The Relationship Between DNS and RANS Modeling

Philippe Spalart

Boeing Commercial Airplanes

with G. Coleman and A. Garbaruk

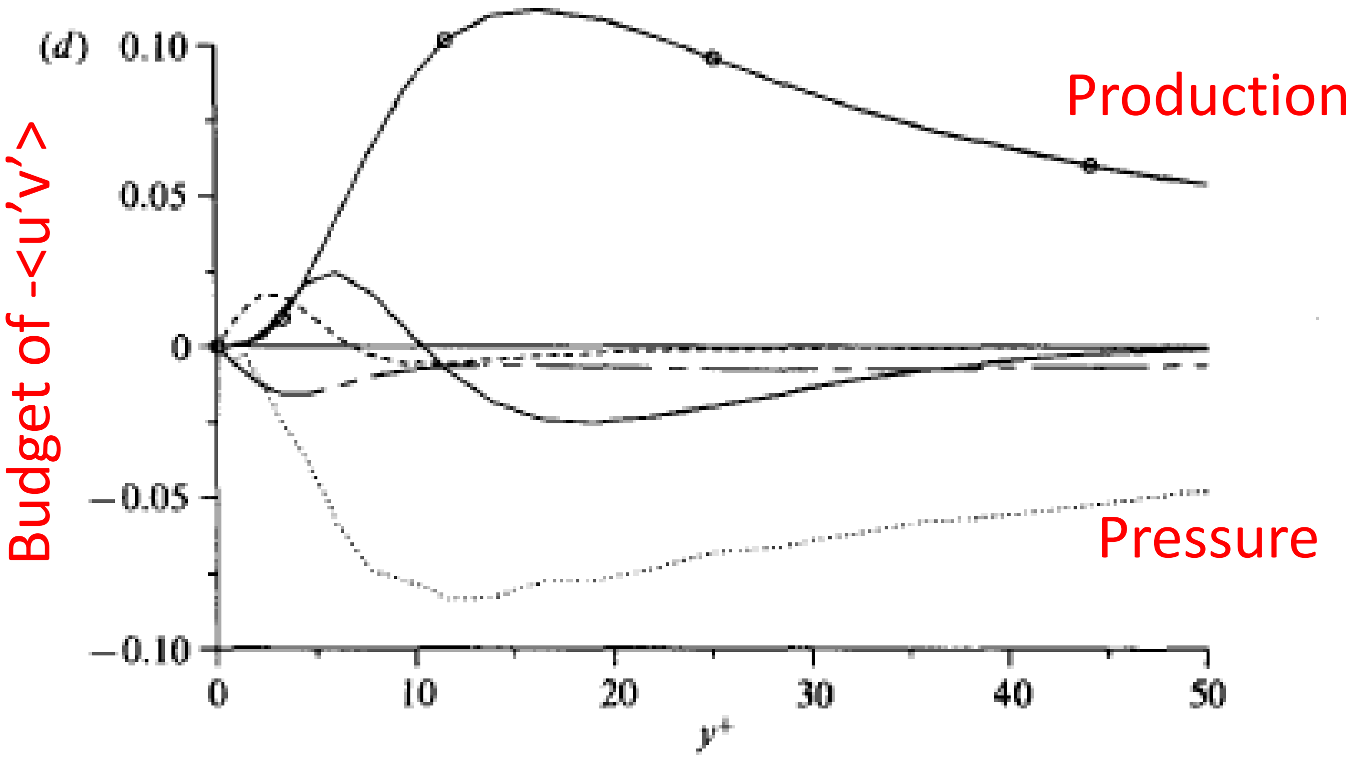
ETMM12, Montpellier, 2018

Outline

- Direct Numerical Simulation as a source of data
  - Advantage over experiments: complete information
  - Potential: new ideas, or calibration of existing constants, or validation of full model?
    - Idea in SA model
    - Validation of Reynolds-Stress model
  - Limitations: Reynolds number and geometry
- Puzzling findings in DNS
  - Log layer and Karman "constant" have been very elusive
  - Luchini's near-theoretical unification of Couette, Poiseuille and pipe flows
- Structural conflicts inherent to RANS models
  - Log-layer behavior of the Reynolds stresses
  - Insensitivity to flow Reynolds number
- Contributions to complex models
- Attempts to concretely steer simple models
  - Effective eddy viscosity
- Artificial intelligence

DNS as Source of RANS Ideas

- DNS of turbulent boundary layer provided budgets for Reynolds stresses
- <u’v’> is dominant, and pressure redistribution opposes production
- SA model mimics this with “wall term”
  - Actually, combined with diffusion term



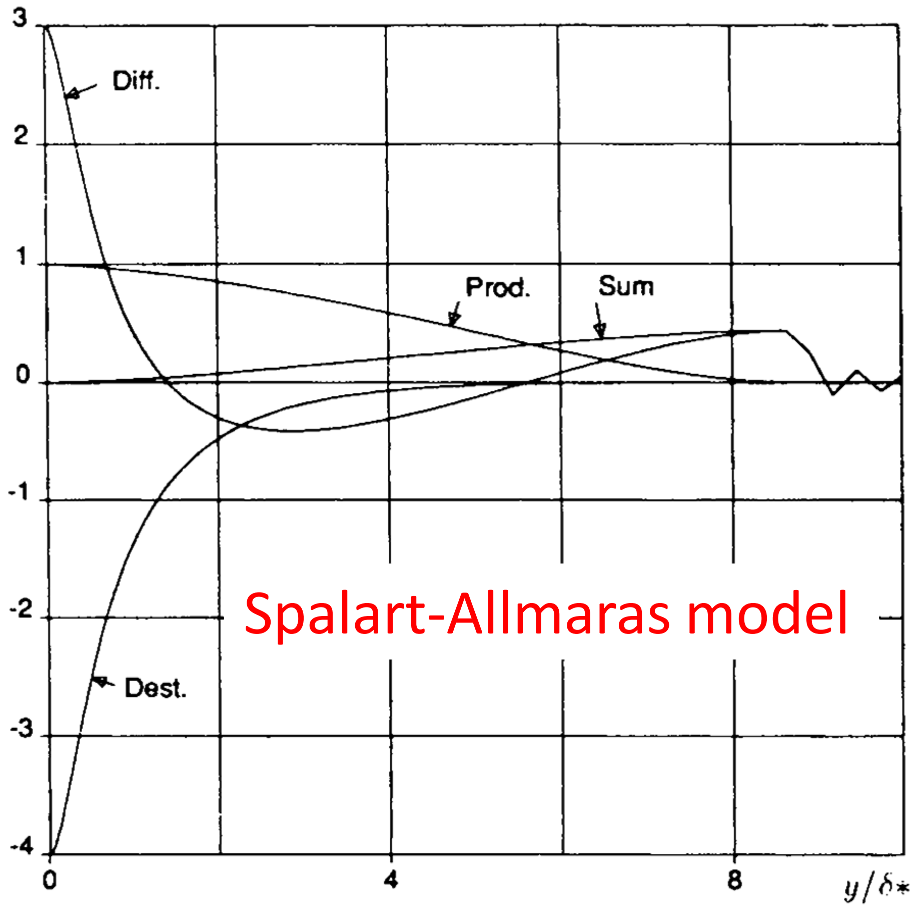
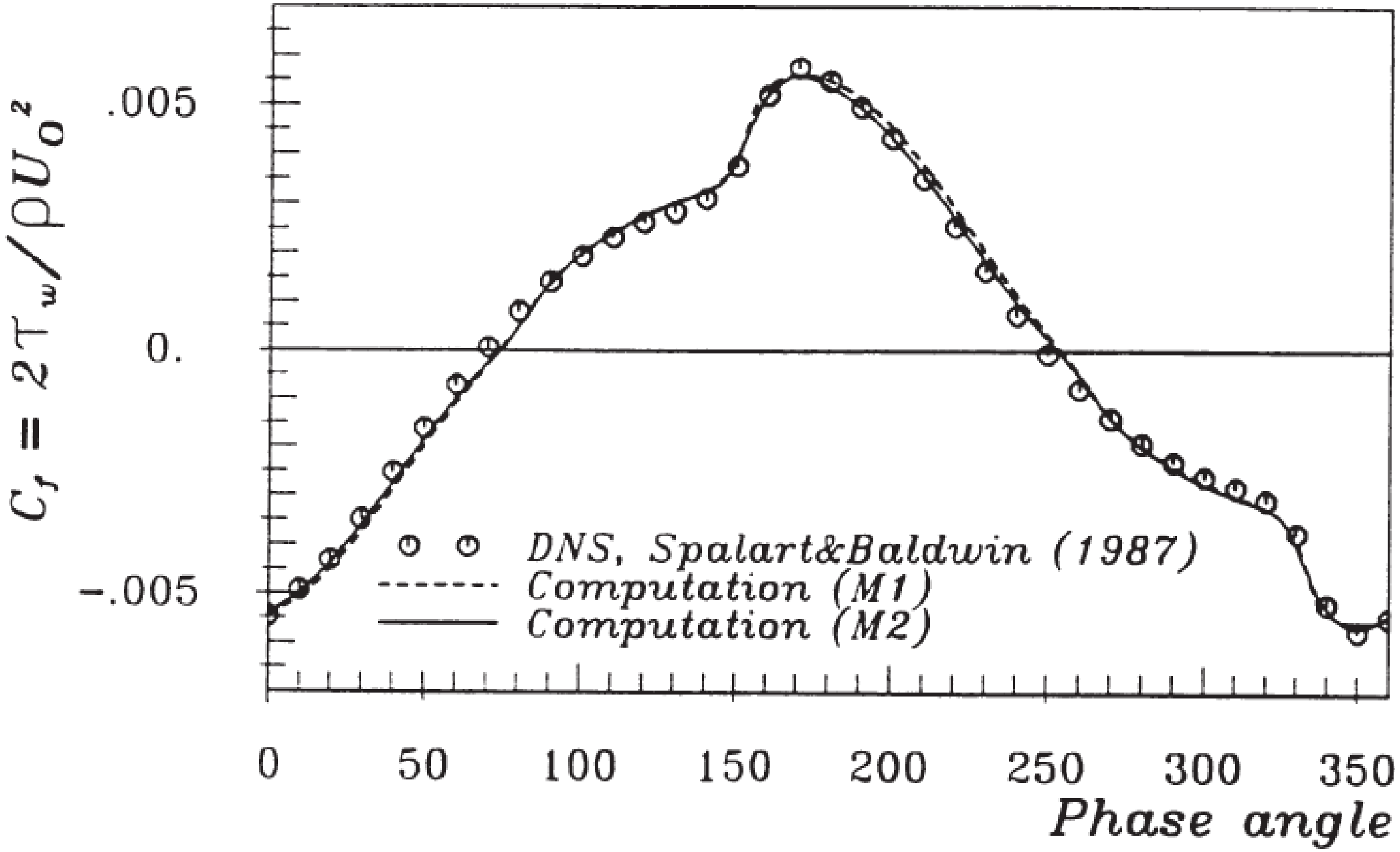


Fig. 6. – Eddy-viscosity budget in a flat-plate boundary layer. Normalized with $c_{b1}\tau_{wall}$.

