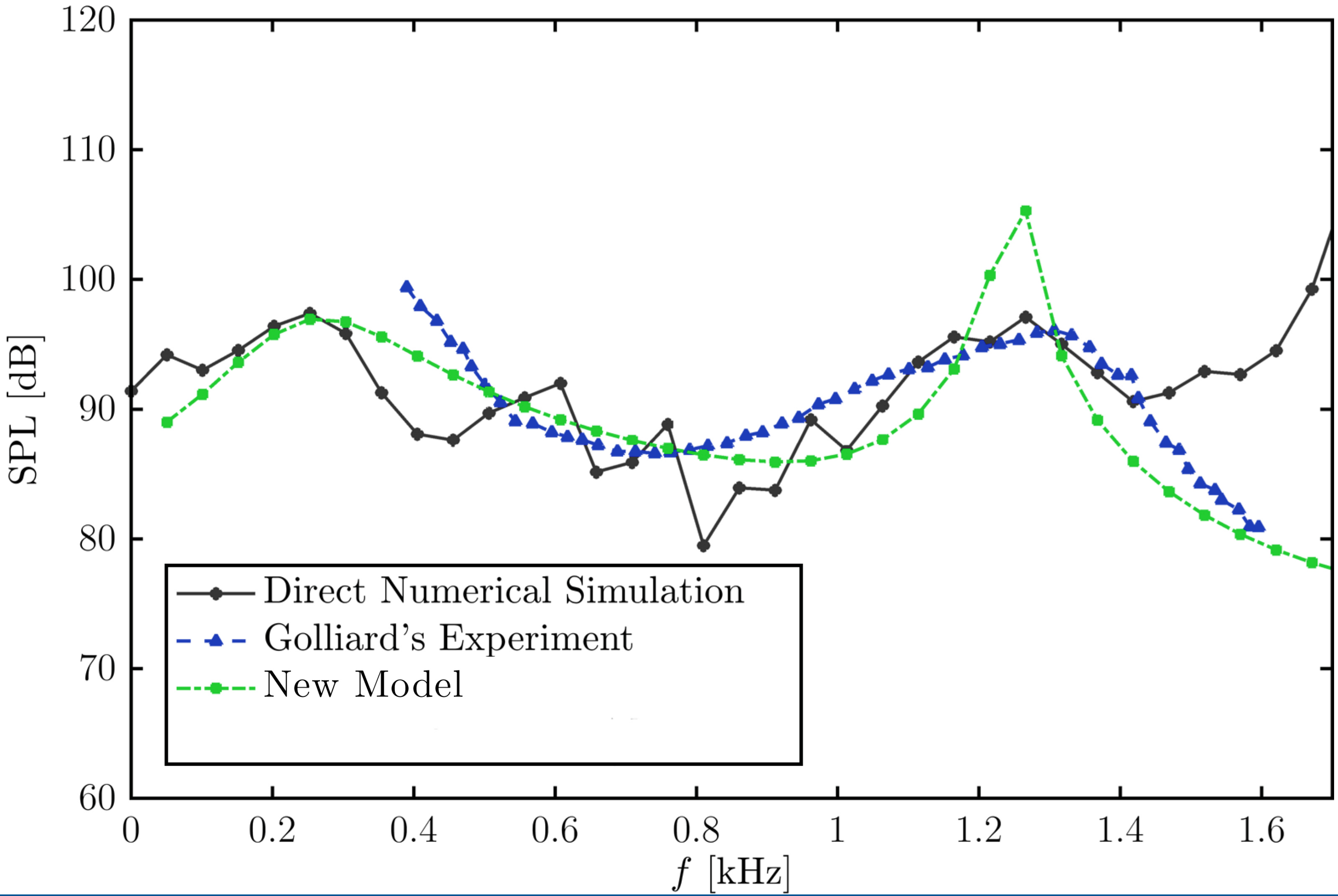


Model, DNS and Experiment



