UNIVERSITAT POLITÈCNICA DE CATALUNYA
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Escola Superior d'Enginyeries Industrial, Aeroespacial i Audiovisual de Terrassa

SEAT

Noise Radiated by an Open Cavity at Low Mach Number

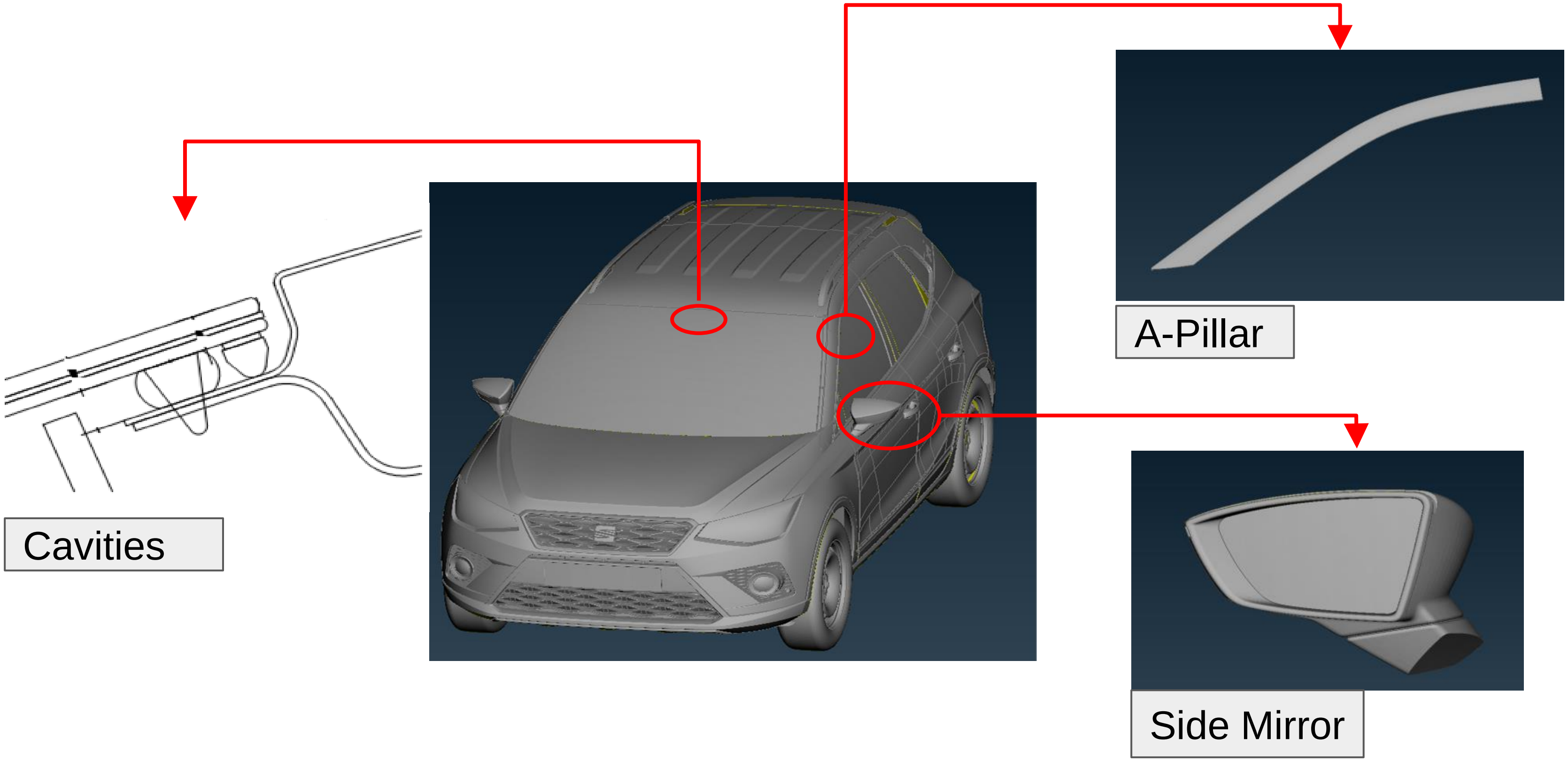
R. Martin[*,**], M. Soria[*] , O. Lehmkuhl[***], A.Gorobets[****] and A.Duben[****]
Corresponding author: manel.soria@upc.edu

[*] Universitat Politècnica de Catalunya, Spain.
[**] SEAT, Spain.
[***] Barcelona Supercomputing Center, Spain.
[****] Keldysh Institute of Applied Mathematics of RAS, Russia.

Aeroacoustics

- **Aeroacoustics in the Automobile Industry**

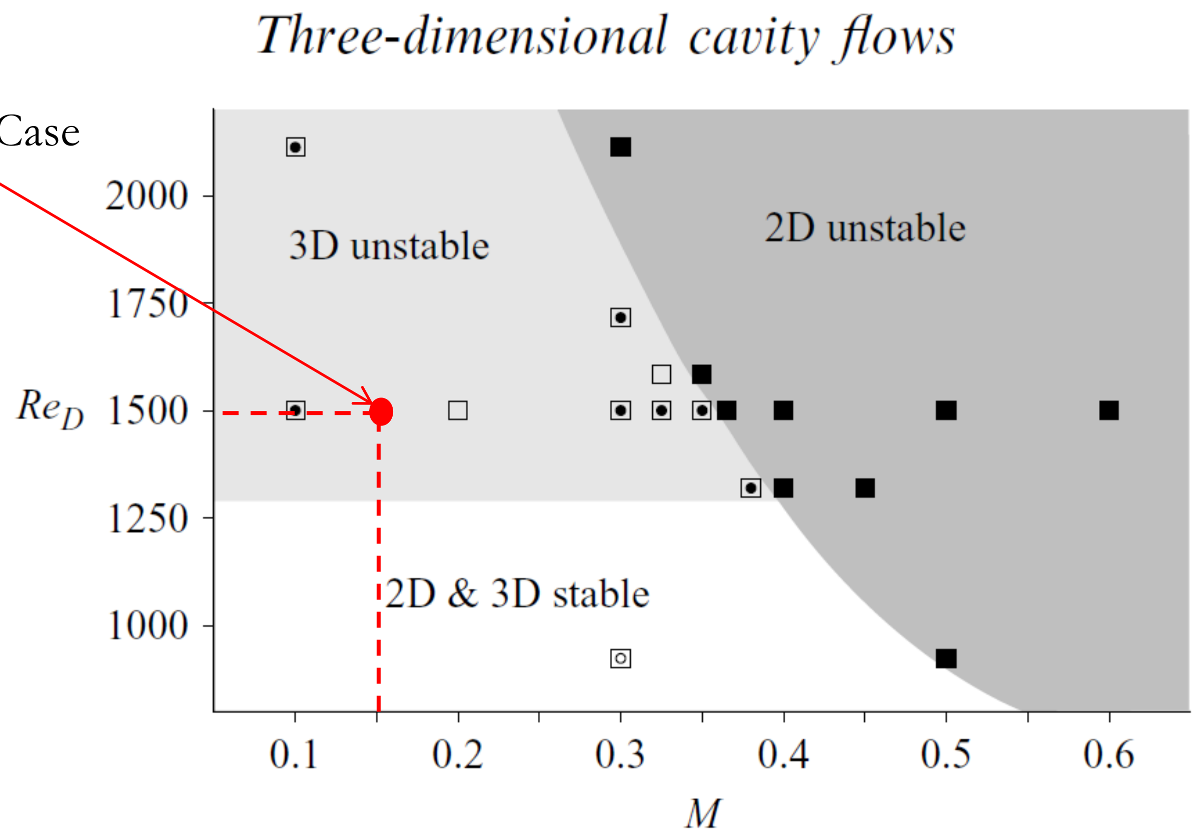
$M \in [0.08, 0.15]$



Previous Works on Flow Past Cavities I

Publication	Reynolds Number	Mach Number	Aspect Ratio (L/D)	Dimension	Acoustic Method
Shieh, Chingwei, and Philip Morris. *Parallel numerical simulation of subsonic cavity noise*, 1999.	5000	0.5	4	2D	DS
X Gloerfelt, C Bailly, and D Juvé. *Direct computation of the noise radiated by a subsonic cavity flow and application of integral methods*, 2003.	$4.1 \cdot 10^4$	0.7	2	2D	DS FW-H
Jonas Ask and Lars Davidson. *Sound generation and radiation of an open two–dimensional cavity*, 2009.	1500	0.15	4	2D	DS Curle
Huanxin Lai and Kai H Luo. *A three-dimensional hybrid les-acoustic analogy method for predicting open-cavity noise*, 2007.	$1.36 \cdot 10^6$	0.85	5 (finite,W/D=1)	3D	FW-H
C Shieh and P Morris. *Comparison of two-and three-dimensional turbulent cavity flows*, 2001	$2 \cdot 10^5$	0.6	4,4 (finite,W/D=1)	3D	DS