DNS as Reference for Validation

- Hanjalić, Jakirlić and Hadzić 1993
  - Oscillating boundary layer: $U_e = U_0 \cos(\omega t)$
  - Excellent comparison with DNS, even for flow with laminarescent phase

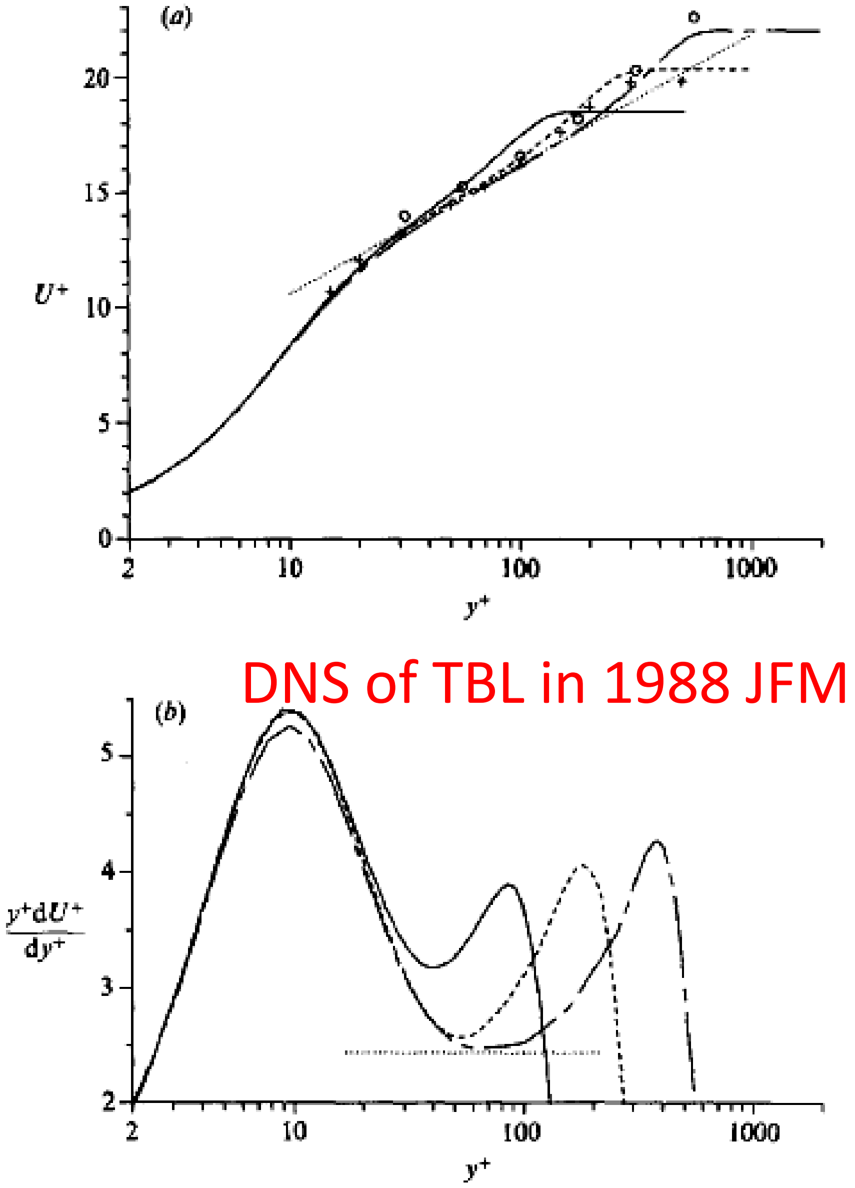


Outline

- Direct Numerical Simulation as a source of data
  - Advantage over experiments: complete information
  - Potential: new ideas, or calibration of existing constants, or validation of full model?
    - Idea in SA model
    - Validation of Reynolds-Stress model
  - Limitations: Reynolds number and geometry
- **Puzzling findings in DNS**
  - **Log layer and Karman “constant” have been very elusive**
  - **Luchini’s near-theoretical unification of Couette, Poiseuille and pipe flows**
- Structural conflicts inherent to RANS models
  - Log-layer behavior of the Reynolds stresses
  - Insensitivity to flow Reynolds number
- Contributions to complex models
- Attempts to concretely steer simple models
  - Effective eddy viscosity
- Artificial intelligence

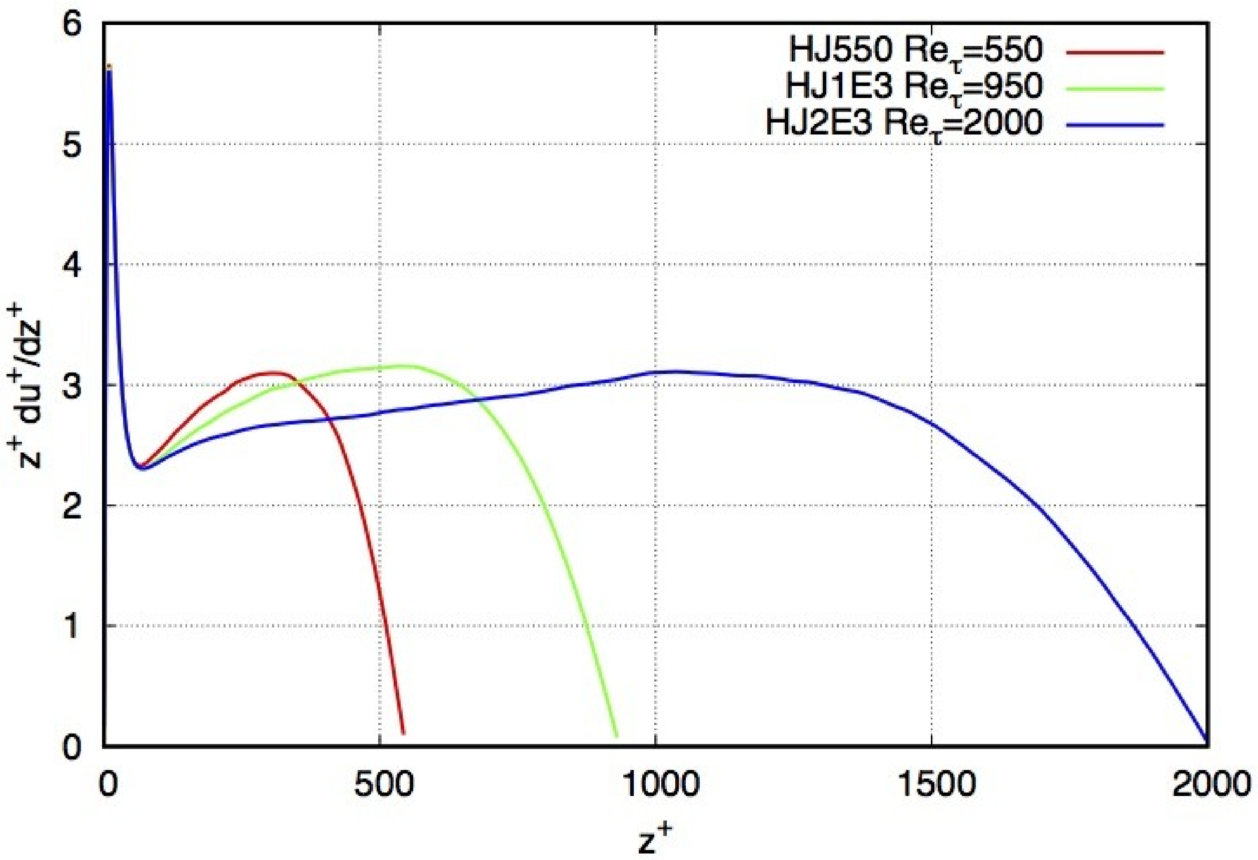
Log Law and Karman Constant

- Early channel and boundary-layer (TBL) DNS had the excuse of "low-Reynolds-number effects"
  - In particular, confirming the log layer and precise value of κ was premature
- Channel Re_τ has risen from 180 to over 5000… and κ is still not found!
  - This is with the "honest" approach of plotting $dU^+/d(\log y^+)$
- Experiments also have conflict between pipe flow (κ ~ 0.42) and TBL (κ ~ 0.385)
- Some people suggest κ is flow-dependent!
  - This would mean the theory fails



Effect of Reynolds-Number

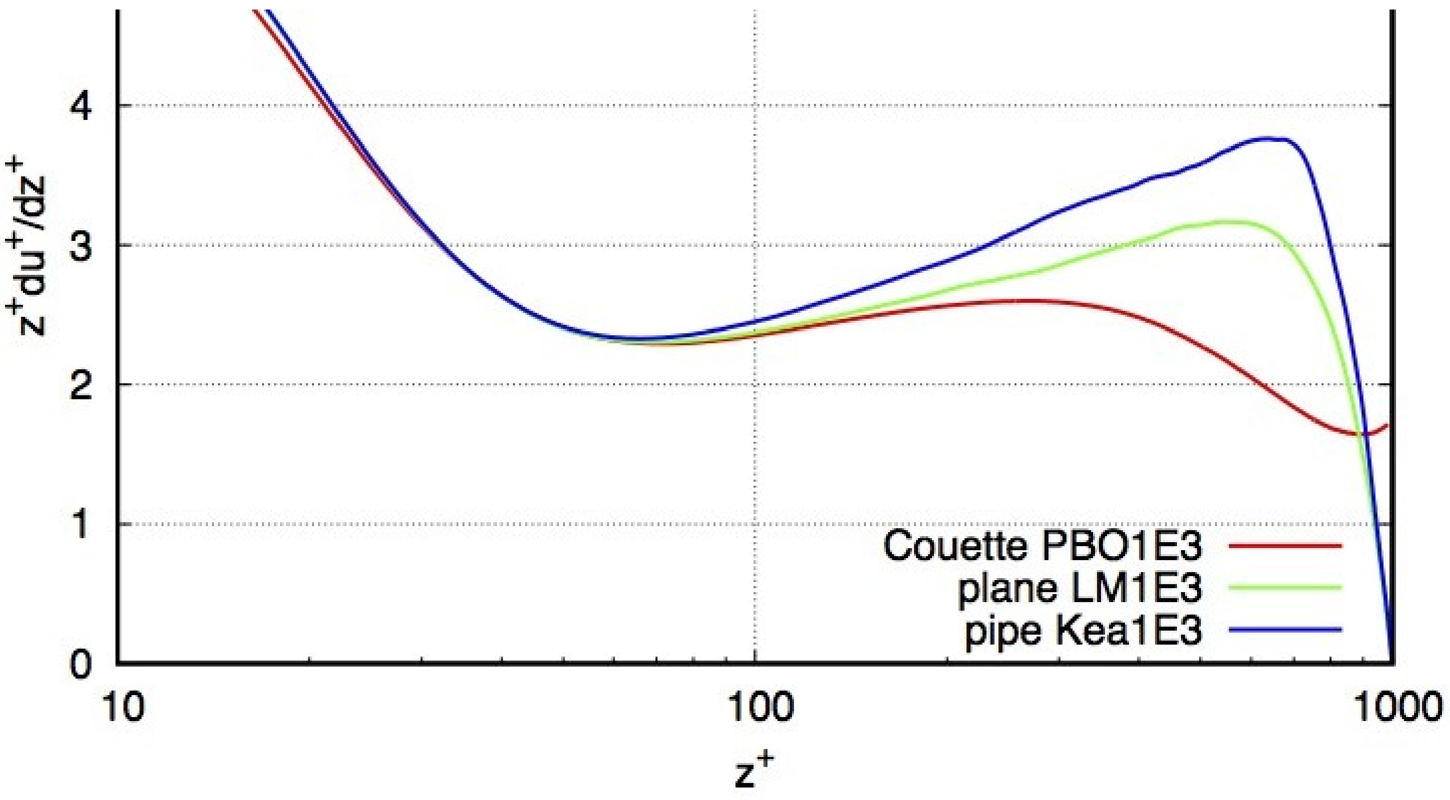
- Channel results of Hoyas and Jimenez up to 2000, rendered by Luchini
- Showing $dU^+ / d(\log y^+)$, which should be $1 / \kappa$