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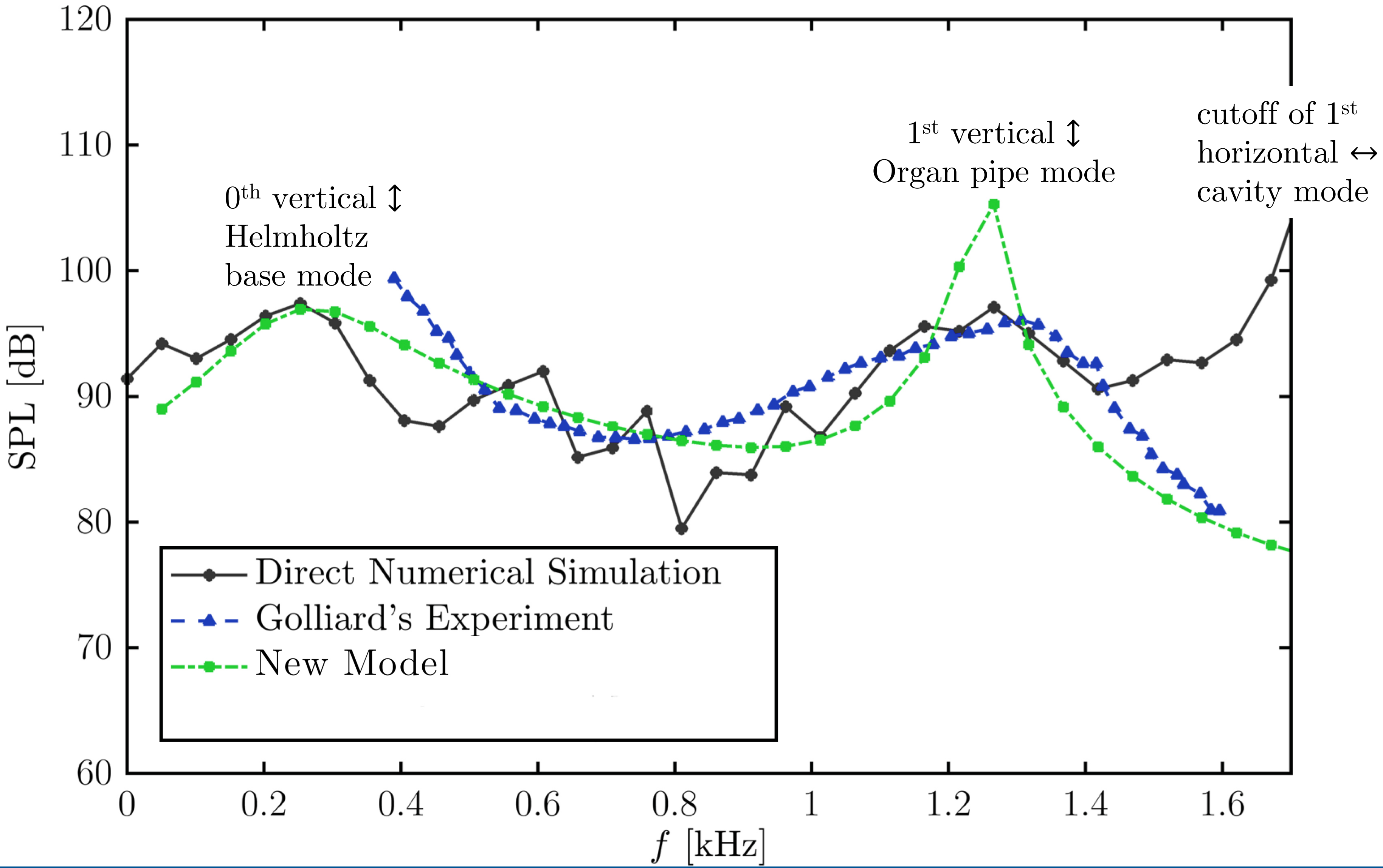