Previous Works on Flow Past Cavities II

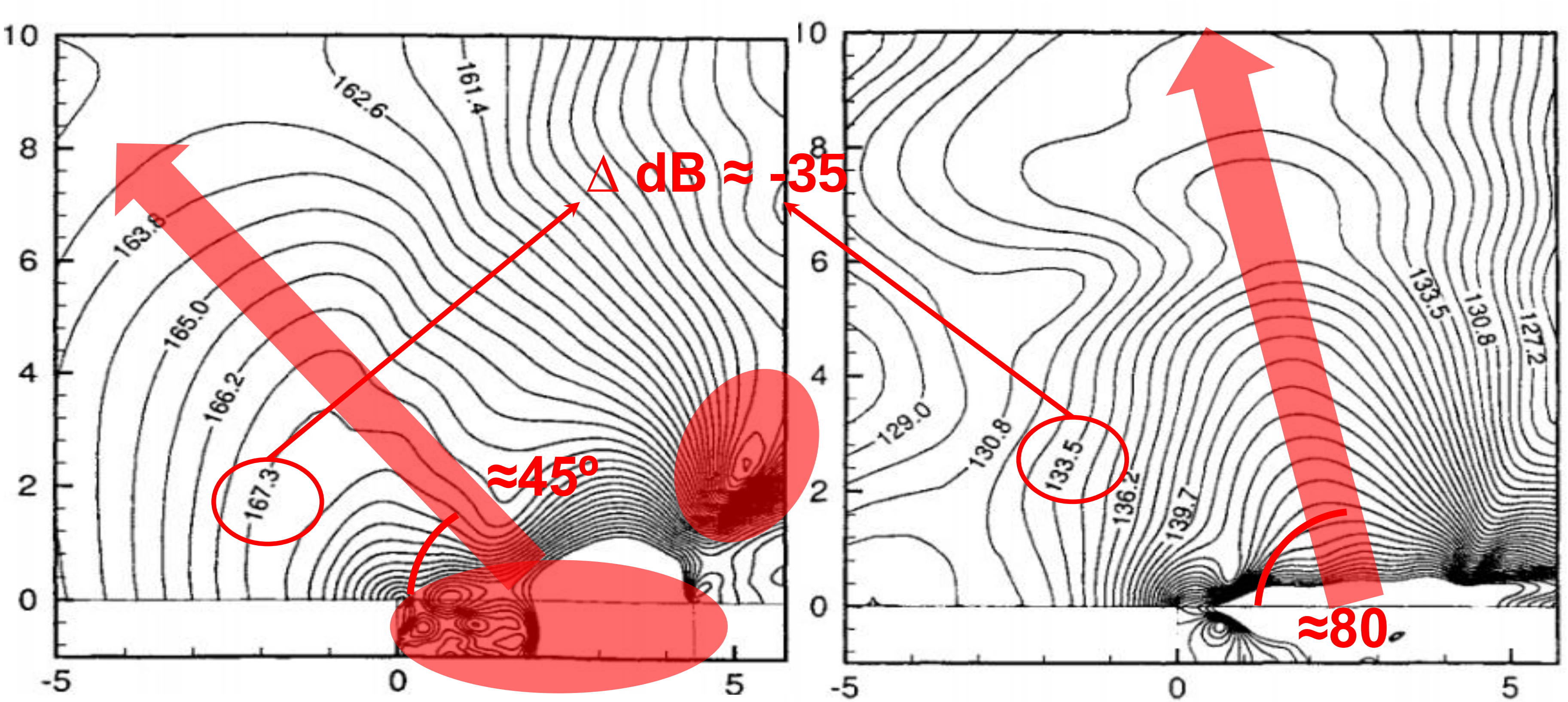


Bres et al, 2008.

How acoustics is influenced by the three dimensional behaviour of the flow?

Previous Works on Flow Past Cavities III

FINITE CAVITY: L/D=4.4, W/D=1



SPL in 2D

SPL in 3D

Shieh et al, 2001.

Previous Works on Flow Past Cavities I

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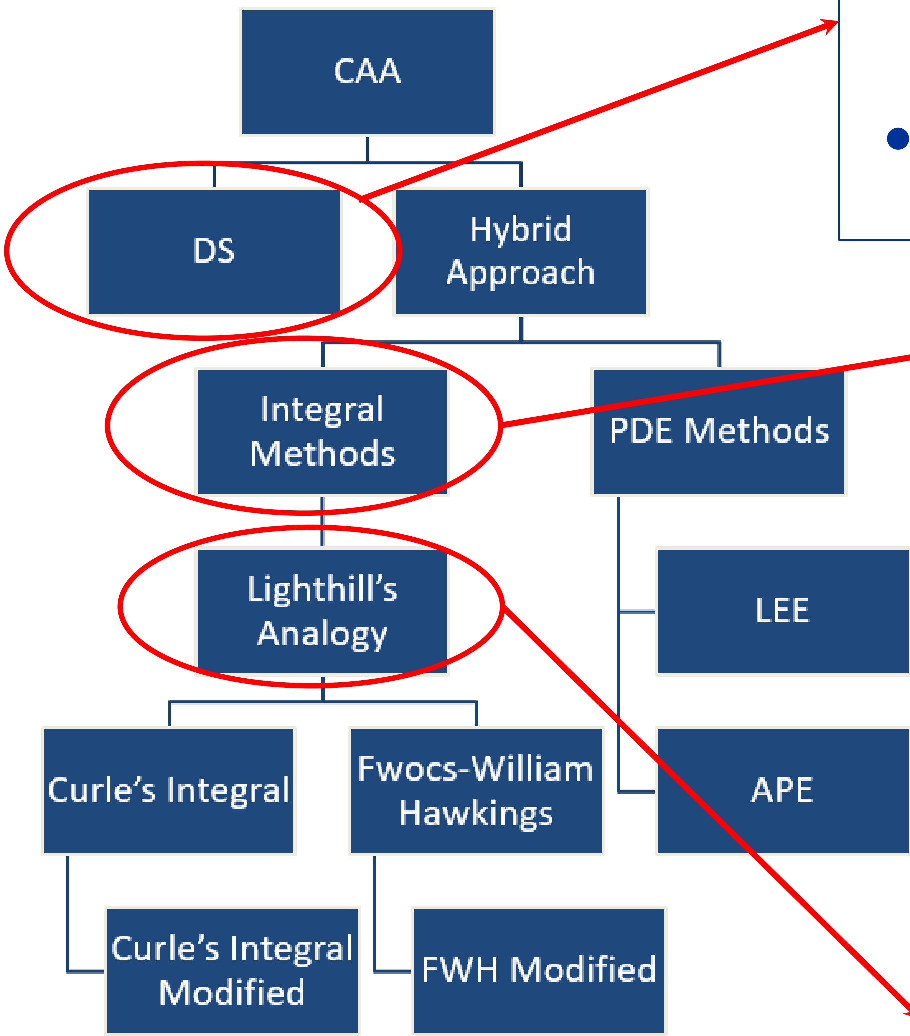
Computational Aeroacoustics Method



- Directly solve Compressible Navier-Stokes Equations.
- Do not include any modeling of the sound.

- Under certain conditions, integral methods can be reduced to a surface integral that can be computed as a post-processing.
- Better for far field.

Lighthill's Equation

$$\frac{\partial^2 \rho}{\partial t^2} - a_0^2 \nabla^2 \rho = \frac{\partial^2}{\partial x_i \partial x_j}(\rho u_i u_j + p_{ij} - a_0^2 \rho \delta_{ij} - \tau_{ij})$$

$$T_{ij} = u_i u_j + p_{ij} - a_0^2 \rho \delta_{ij} - \tau_{ij}$$

Computational Aeroacoustics: Lighthill's Analogy

Lighthill's Equation

$$\frac{\partial^2 \rho}{\partial t^2} - a_0^2 \nabla^2 \rho = \frac{\partial^2}{\partial x_i \partial x_j}(\rho u_i u_j + p_{ij} - a_0^2 \rho \delta_{ij} - \tau_{ij})$$

$$T_{ij} = u_i u_j + p_{ij} - a_0^2 \rho \delta_{ij} - \tau_{ij}$$

J.Lighthill, 1952.

Curle's Solution for Lighthill's System

* [f]:= Evaluation of f at retarded time τ=t-r/a0

$$\rho(\mathbf{x}) - \rho_0 = \frac{1}{4\pi a_0^2} \frac{\partial^2}{\partial x_i x_j} \int_V \frac{[T_{ij}]}{r} dV + \frac{1}{4\pi a_0^2} \frac{\partial}{\partial x_i} \int_S \frac{n_j}{r} [p_{ij} - \tau_{ij}] dS$$

Quadrupole Source

Dipole Source

Negligible for low Mach number!

$$\frac{P_{quad}}{P_{dip}} \approx M$$

N. Curle, 1955

…three hours ago…

Computational Aeroacoustics: Curle's Analogy I

Curle's Solution for Lighthill's System

$$\rho(\mathbf{x}) - \rho_0 = \frac{1}{4\pi a_0^2}\frac{\partial^2}{\partial x_i x_j}\int_V \frac{[T_{ij}]}{r}dV + \frac{1}{4\pi a_0^2}\frac{\partial}{\partial x_i}\int_S \frac{n_j}{r}[p_{ij} - \tau_{ij}]dS$$

Only pressure on surface needs to be recorded for post-processing!

- Isentropic Fluid: $\rho - \rho_0 = (p - p_0)/a_0^2$
- Spatial derivatives to time derivatives:

$$\frac{\partial q(\tau)}{\partial x_i} = \frac{\partial q}{\partial \tau}\frac{\partial \tau}{\partial x_i} = \frac{\partial q}{\partial \tau}\frac{\partial \tau}{\partial r}\frac{\partial r}{\partial x_i} = \frac{-1}{a_0} l_i \frac{\partial q}{\partial \tau}$$

Myers et al, 1988.

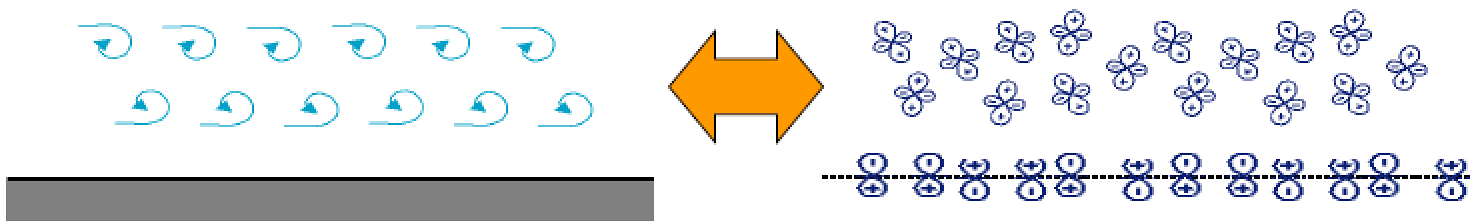
- Surface does not depend on time: derivatives inside integral.
- Chain rule.
- Viscous terms neglected.