- There is no plateau, and even the local maxima are much too high

Effect of Flow Type: Pipe, Poiseuille, Couette

- The three flows are “justified” to disagree in the core region
- At z+ = 100 (out of Re_τ = 2000), the disagreement is already palpable
- None of the flows have a plateau anyway



Luchini's Theory

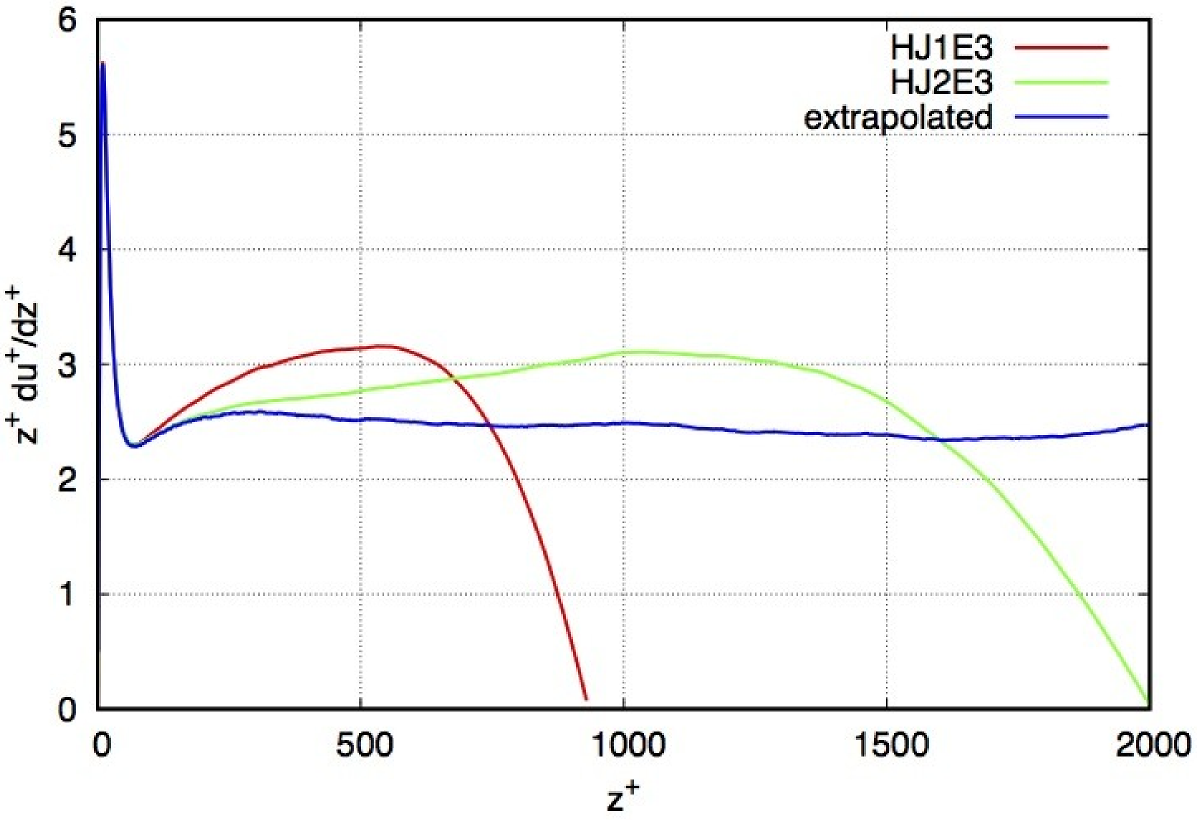
- Luchini in 2017 and 2018 papers proposes a unified correction of velocity profile for pipe, Poiseuille (channel) and Couette flow
  - He extrapolates from two Re values to ∞ in mathematical fashion
  - He adds a linear function of y^+ to U^+:

$$U^+ = U_0^+(y^+) + A_1\,(dp/dx)^+\,y^+$$

  - In channel, $(dp/dx)^+ = 1 / Re_\tau$
- It's empirical, but considerably improves consistency between the three flows and across Reynolds numbers, using only ONE constant, A_1
- My issue:
  - I normally exclude the pressure gradient from models and theory
  - Pressure does not influence vorticity
  - Ongoing discussions with Luchini
  - In steady flows, $\partial p / \partial x_i = \partial \tau_{ij} / \partial x_j$, the "turbulence force"
    - I "prefer" a term based on stresses
    - We have unpublished evidence that this correction works better in boundary layers

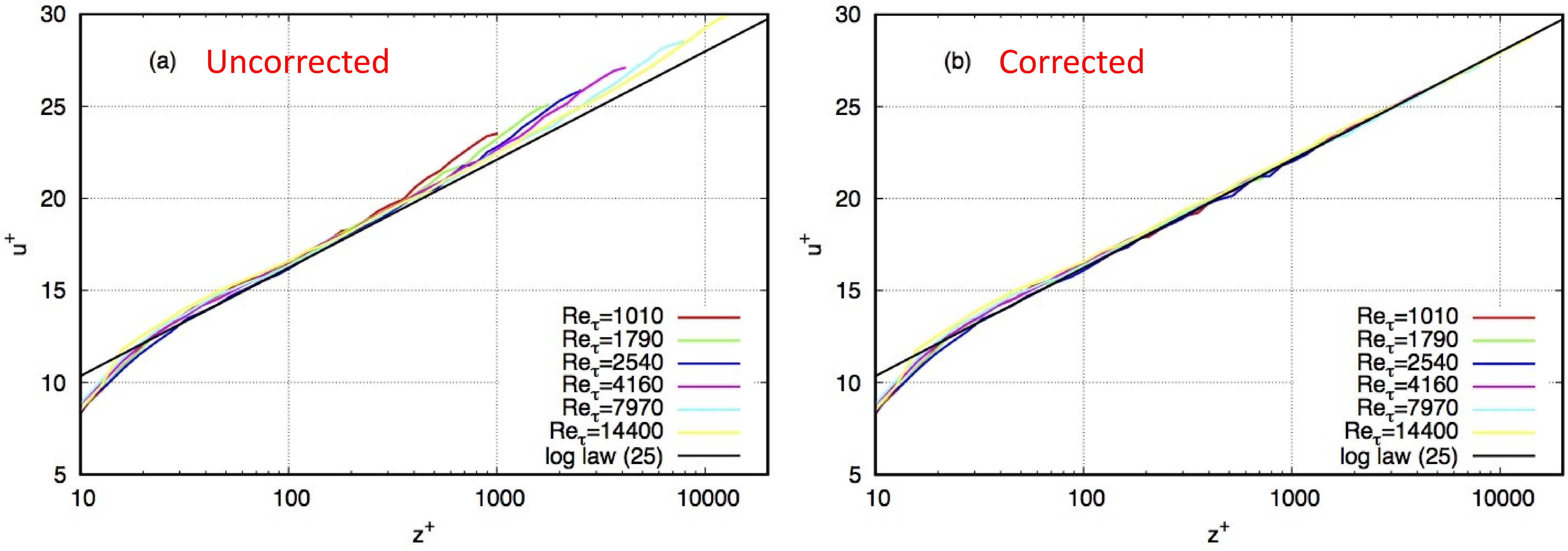
Extrapolation in Channel from Re_τ = 1000/2000 to ∞

- Removal of "wake component" is rigorous
- The curve is considerably closer to a plateau
- It's still not flat enough to really determine κ, say better than 10%