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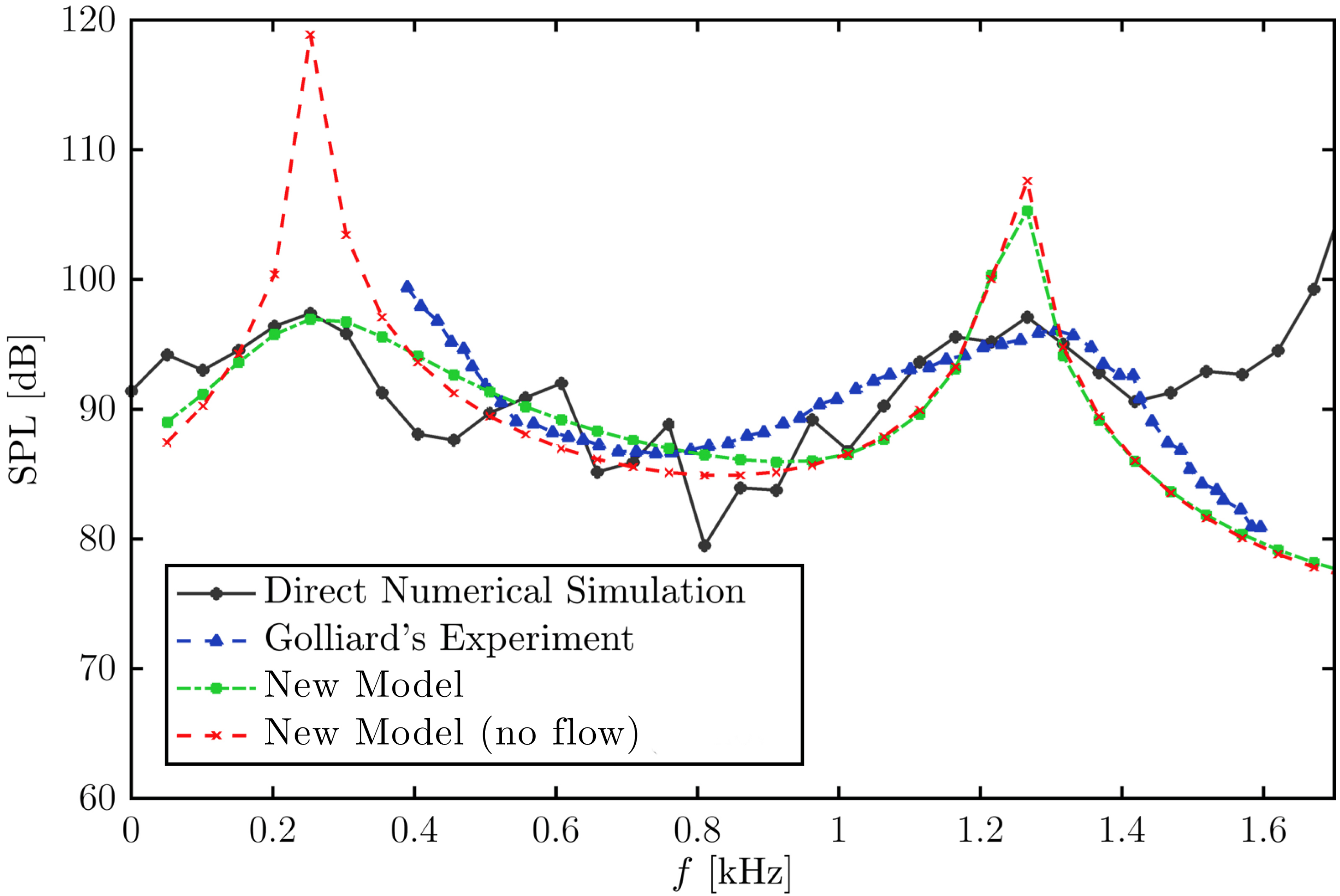
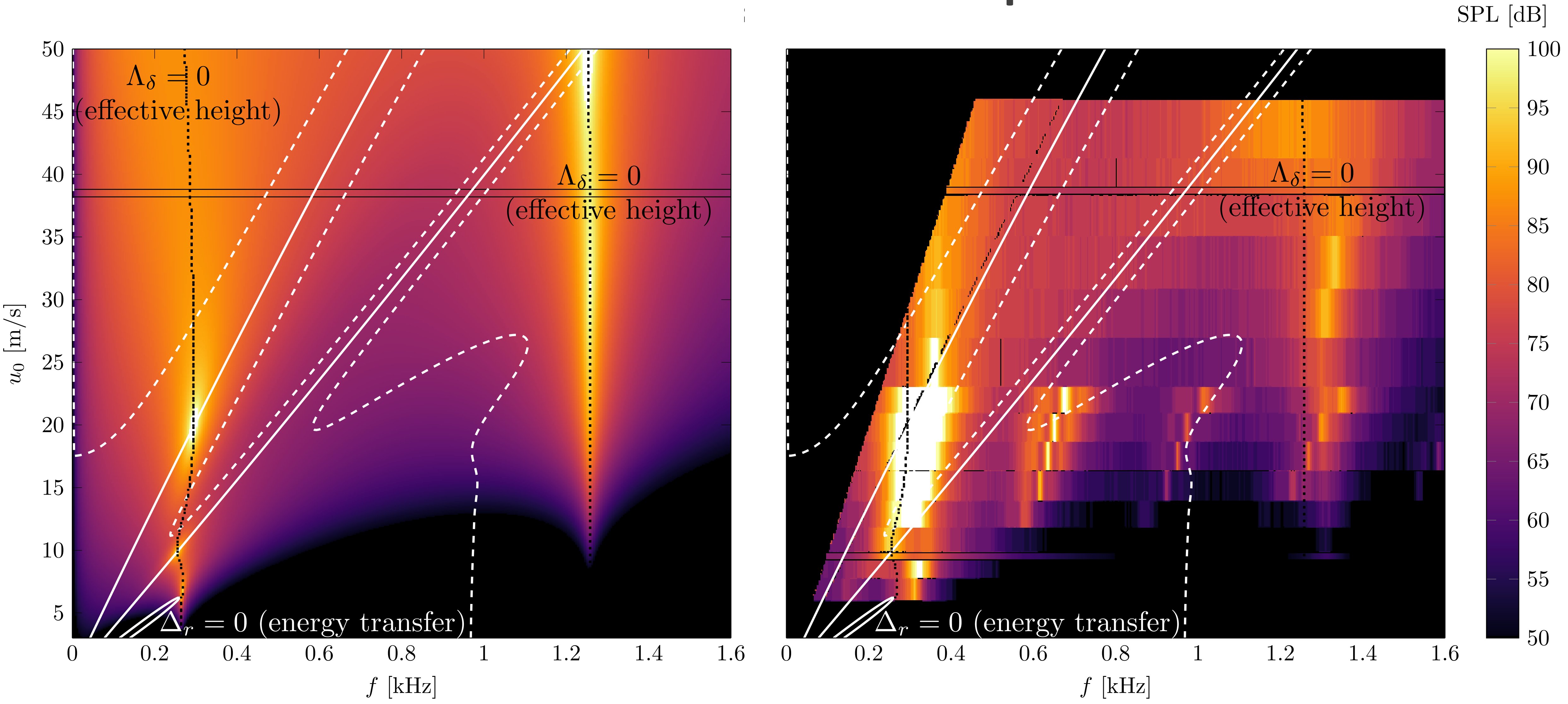
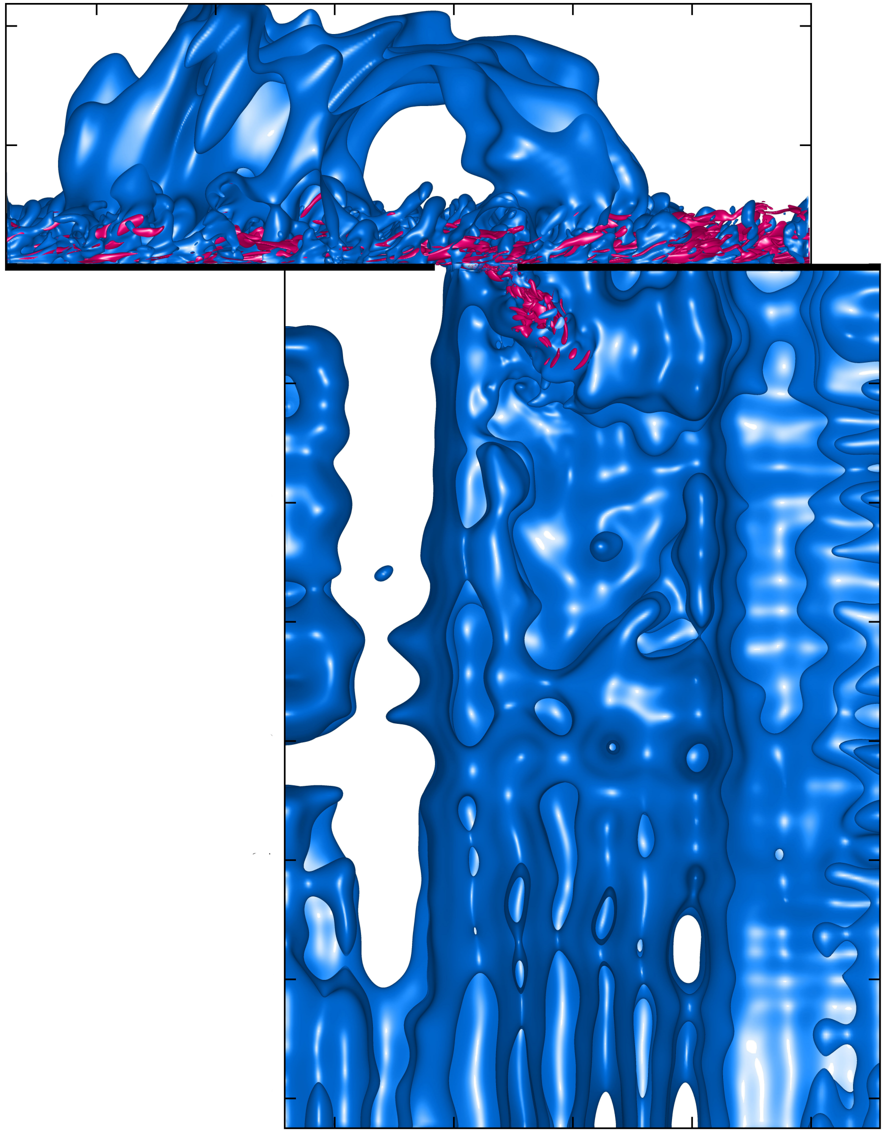


Model | Experiment





Summary

- **1st DNS** of a Helmholtz Resonator with turbulence.
- **Helmholtz Resonator Model** developed

  for typical range of operation.
- New **definition of the convection velocity** proposed,

  independent of empirical fits.

Thank You