Curle's Modified Solution

$$p - p_0 = \cancel{\frac{1}{4\pi}\int_V \left(\frac{l_i l_j}{a_0^2 r}[\ddot{T}_{ij}] + \frac{3l_i l_j - \delta_{ij}}{a_0 r^2}[\dot{T}_{ij}] + \frac{3l_i l_j - \delta_{ij}}{r^3}[T_{ij}]\right) dV} +$$

$$\frac{1}{4\pi}\int_S -l_i n_j \left(\frac{1}{r a_0}[\dot{p}_{ij}] + \frac{1}{r^2}[p_{ij}]\right) dS$$

Computational Aeroacoustics: Curle's Analogy II



- The replacement of the surface by a dipole distribution makes the surface acoustically transparent.
- Compact bodies do not scatter its own acoustic field, i.e., they are acoustically transparent.
- Acoustical compactness is expressed by Helmholtz number:

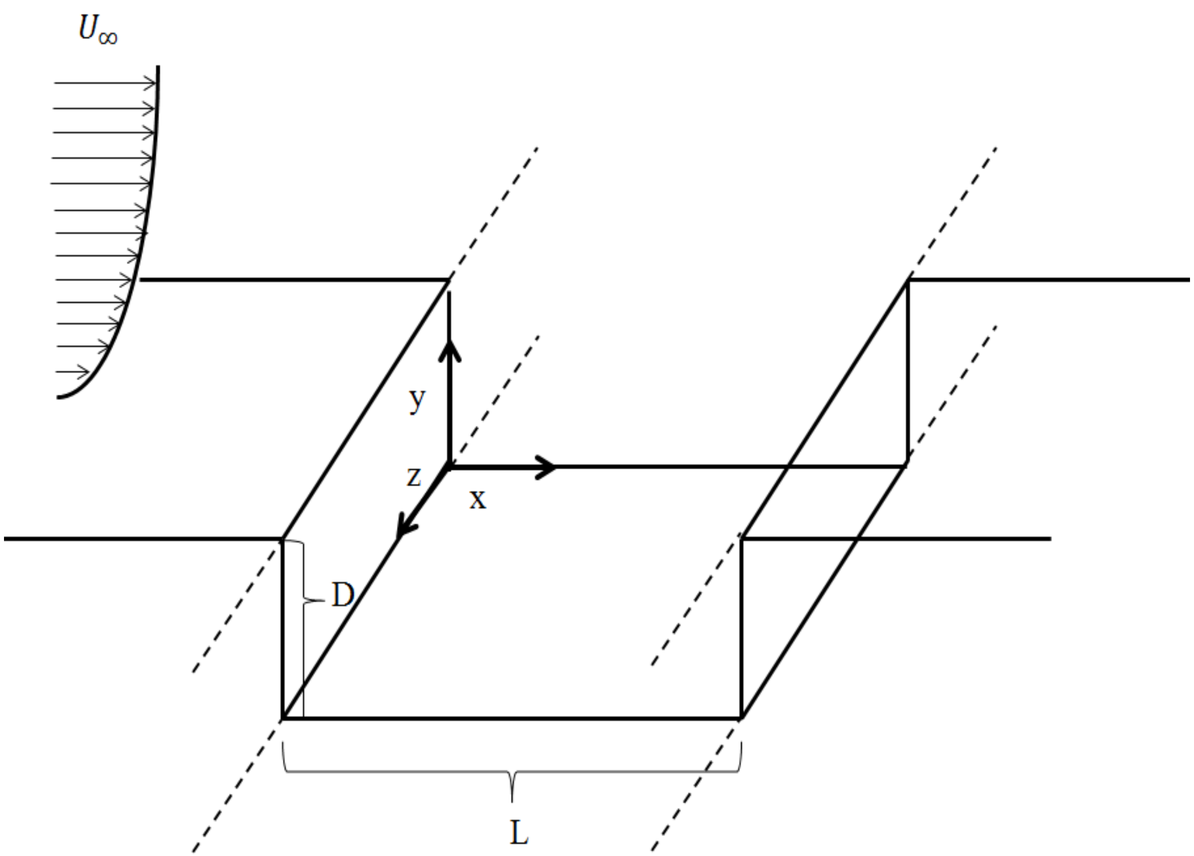
$$He = \frac{2\pi f D}{a_0} < 1$$

SW. Rienstra, 2013.

For low Mach numbers + compact source regions

→ **Incompressible simulation!**

Problem Statement I



$$Re = \frac{\rho_\infty U_\infty D}{\mu} = 1500$$

$$M = \frac{U_\infty}{a_0} = 0.15$$

$$\frac{\delta_{0.99}}{D} = 0.4$$

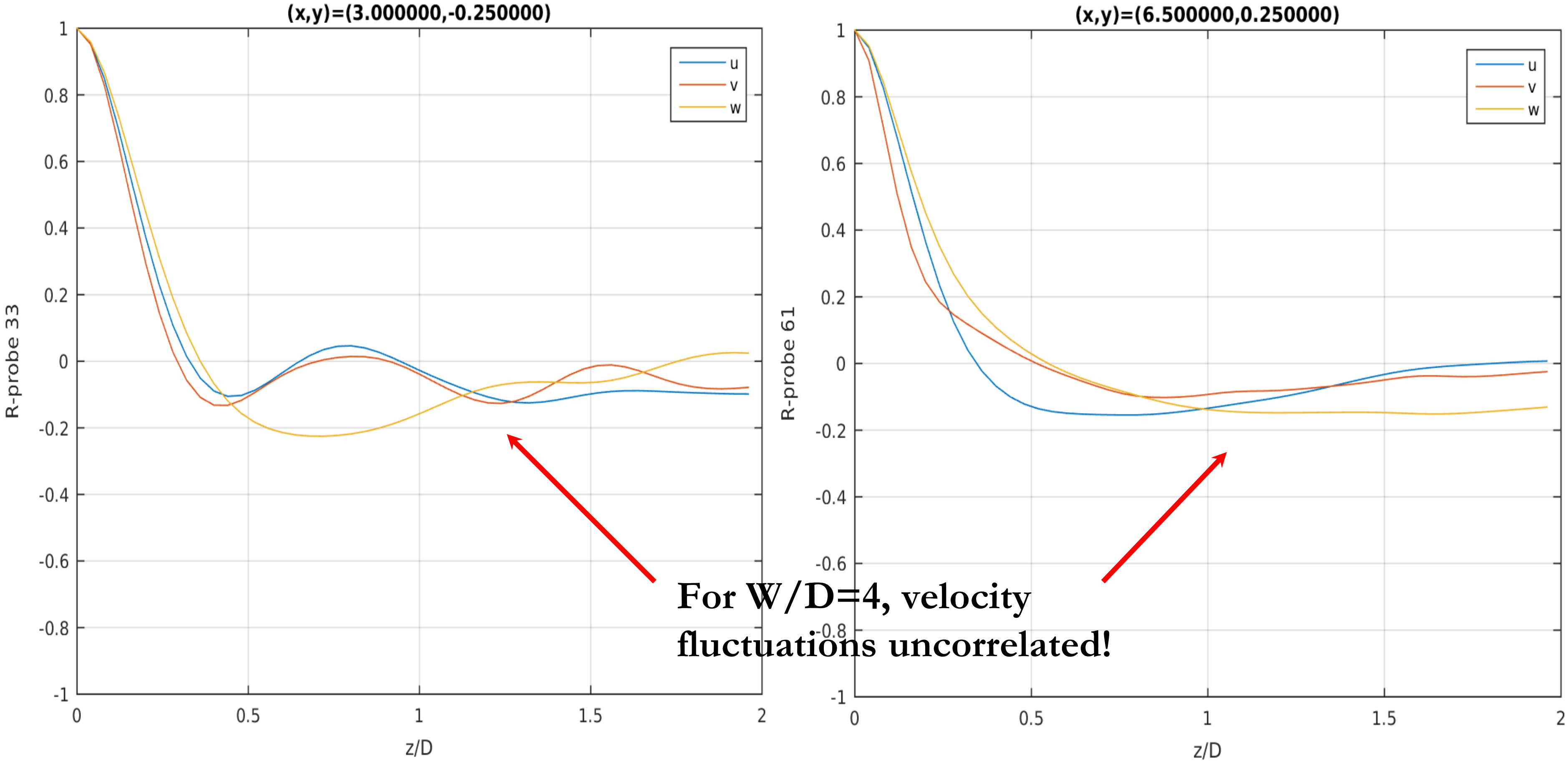
$$\frac{L}{D} = 4$$

Infinite in spanwise direction

Problem Statement II

Width of the domain: the correlation coefficient for the velocity fluctuations has to tend to zero as it approaches the half-size of the domain:

$$R_{ii} = \frac{< u_i'(x_i,t)u_i'(x_i+\delta,t) >}{< u_i' u_i' >}$$



For W/D=4, velocity fluctuations uncorrelated!

CFD and CAA Approach

- **Compressible Simulation (DS)**

  CFD simulation: solving of compressible Navier Stokes equations using DNS with NOISEtte code, developed by Keldysh Institute of the Russian Association of Mathematics.

  A Gorobets, 2018.

  CAA simulation: acoustic pressure, p', directly obtained from CFD simulation.

- **Incompressible Simulation (Curle's Analogy)**

  CFD simulation: solving of incompressible Navier Stokes equations using DNS with Alya code, developed by Barcelona Supercomputing Center.