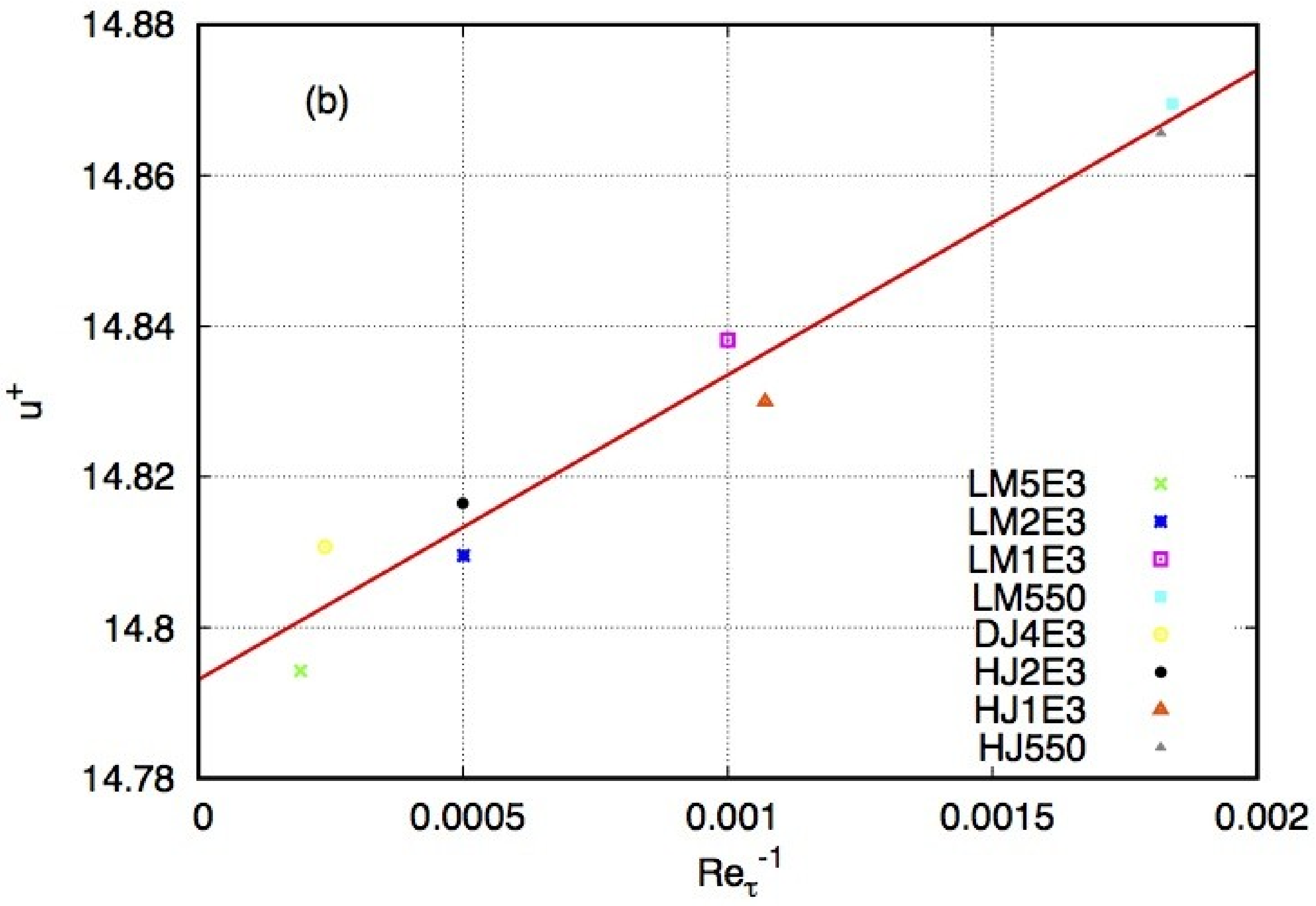
Pipe Flow

- Superpipe velocity profiles (McKeon, Hultmark, Smits) with Luchini correction



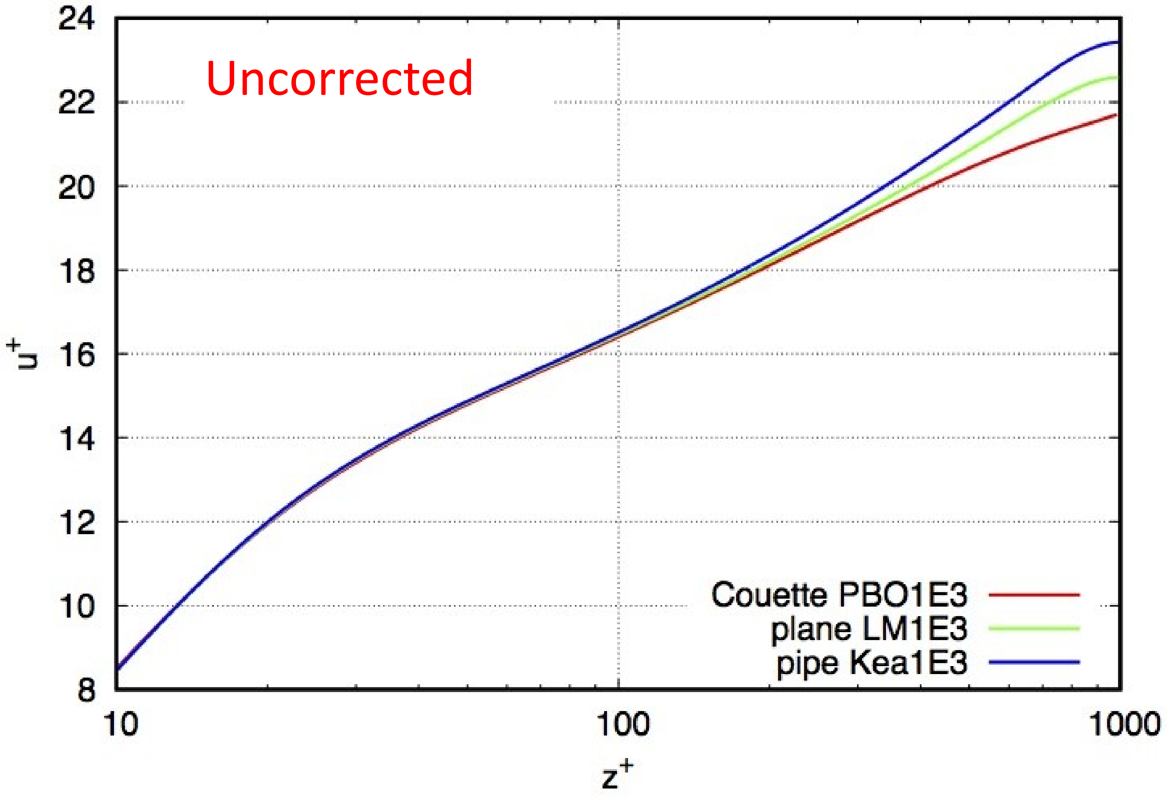
Luchini's Theory

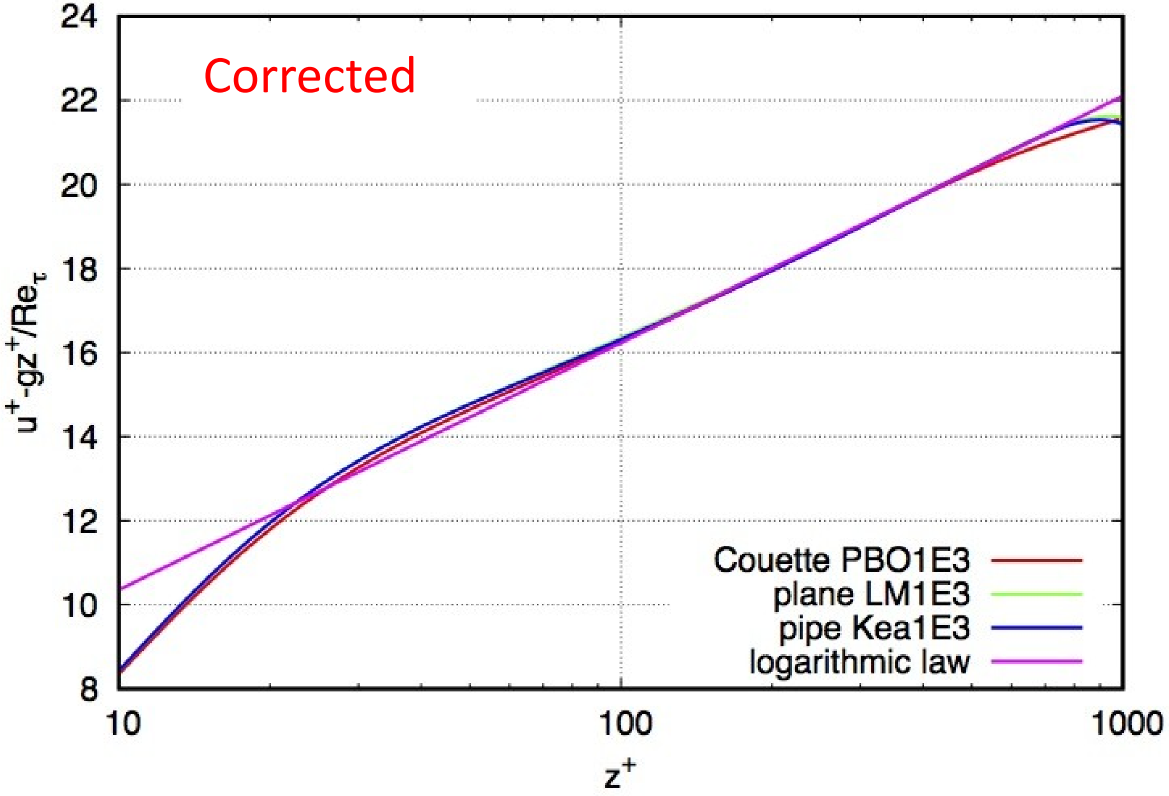
- DNS evidence for a linear dependence on $(dp/dx)^+ = 1 / Re_\tau$
- This is a conjecture!
- Shows U^+ at $y^+ = 50$



Luchini Correction, $Re_\tau = 1000$

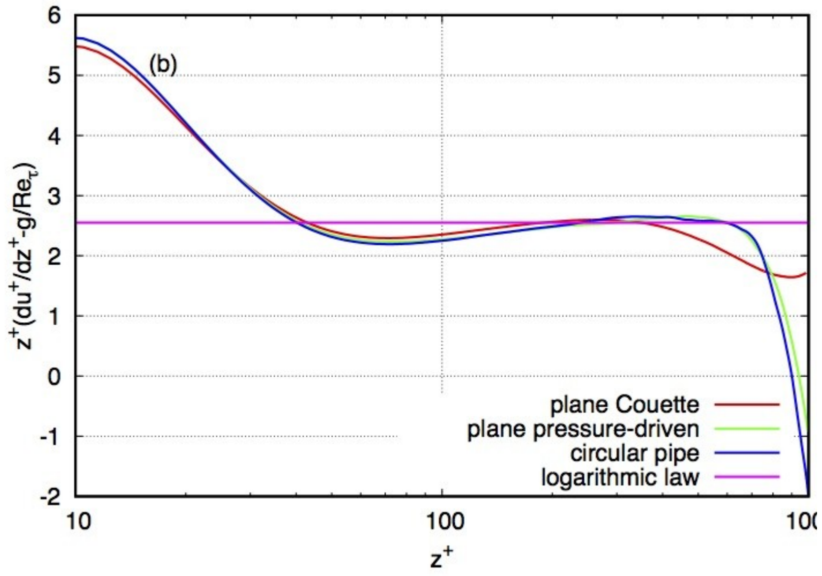
- The three flows are essentially unified in U^+ terms