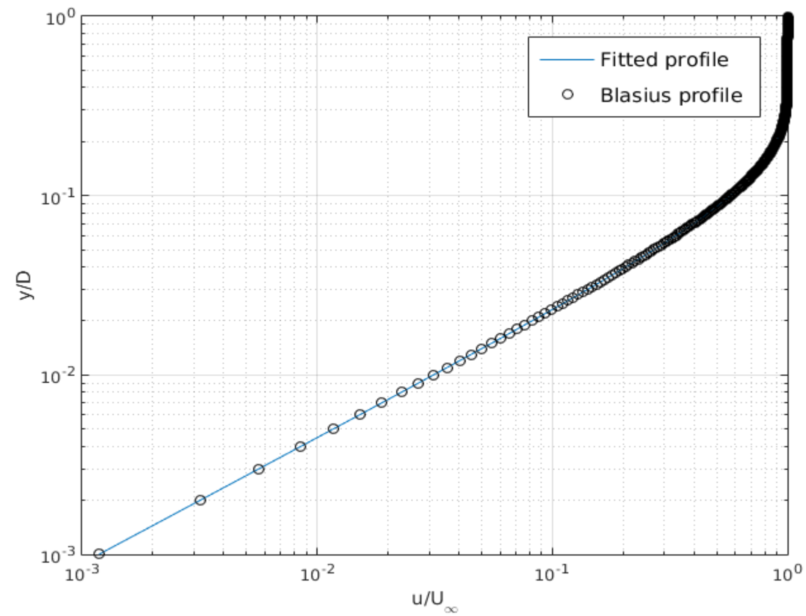
  M Vázquez et al, 2016.

  CAA simulation: acoustic pressure, p', obtained from Curle's postprocess of incompressible CFD simulation data.

Boundary Conditions

- **Inlet**:
  Blasius solution developed during 5 length units

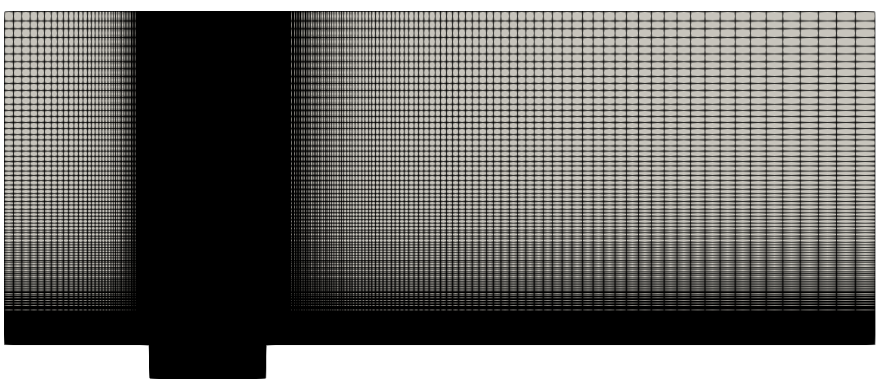
$$\frac{u}{U_\infty}(y/D) = 1.0 - e^{-22.837905(y/D)^{1.4288577}}$$



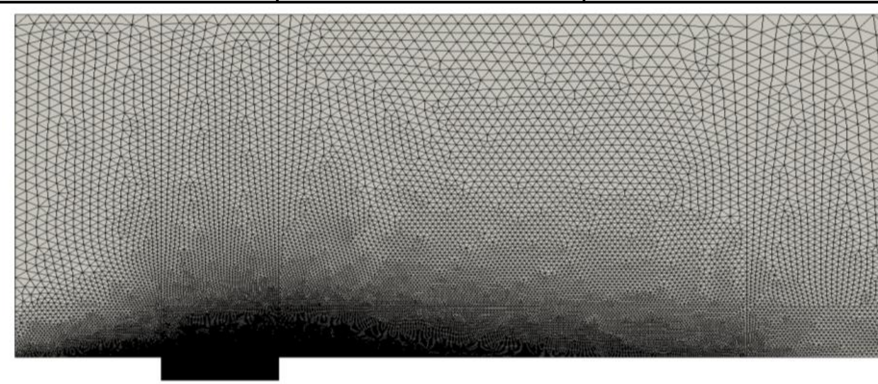
- **Outlet**: Zero normal derivatives.
  Buffer zone:
  Compressible: last 6 length units of the domain.
  Incompressible: 10 mesh elements of the domain (2 length units approximately at plane y = 0).
- **Solid Walls**: No slip boundary confitions.
- **Front and Back Walls**: Periodic boundary conditions.

Mesh Convergence

Case	Number of Elements	Number of planes	Mean Drag Value	Fundamental Frequency
2D Incompressible	$9.6 \cdot 10^4$	1	0.419	0.061
	$11.9 \cdot 10^4$	1	0.417	0.061
	$13.6 \cdot 10^4$	1	0.418	0.061
3D Incompressible	$9.8 \cdot 10^4$	100	0.058	0.220
	$11.9 \cdot 10^4$	100	0.058	0.220
	$18.4 \cdot 10^4$	100	0.059	0.220
3D Compressible	$8.7 \cdot 10^4$	150	0.055	0.219



Compressible Case



Incompressible Case

Curle's Postprocess Implementation I

- **Convergence**

$$p(\mathbf{x},t) - p_0 = \frac{1}{4\pi}\int_S -l_i n_j \left(\frac{1}{r a_0}[\dot{p}_{ij}] + \frac{1}{r^2}[p_{ij}] \right) dS$$

Inverse dependance with distance
→ convergence assured!

p has to be stored every time step
→ high amount of disk space!

- **Time Treatment**

Linear interpolation for evaluation at retarded time: $\tau = t - r/a_0$

Total time range: $T = 100$

Minimum frequency: $f_{min} = \frac{10}{T}$

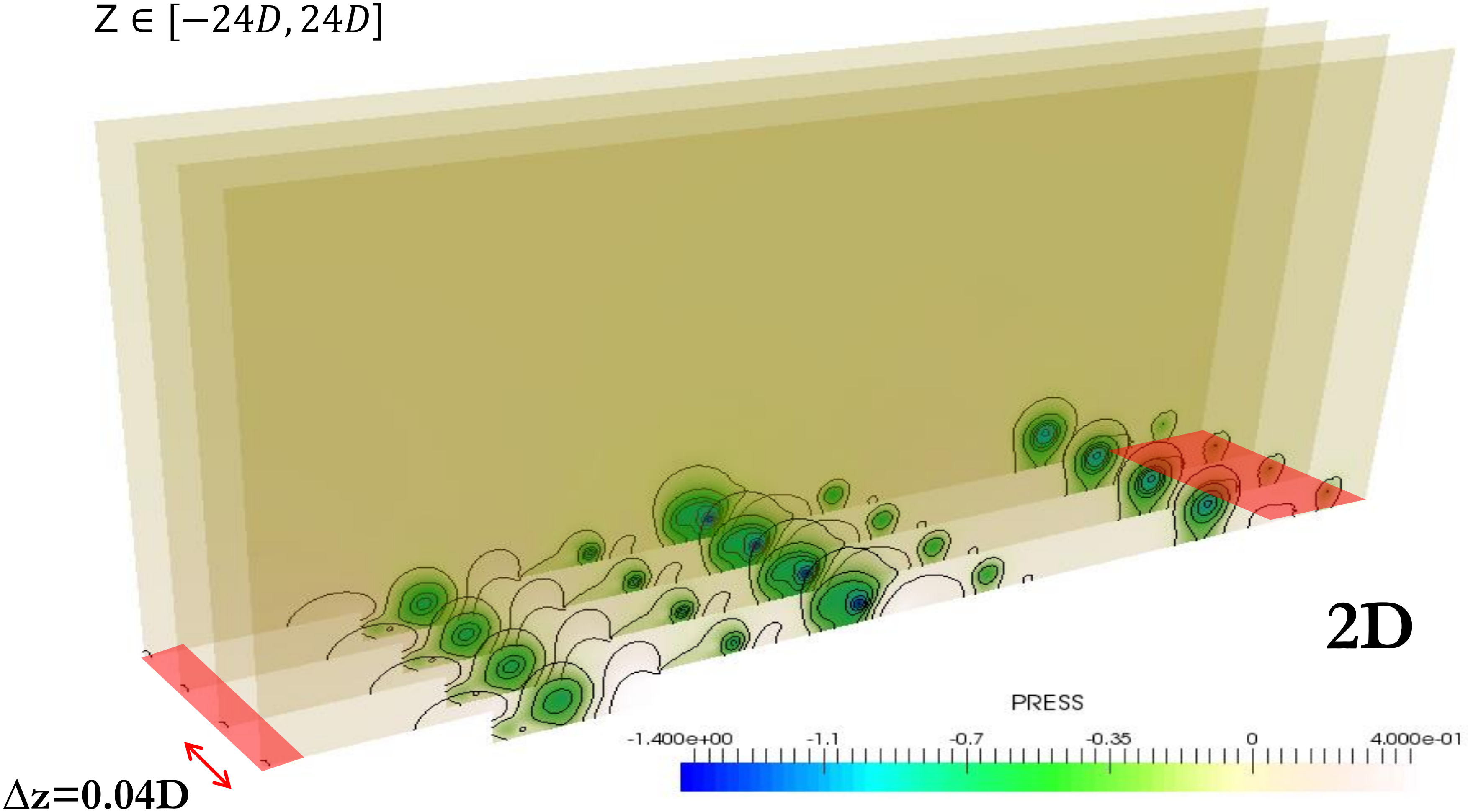
Sample rate: $\Delta t = 0.1\frac{D}{U_\infty}$

Maximum frequency: $f_{max} = min\left\{\frac{1}{\Delta t}, \frac{a_0}{2\pi D}\right\} = 1$

Curle's Postprocess Implementation II

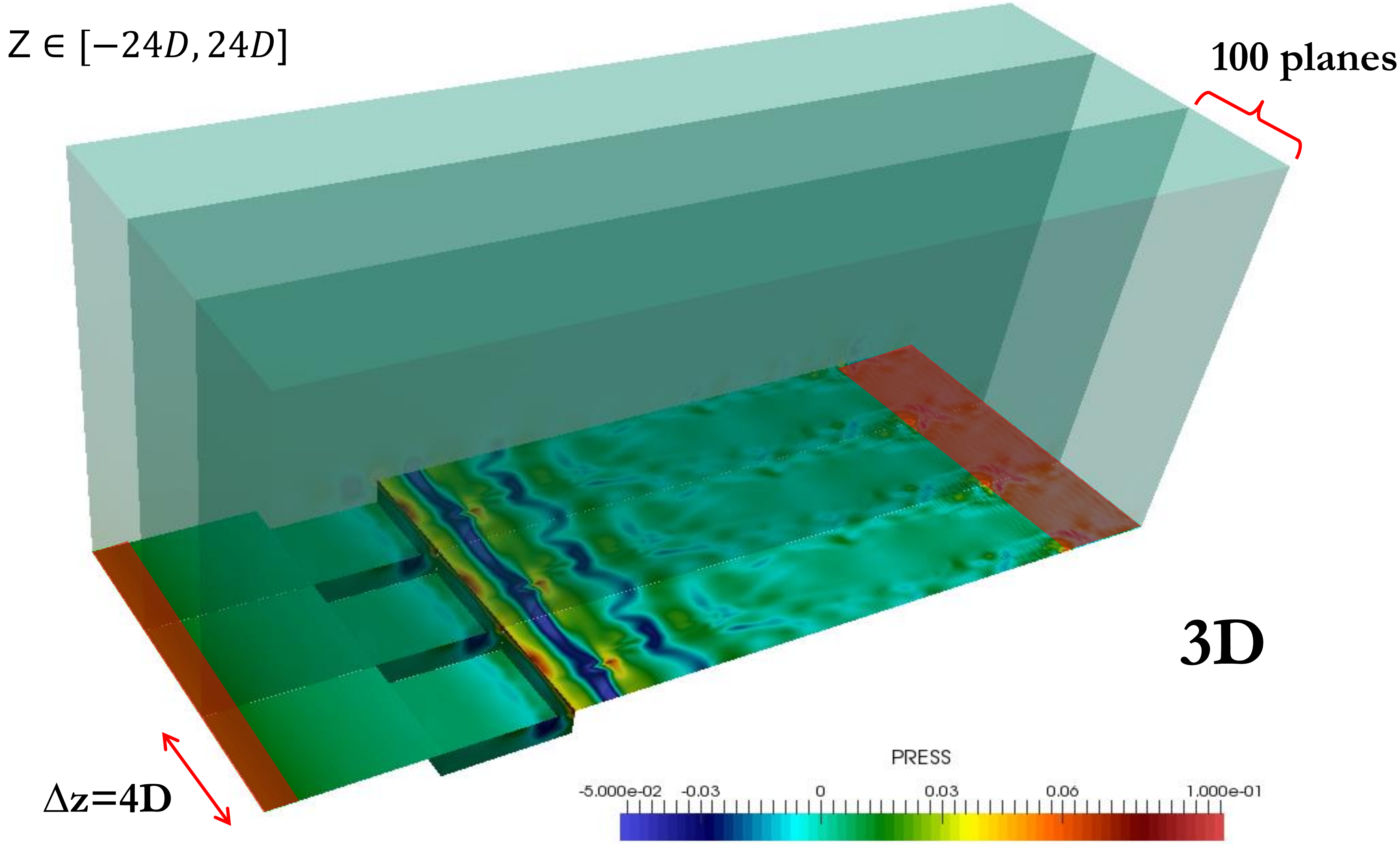
- **Surface of Integration**

$Z \in [-24D, 24D]$



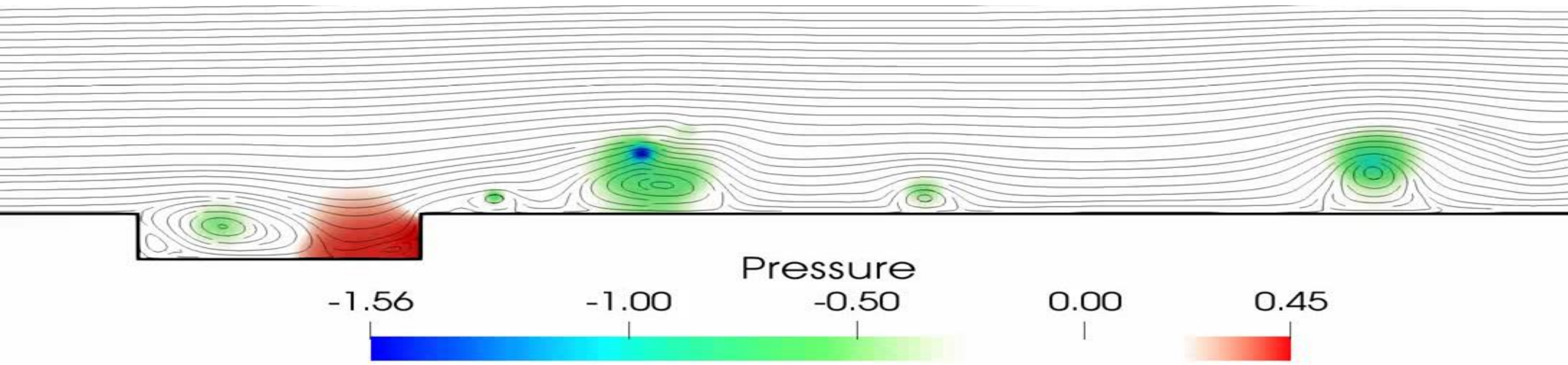
Curle's Postprocess Implementation III

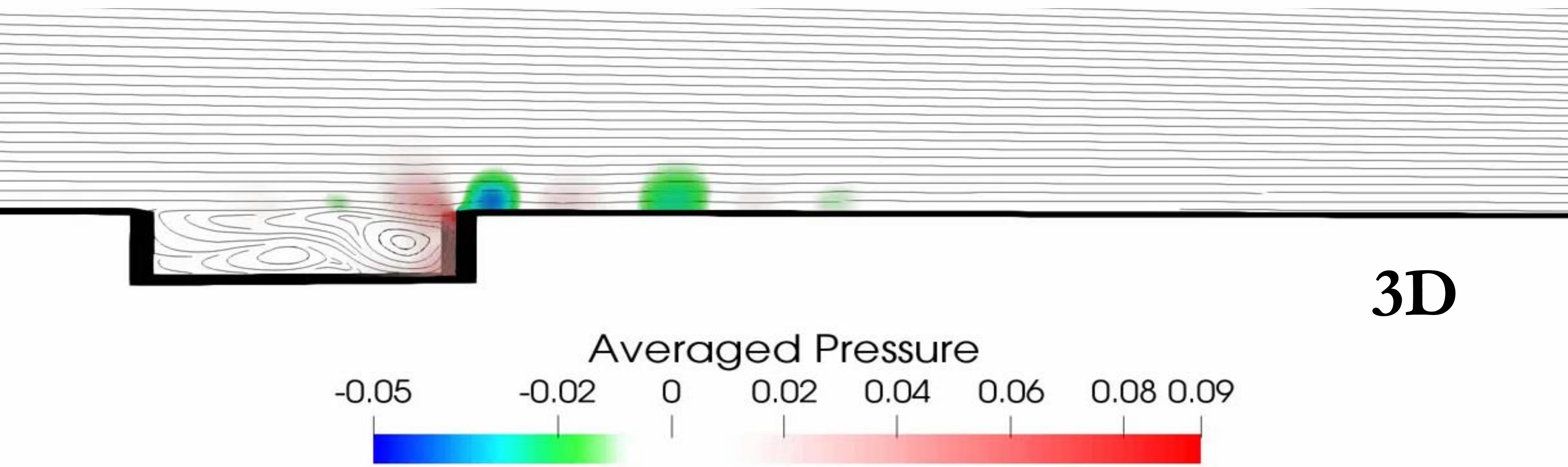
- **Surface of Integration**