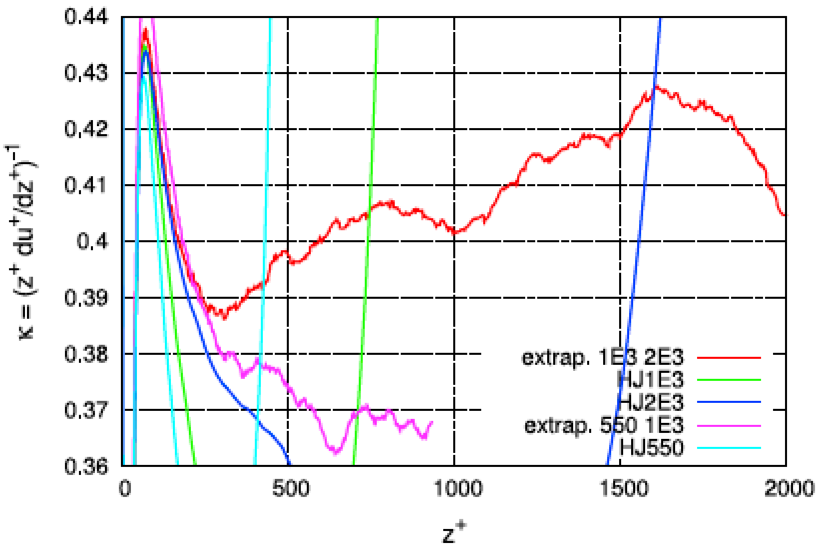


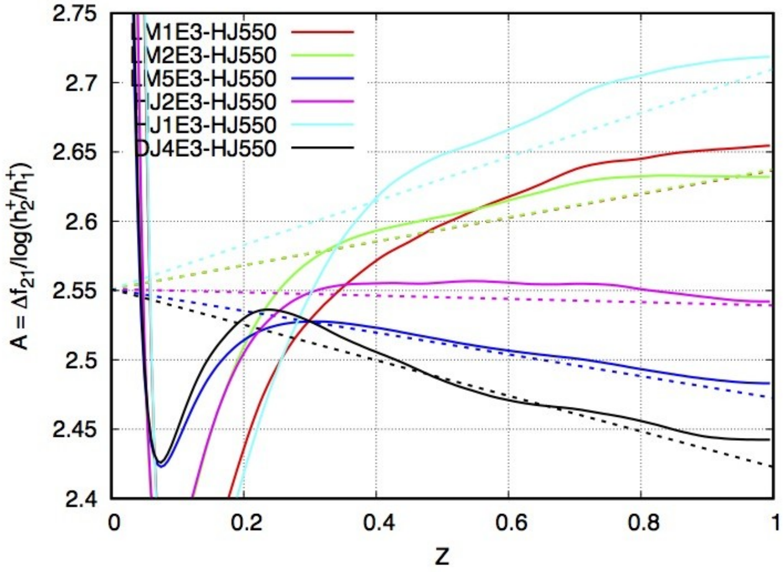


Luchini Correction

- The three flows are essentially unified in $dU^+ / d(\log y^+)$ terms
- The precise value of the Karman constant is still not obvious after extrapolation to ∞
- Luchini estimates that 0.392 is best





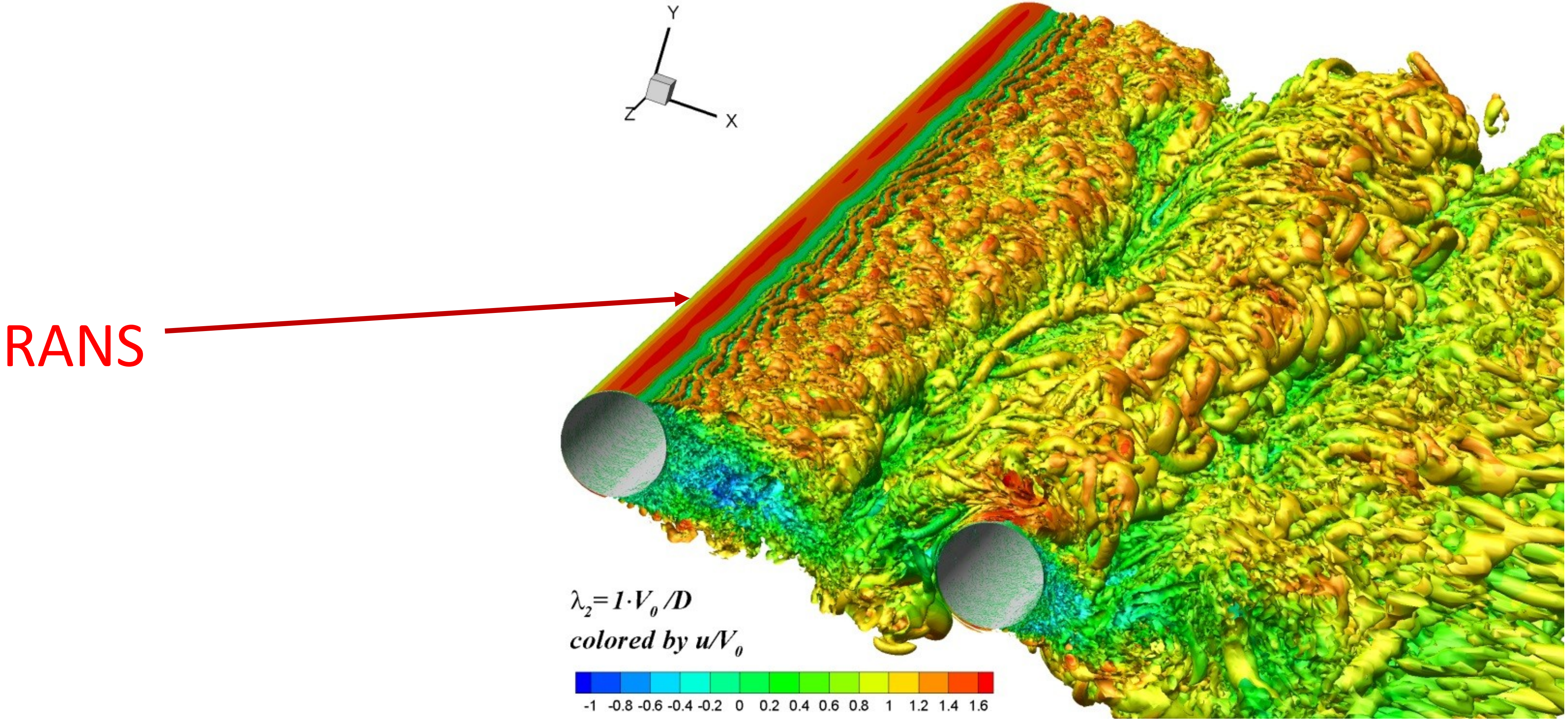


Outline

- Direct Numerical Simulation as a source of data
  - Advantage over experiments: complete information
  - Potential: new ideas, or calibration of existing constants, or validation of full model?
    - Idea in SA model
    - Validation of Reynolds-Stress model
  - Limitations: Reynolds number and geometry
- Puzzling findings in DNS
  - Log layer and Karman “constant” have been very elusive
  - Luchini’s near-theoretical unification of Couette, Poiseuille and pipe flows
- **Structural conflicts inherent to RANS models**
  - **Log-layer behavior of the Reynolds stresses**
  - **Insensitivity to flow Reynolds number**
- Contributions to complex models
- Attempts to concretely steer simple models
  - Effective eddy viscosity
- Artificial intelligence

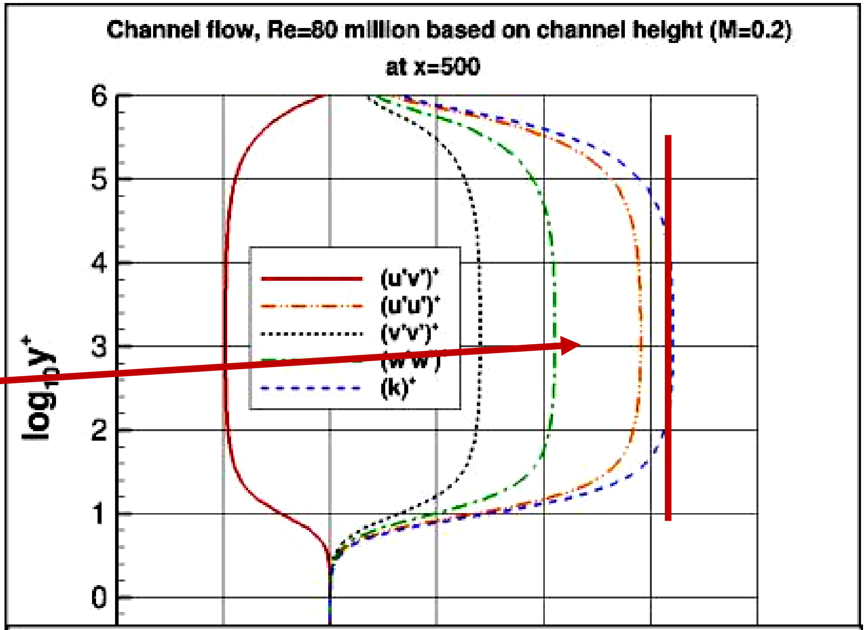
Structural Conflicts Inherent to RANS Models

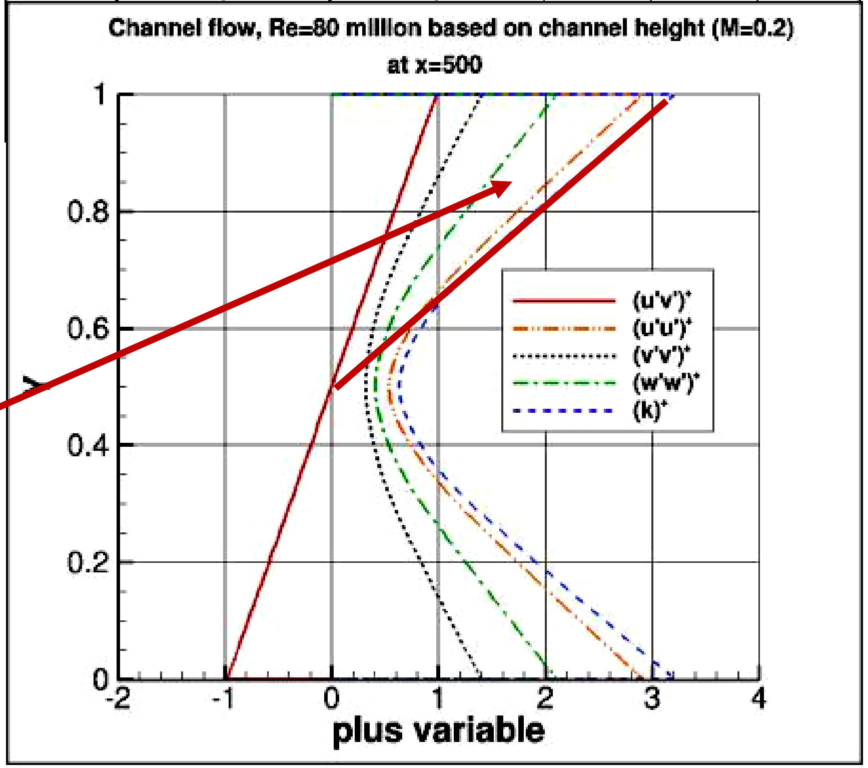
- In one school of thought (DES), pure RANS is used only in boundary layers
  - The flow after massive separation is treated by LES
- For this reason, we focus on channel and boundary-layer cases