Flow Field Results I

2D

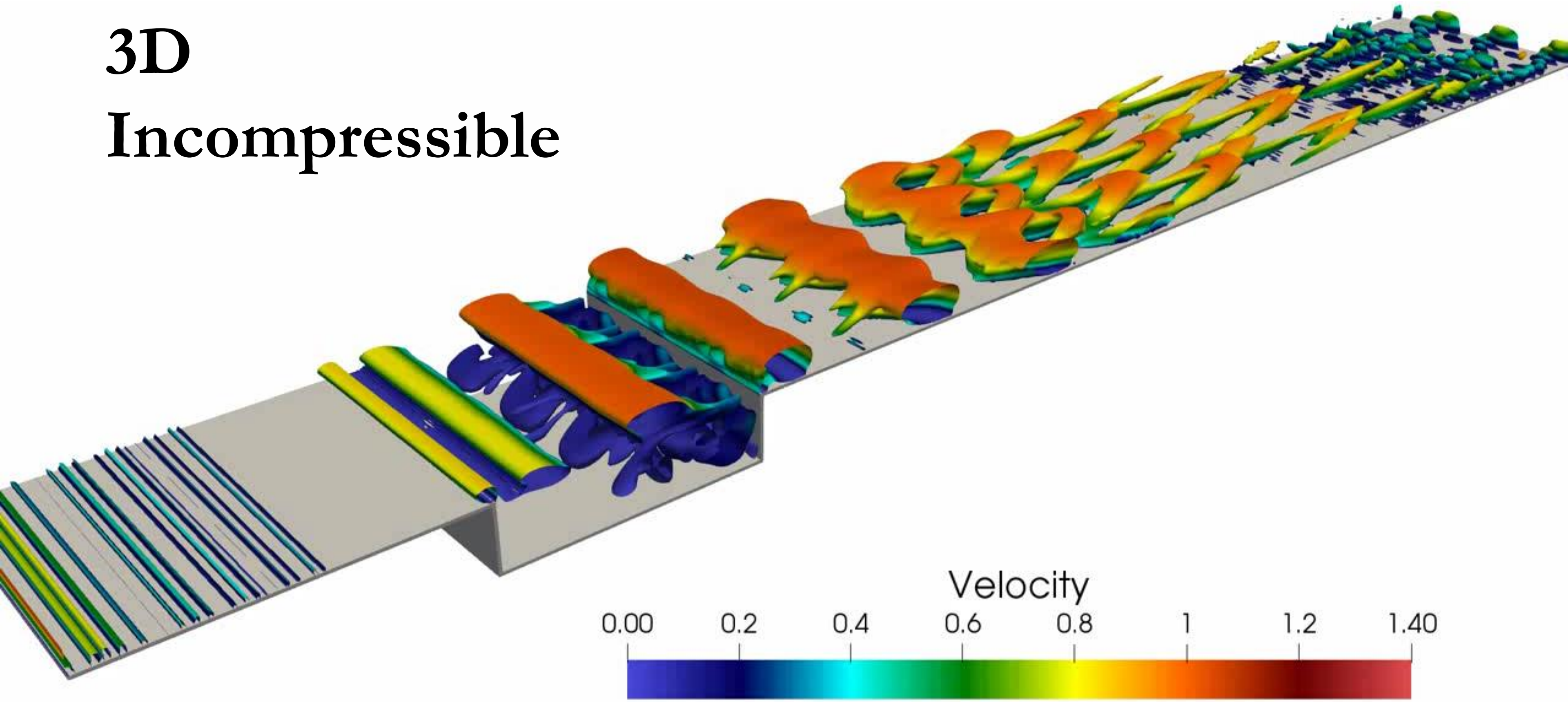




3D

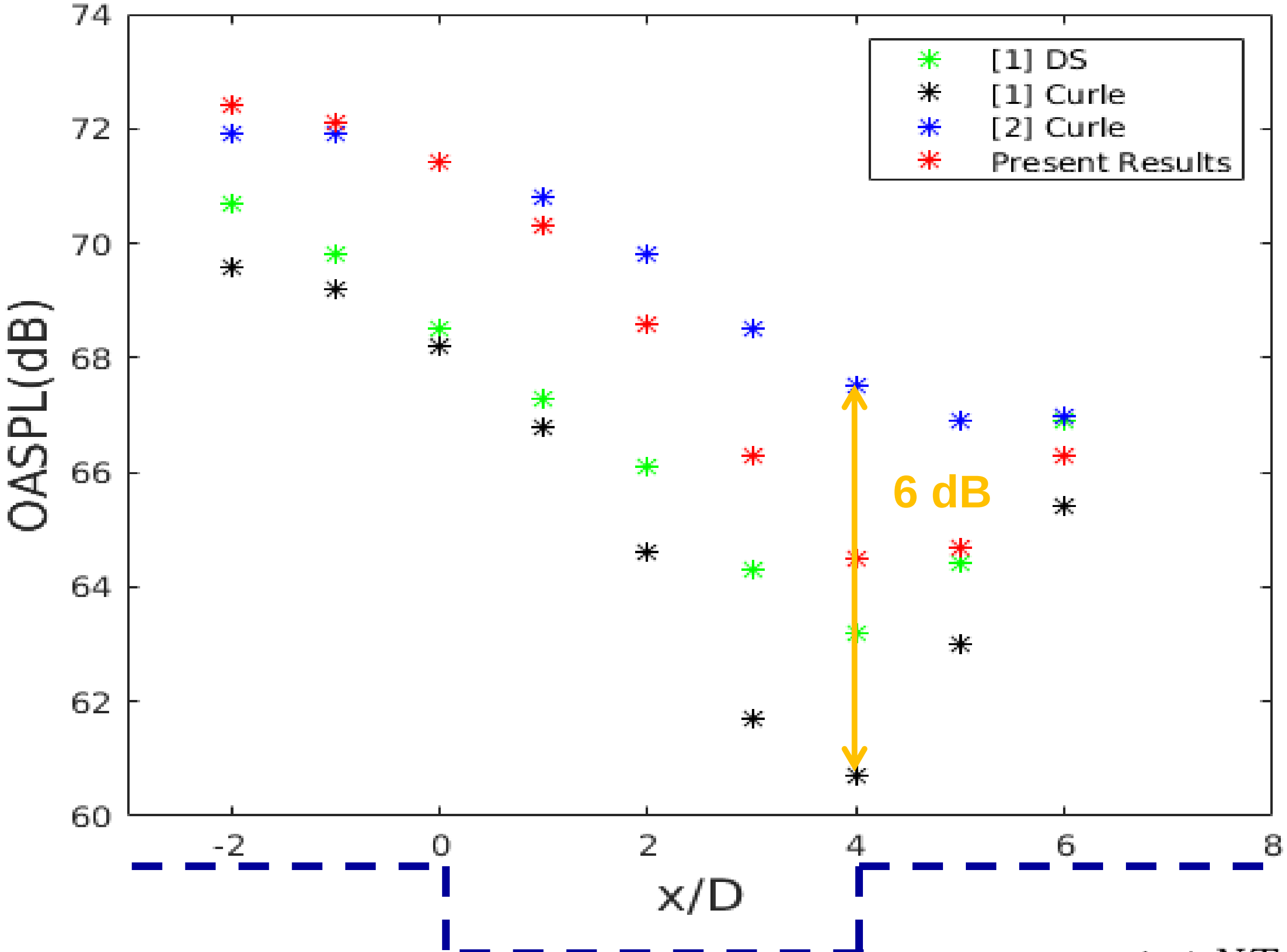
Flow Field Results II

3D Incompressible



Isosurfaces of $\mathcal{Q} = -\frac{1}{2}\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} = 0.01$ colored by velocity magnitude