Reynolds Stresses in Channel Flow at High Re

- Work of Rumsey, posted on Turbulence Modeling Resource
- RSM at very high Re_τ (two-equation models do the same, $k^+ = 1 / \sqrt{C_\mu}$)
- In region with $\tau^+ = 1$, all Reynolds stresses are constant, which "theory" would have predicted
  - $d\,\tau_{ij} / d\,y = 0$
- Model is purely driven by $\partial U / \partial y$, which obeys the Law of the Wall. u_τ controls all stresses
- This conflicts with DNS and experiment
- Plateaus on the stresses in high-Re pipe experiments are still controversial

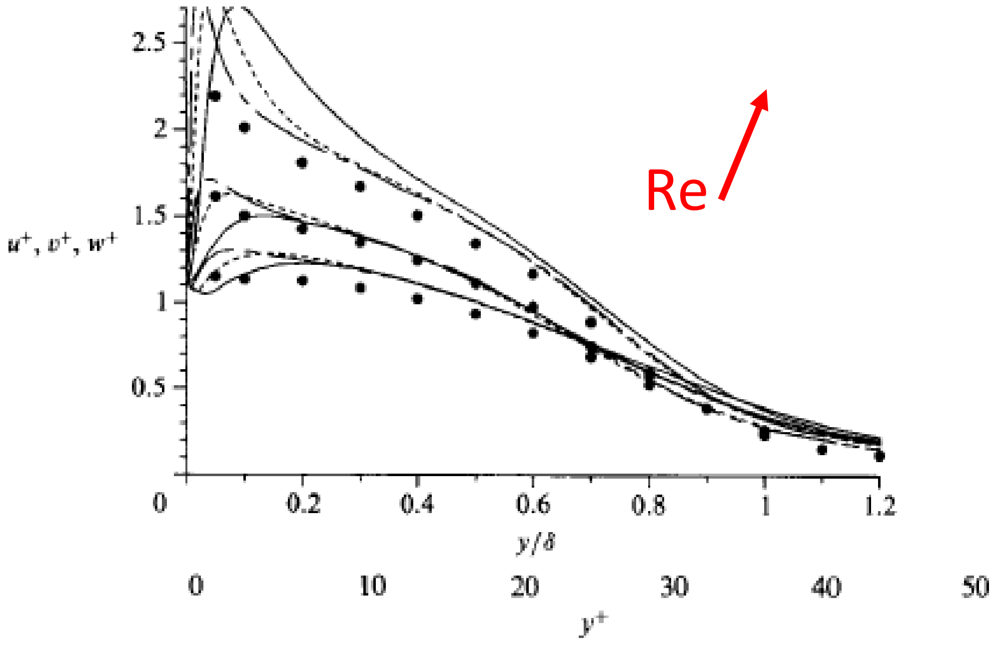


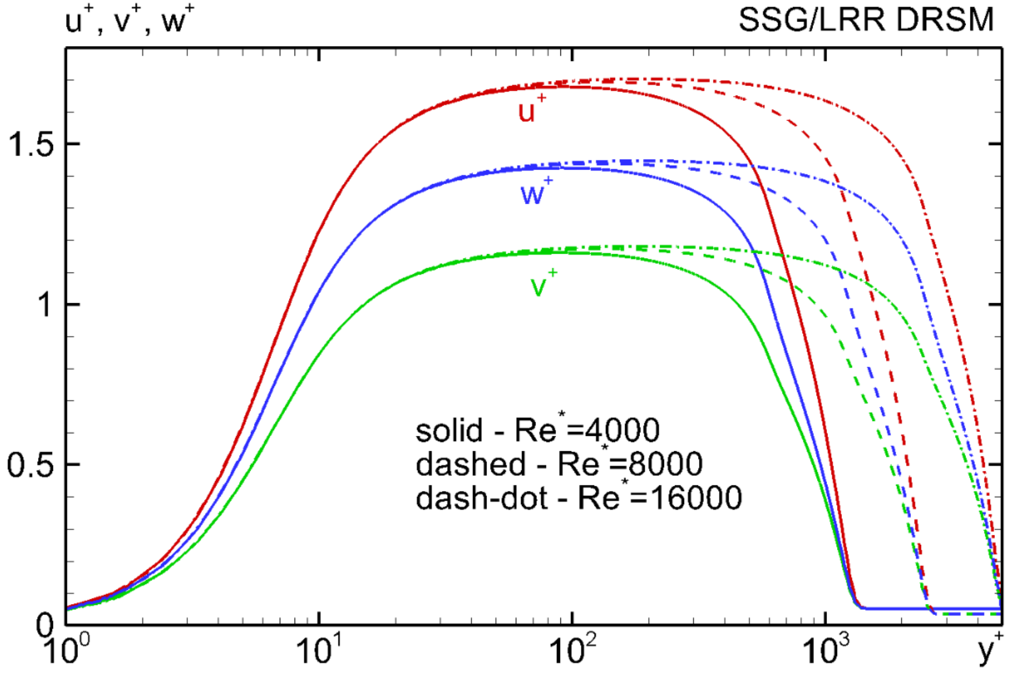


- Except in center region, anisotropy of tensor is constant: all stresses are proportional to (Y – ½), like the shear stress
  - $d\,a_{ij} / d\,y = 0$
  - This may allow an analytical solution, but is not Real Life
- Model is here driven by $\partial U / \partial y$, which obeys the Law of the Wake. u_τ, combined with y, again controls stresses!

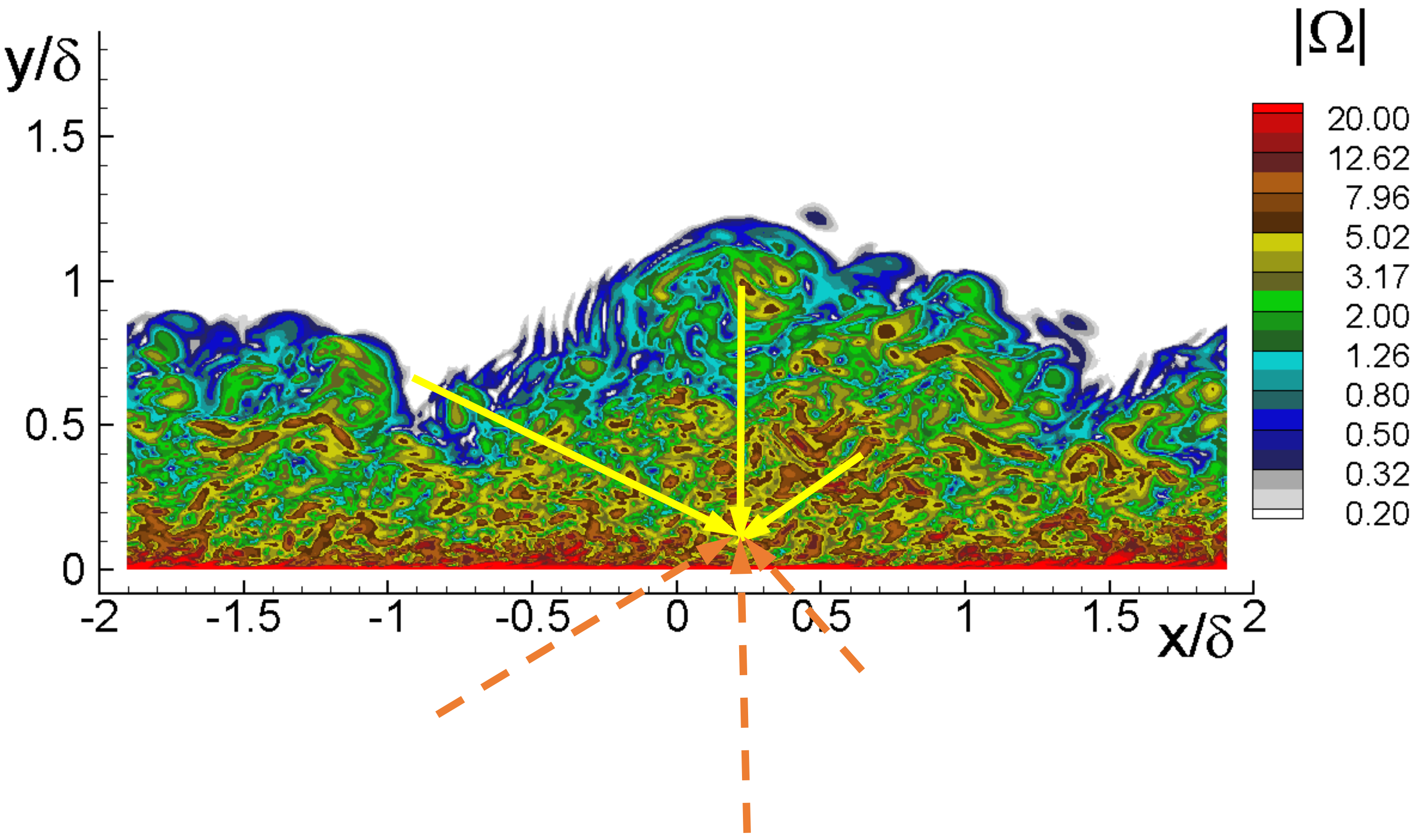
Reynolds Stresses Near Wall, Re Effect

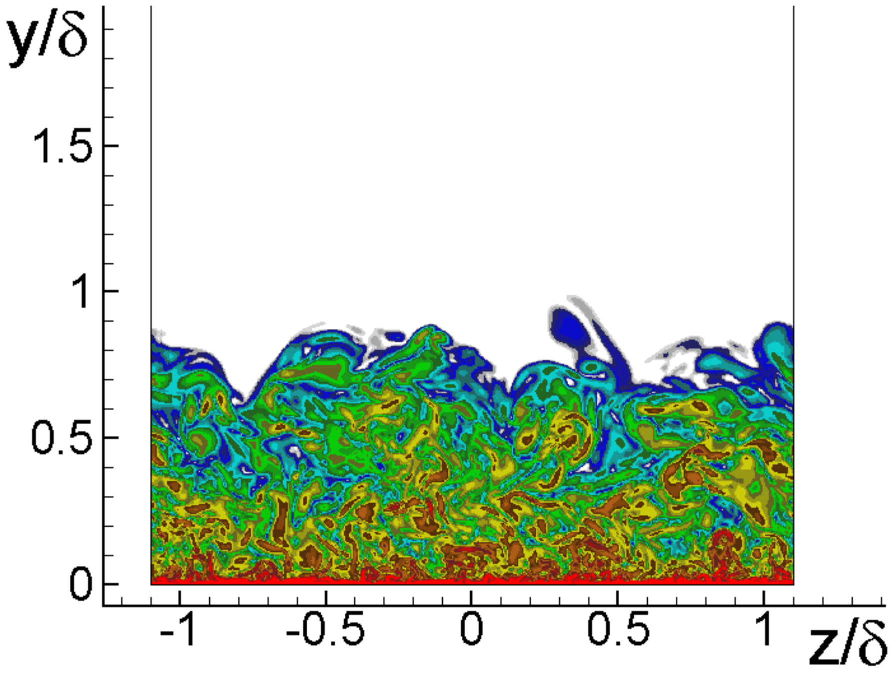
- Old TBL DNS data: 1988!
- Again, the model is driven by $\partial U^+ / \partial y^+$, which very closely obeys the Law of the Wall
- DNS shows a Reynolds-number effect all the way to the wall. The slope of w'^+ is especially sensitive
- Wall values such as ε^+ or $p_{rms}{}^+$ are definitely not constant in the DNS Re range
- The Reynolds-Stress Model fails to predict any similar Reynolds-number dependence
  - Or even the near-wall peaks
- This is arguably related to "Inactive Motion" with wall—parallel scales >> y
  - See Bradshaw, JFM 1967, `Inactive motion and pressure fluctuations in TBL'
  - And thinking of Wilcox and Durbin (leading to v2f)
- One-equation models, by chance, avoid this issue





Interactions at a Distance

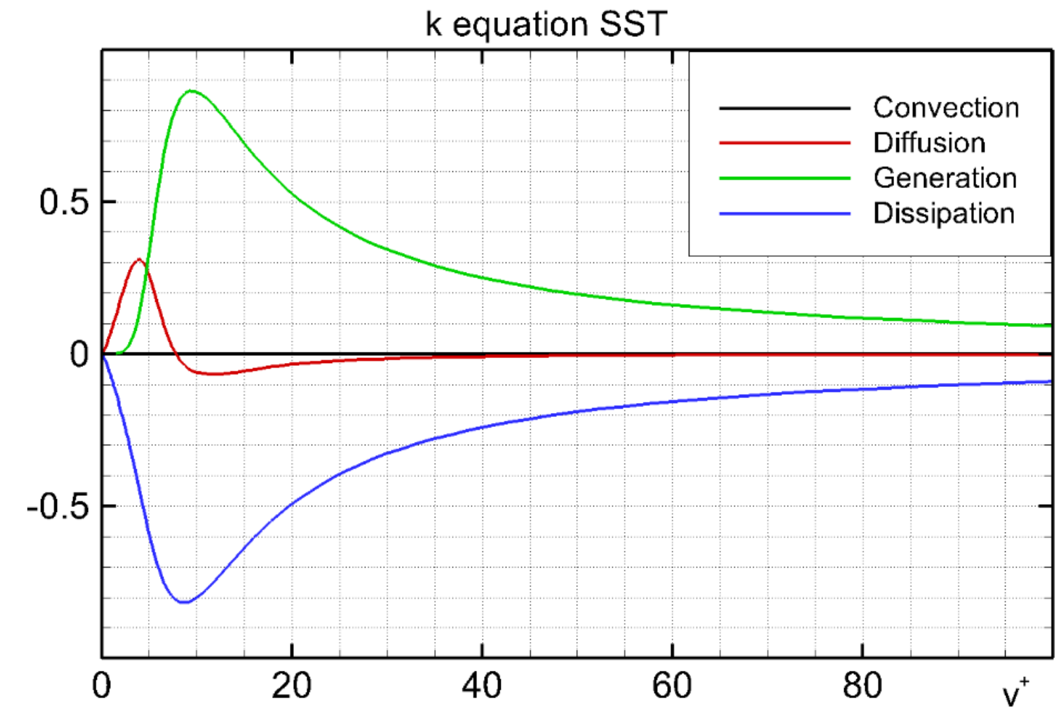




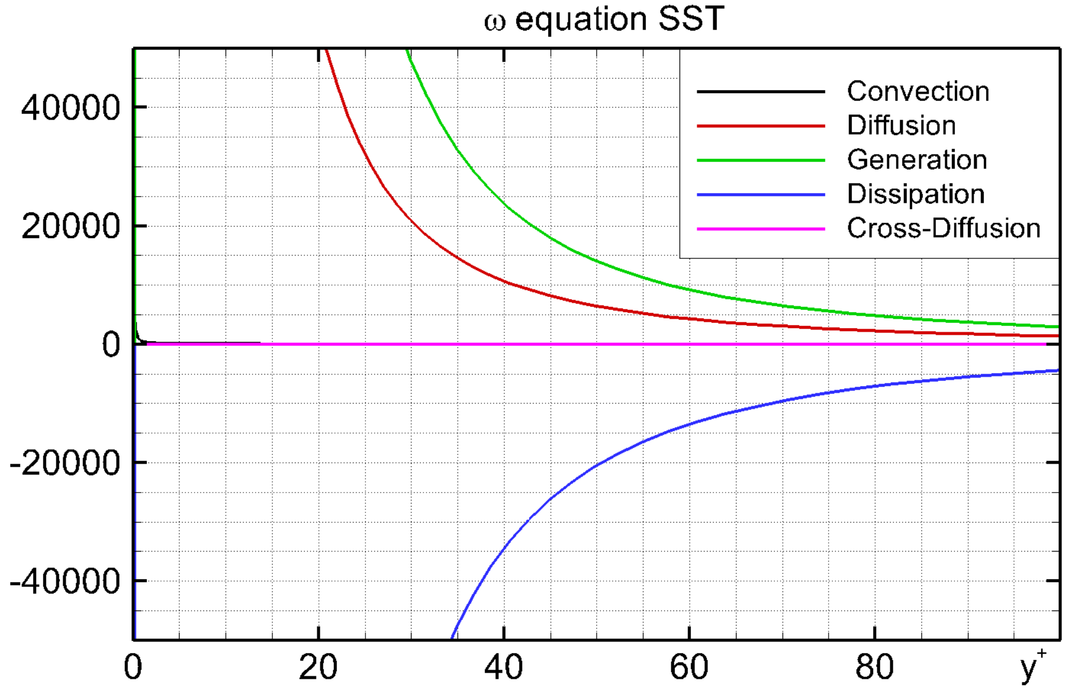
$\lambda >> y$

SST Budgets

- Boundary layer, region up to $y^+ = 100$
  - Courtesy A. Stabnikov and A. Garbaruk



- For k, the diffusion is negligible
  - Except in the buffer layer



- For ω, the diffusion is significant, but dominated by the near-wall region

Outline

- Direct Numerical Simulation as a source of data
  - Advantage over experiments: complete information
  - Potential: new ideas, or calibration of existing constants, or validation of full model?
    - Idea in SA model
    - Validation of Reynolds-Stress model
  - Limitations: Reynolds number and geometry
- Puzzling findings in DNS
  - Log layer and Karman “constant” have been very elusive
  - Luchini’s near-theoretical unification of Couette, Poiseuille and pipe flows
- Structural conflicts inherent to RANS models
  - Log-layer behavior of the Reynolds stresses
  - Insensitivity to flow Reynolds number
- **Contributions to complex models**
- **Attempts to concretely steer simple models**
  - **Effective eddy viscosity**
- Artificial intelligence

Contributions to Complex Models

- DNS normally provides all terms in any budget that is desired
- In theory, we make each term (e.g., pressure-strain and dissipation tensors) play the correct role
  - DNS then opens a new door, relative to experiments
- In reality, we live with compensating errors
  - Example: modeling the dissipation tensor as isotropic
- The modeled budget of the highest moment of turbulence is empirical
  - “Reynolds-Stress Models have more truth in them, and more lies” (anonymous…)
  - The data do not separate “rapid” and “slow” pressure terms
  - Some models use wall distance or wall-normal vector, which are not in the equations
- Another issue is that the true budget of dissipation (ε, or even ε_{ij}) is dominated by small eddies, but real models are dominated by large-eddy quantities (and mean-flow gradients)
  - Richardson-Kolmogorov energy-cascade arguments are effective, but imperfect
  - This was known in 1975

Attempts to Concretely Steer Simple Models

- DNS provides accurate k and ε. Ergo, we can make a better k-ε model!
- This would be true if the equation

$$\nu_t = \frac{C_\mu \quad k^2}{\epsilon}$$

were exact

- In reality, in a log layer the k-ε model gives a correct $\varepsilon^+=1/(\kappa y^+)$, an erroneous $k^+=1 / \sqrt{C_\mu}$, an erroneous C_μ, and a correct ν_t!

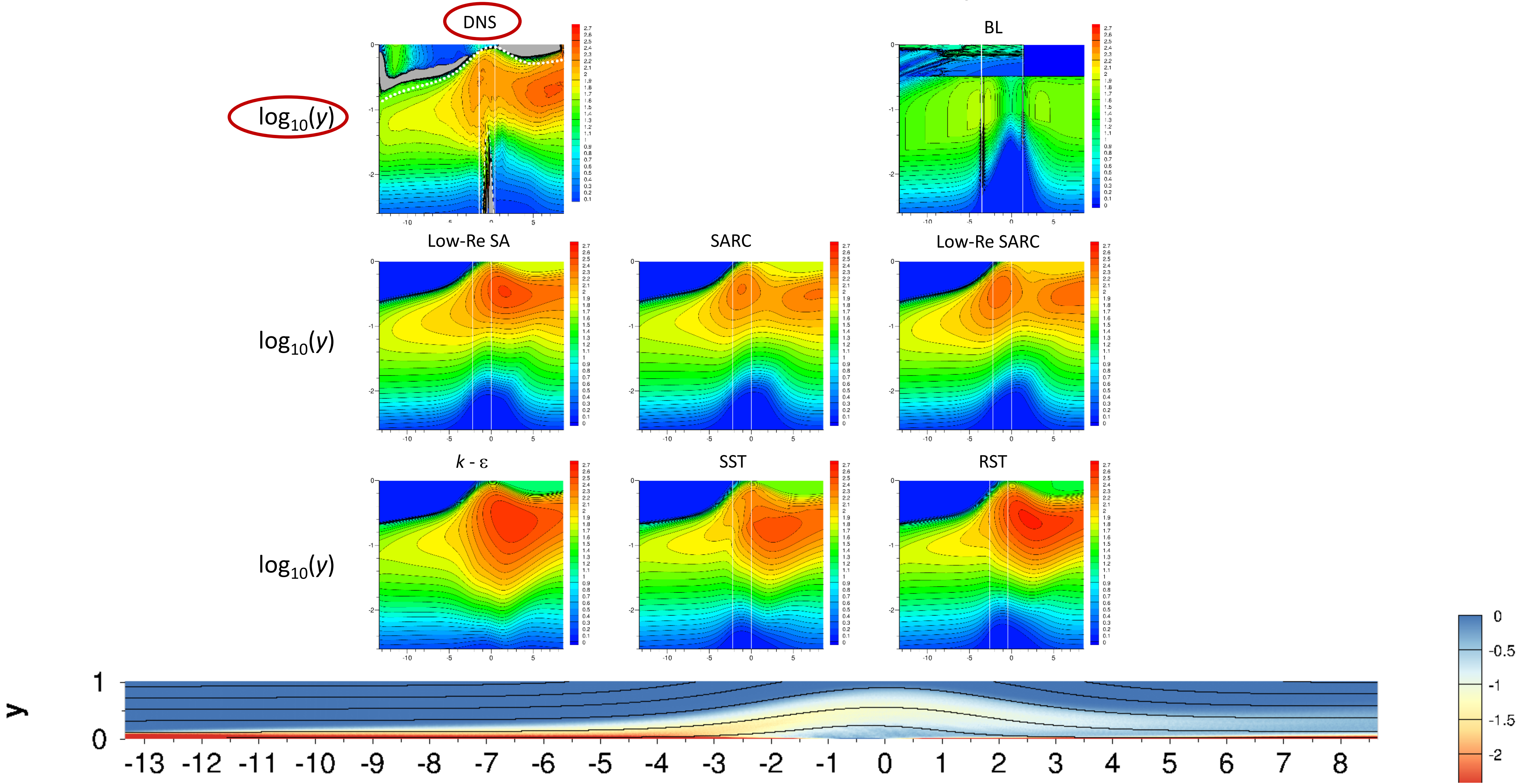
Attempts to Concretely Steer Simple Models