Acoustic Field Results I



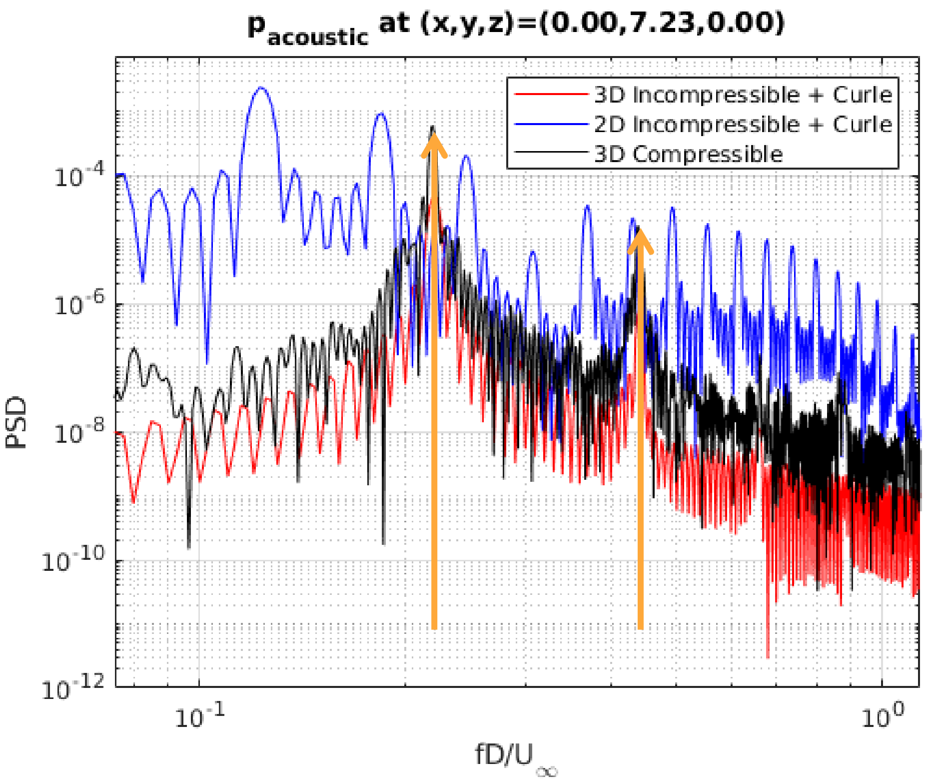
[1] J. Larsson & L. Davidson, 2003

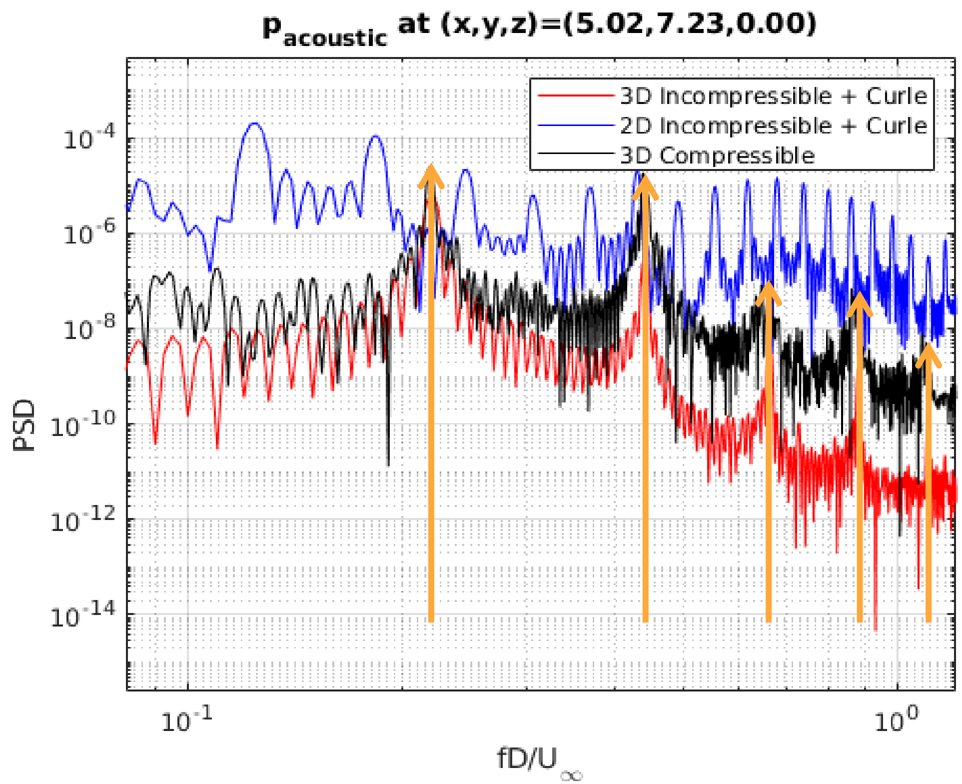
[2] J. Ask & L.Davidson, 2009

x/D	y/D
-2	7.18
-1	7.18
0	7.18
1	7.18
2	7.18
3	7.18
4	7.18
5	7.18
6	7.18

$$OASPL = 20log_{10}\left(\frac{p_{rms}}{p_{ref}}\right) \qquad p_{rms}^2 = \frac{1}{NT}\int_{t_0}^{t_0+NT} p_a(\mathbf{x},t)\cdot p_a(\mathbf{x},t)dt$$

Acoustic Field Results II

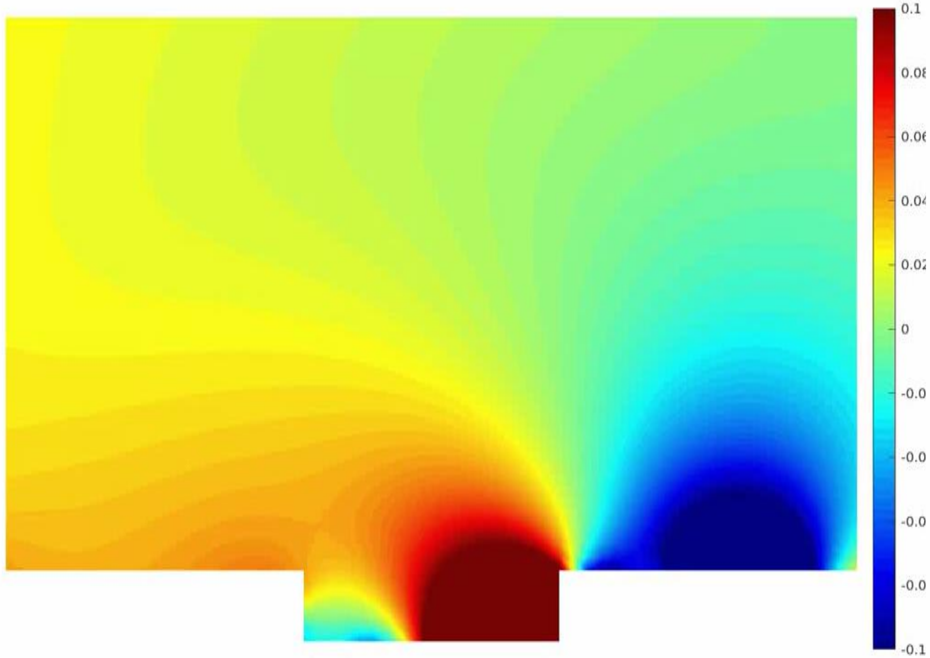




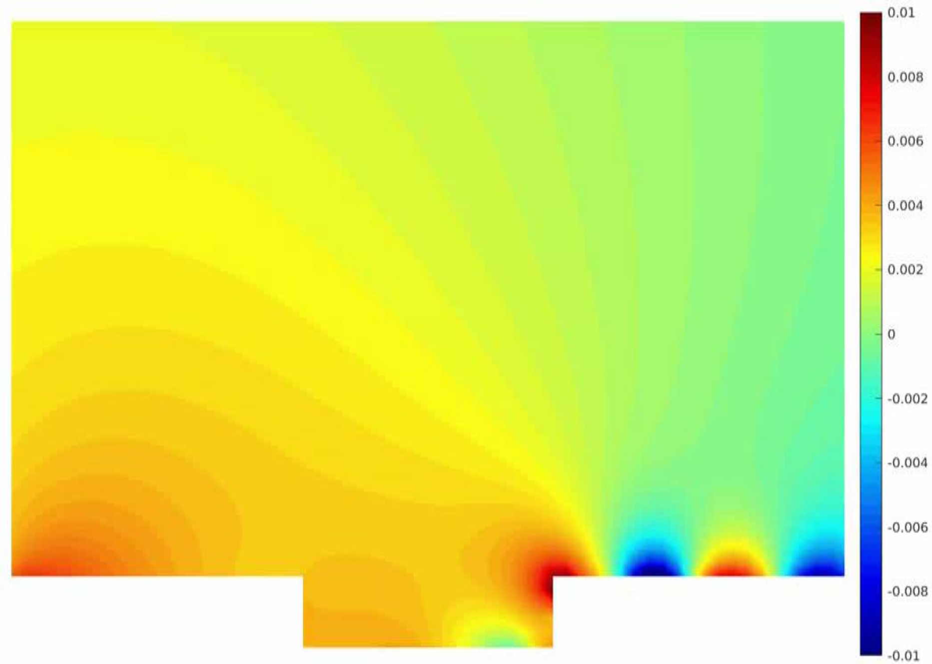
Correctly captured the main frequencies of the spectra!