- Define effective eddy viscosity

$$\nu_{t_{eff}} \equiv -\frac{S_{ij} < u_i u_j >}{2 S_{kl} S_{kl}}$$

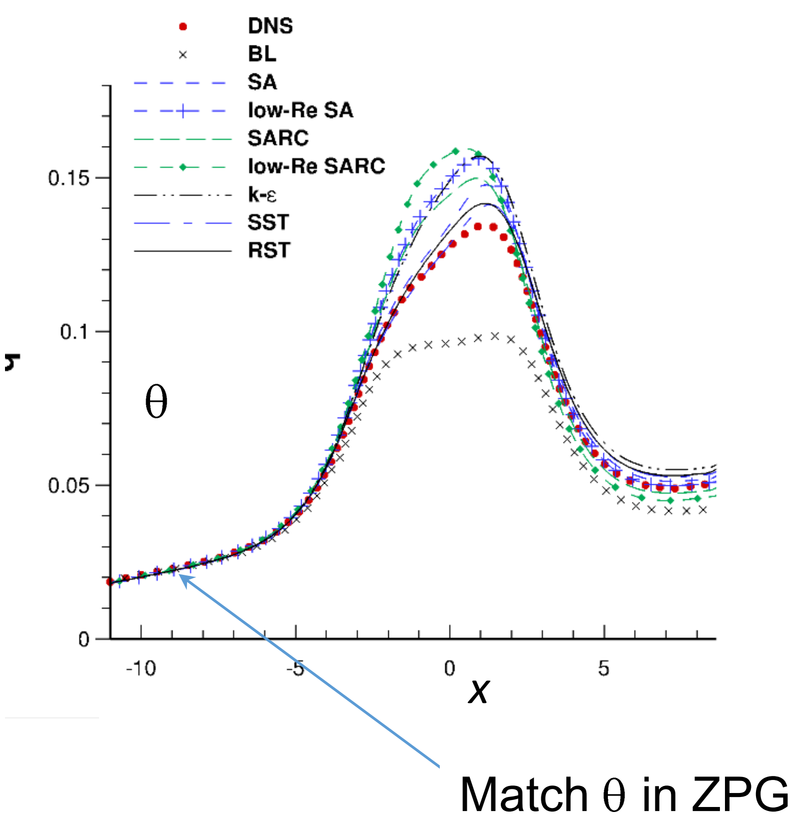
- This can be seen as a least-squares fit of a scalar to the stress tensor, or as the eddy viscosity that would give the correct TKE production
- This gives us a *local* target when working on eddy-viscosity models
- The results to date are mixed: the mean-flow improvement from an improved eddy viscosity is not reliable
    - There is a "norm problem." Thin regions, especially near the wall, may over-ride much larger regions (point made by P. Durbin)
- The turbulence equations may be solved in the "frozen DNS flow field"
- This concept is in our "tool box," and we may find fruitful uses for it

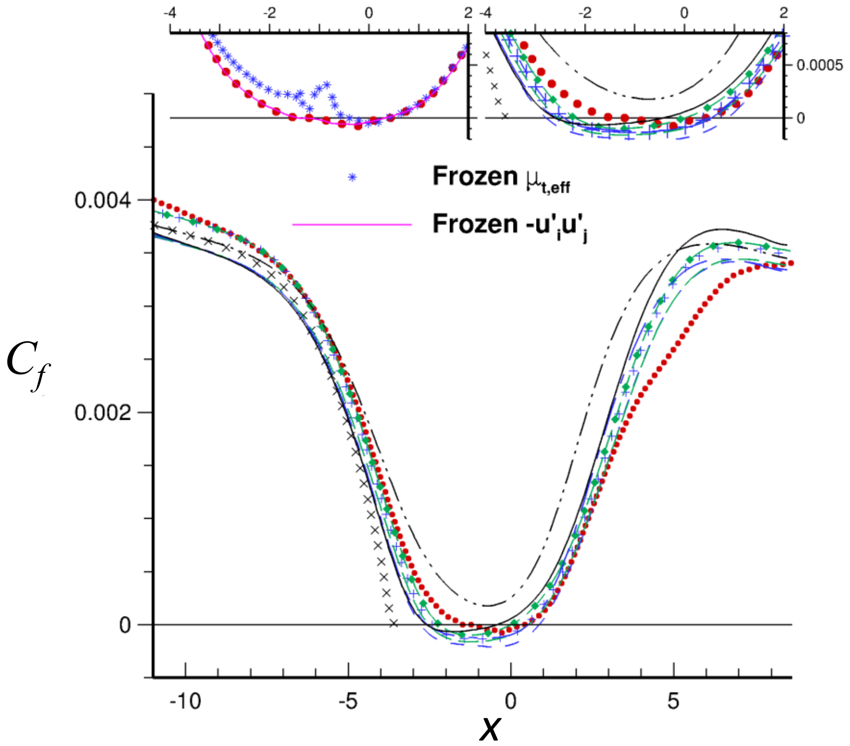
Compare Models and DNS in Separation Bubble
Work with Coleman and Rumsey, in JFM

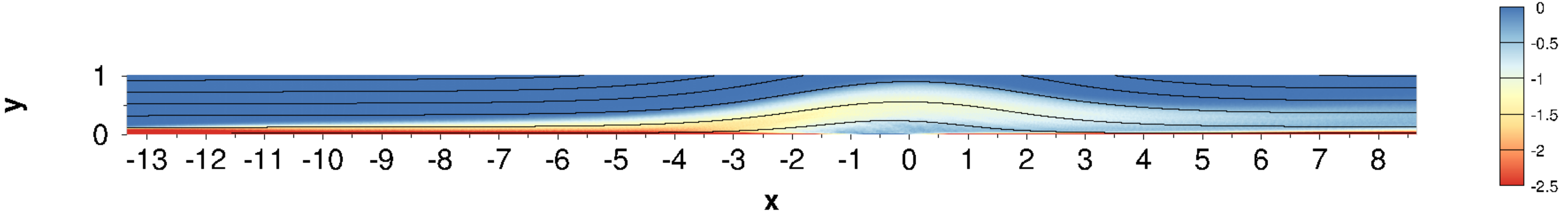


Compare Models and DNS in Separation Bubble

RANS Solutions via CFL3D, using DNS as inflow BC







Outline

- Direct Numerical Simulation as a source of data
  - Advantage over experiments: complete information
  - Potential: new ideas, or calibration of existing constants, or validation of full model?
    - Idea in SA model
    - Validation of Reynolds-Stress model
  - Limitations: Reynolds number and geometry
- Puzzling findings in DNS
  - Log layer and Karman "constant" have been very elusive
  - Luchini's near-theoretical unification of Couette, Poiseuille and pipe flows
- Structural conflicts inherent to RANS models
  - Log-layer behavior of the Reynolds stresses
  - Insensitivity to flow Reynolds number
- Contributions to complex models
- Attempts to concretely steer simple models
  - Effective eddy viscosity
- **Artificial intelligence**

Artificial Intelligence

- AI has made great strides in extremely difficult areas such as translation
  - Tools include Machine Learning, Big Data, Deep Neural Networks, etc.
- RANS modeling arguably has stagnated for decades
  - It’s possible that RANS modeling faces a “Fundamental Paradox,” and the community’s expectations/the demands of CFD are not realistic (because of local model formulation)
  - The value of RANS to industry and society is very high
- The SA and SST models are very useful, but not founded on theory or DNS
- There is logic in hoping AI can end the stagnation, with two threads:
  - 1. New thinking, new terms, new physics, some based on DNS data
  - 2. More powerful optimization of existing models over a wide range of flows
- It is debatable whether such efforts should include “historical” modelers, or start from a “clean sheet of paper”
  - Many “clean sheet” efforts violate Galilean Invariance, or have similar defects
  - Note that Symbolic Manipulation of equations has not caused much progress in RANS
  - Careful studies involve much “human” intervention (e.g., specify training region)
- A large European proposal, HiFi-Turb, hinges on this hope

Summary and Future

- Since the 1980’s, Direct Numerical Simulation has made great progress
  - Reynolds number: Channel Re_τ from 180 to 5000, cylinder Re_D from 3900 to $6\ 10^5$
  - Geometric complexity: from channel to TBL, cylinders, golf balls, high Mach numbers, separation bubbles including shock-induced
- Yet, its impact on everyday turbulence models is still almost invisible
  - One key factor is the empirical nature of these models
  - Even the Reynolds-Stress models suffer from compensating errors
  - Another factor is the probable “structural” inability of RANS models to track DNS (i.e., reality!) for the y- and Re-dependence of the Reynolds stresses
  - This is not exactly the same as the “Fundamental Paradox”
- It’s not that the DNS and RANS communities ignore each other
- The value of RANS to society justifies sustained efforts
  - Breakthroughs are not likely
  - Artificial Intelligence might help
  - It is very hard, for me, to develop new RANS modelers