Acoustic Field Results III

2D

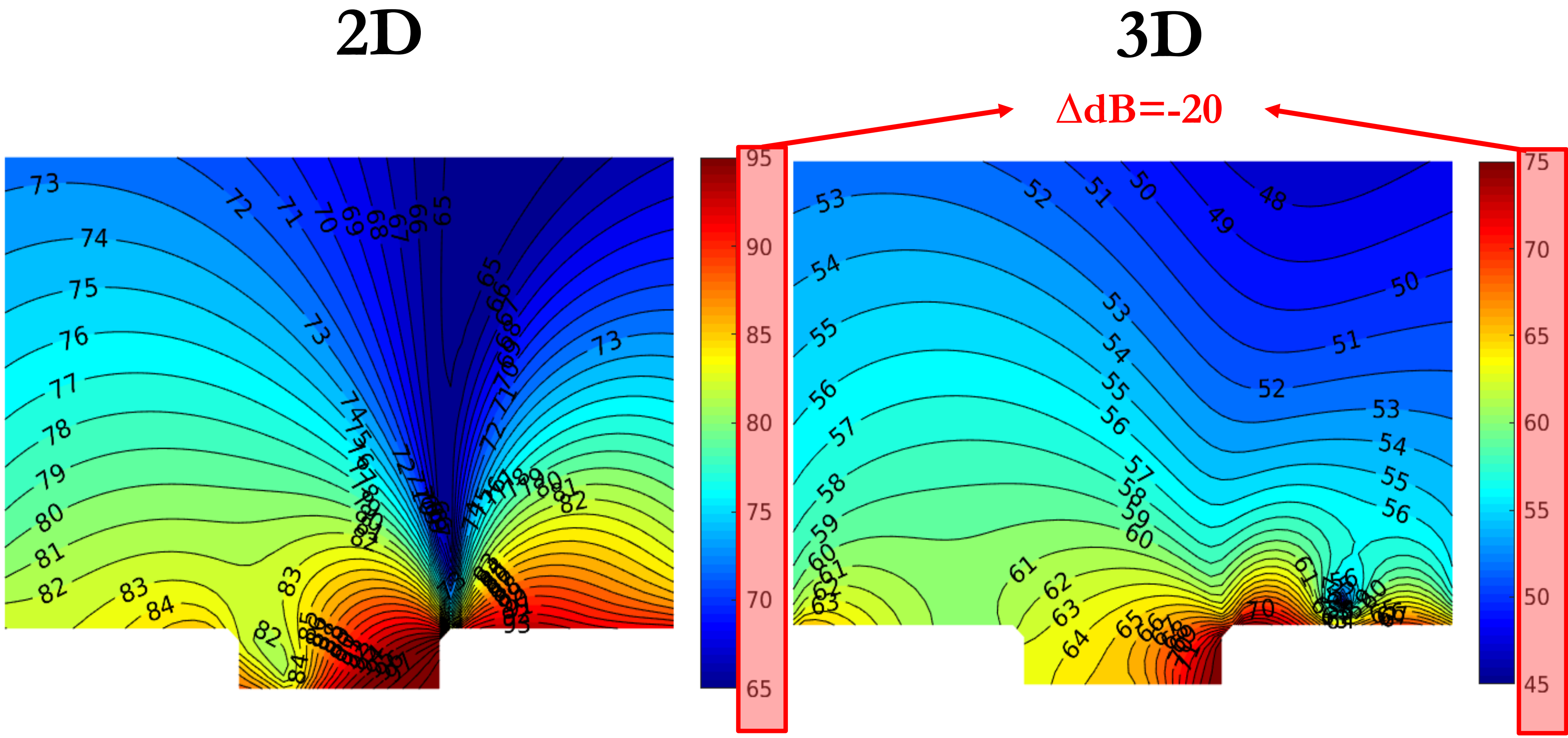


3D



$$C_{p'} = \frac{p' - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$$

Acoustic Field Results III

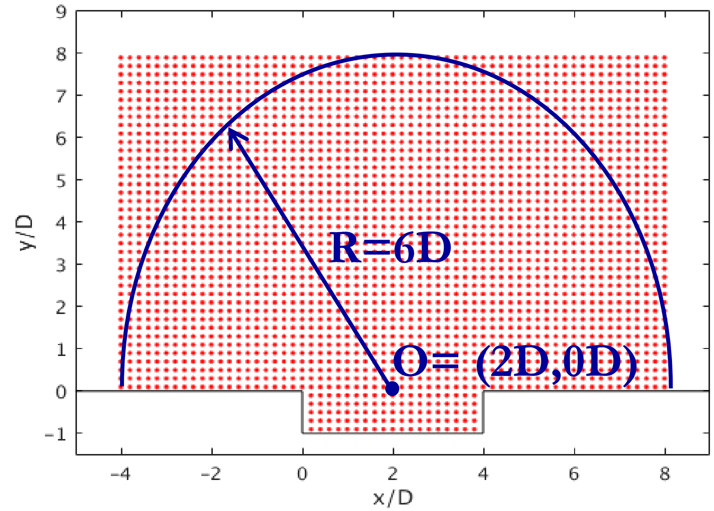


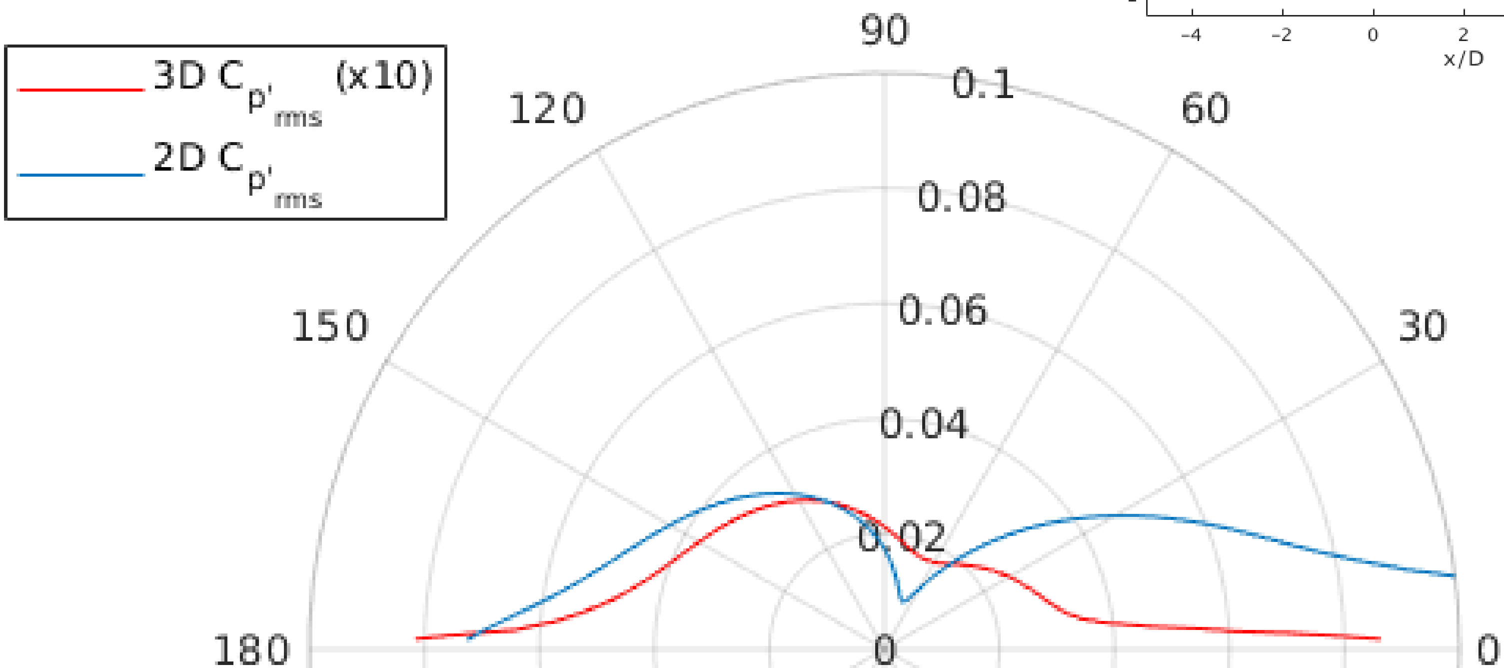
$$OASPL = 20log_{10}\left(\frac{p_{rms}}{p_{ref}}\right) \qquad p_{rms}^2 = \frac{1}{NT}\int_{t_0}^{t_0+NT} p_a(\mathbf{x},t)\cdot p_a(\mathbf{x},t)dt$$

Acoustic Field Results III

Directivity Pattern

$$C_{p'_{rms}} = \frac{p'_{rms} - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$$

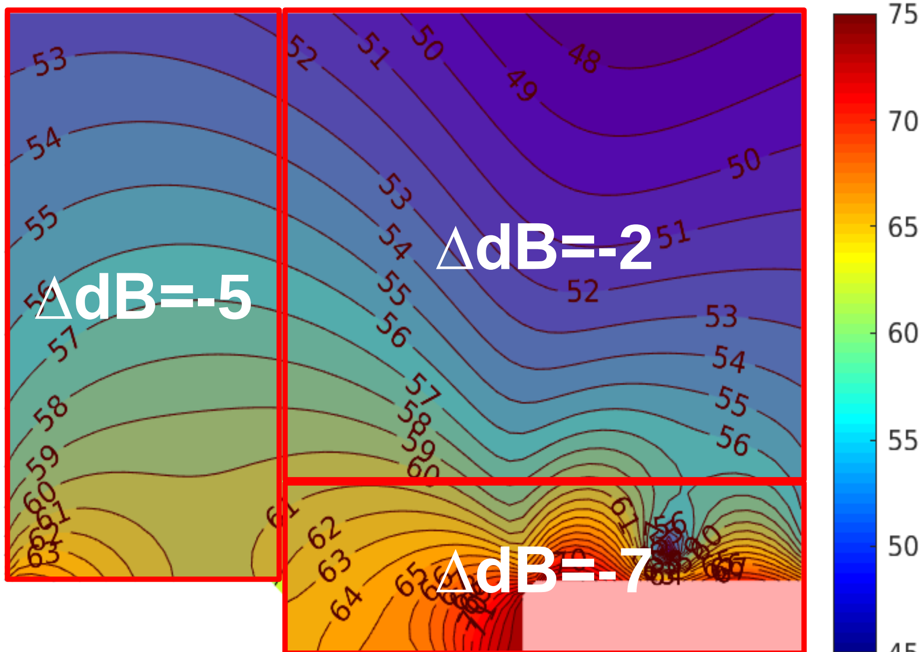


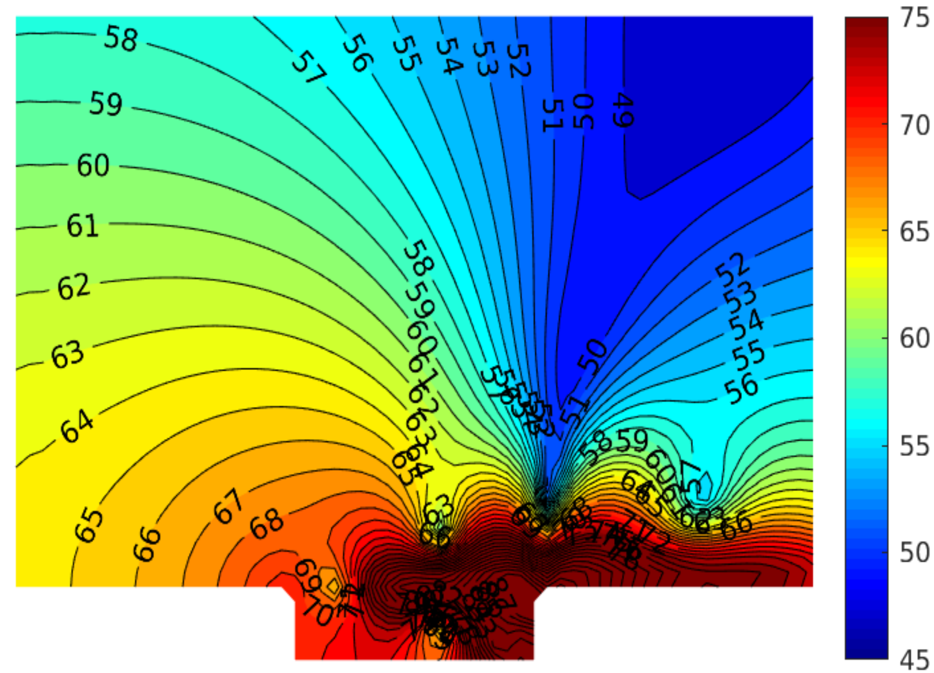


Acoustic Field Results III

Curle

DS





$$OASPL = 20log_{10}\left(\frac{p_{rms}}{p_{ref}}\right) \qquad p_{rms}^2 = \frac{1}{NT}\int_{t_0}^{t_0+NT} p_a(\mathbf{x},t)\cdot p_a(\mathbf{x},t)dt$$

Conclusions

- Two-dimensional and three-dimensional incompressible flow results are significantly different for Re= 1500.
- Due to the vortex stretching mechanism, the pressure value over the wall changes completely.
- Instantaneous $C_{p'}$ shows higher frequencies for the acoustic waves.
- $C_{p'_{rms}}$ acoustic directivities are also in disagreement.
- Three dimensional OASPL is about 20 dB lower than two dimensional OASPL.
- Frequency spectra is in good agreement between the comparison of the Curle formulation with a direct acoustic simulation.
- Highest differences in OASPL between DS and Curle occur at turbulent area, where volumen sources are probably not negligible.

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THANK YOU FOR YOUR ATTENTION