Mismatching Eigen-mode Bases in Induct Sound Field Analysis with Inhomogeneous Mean Flow

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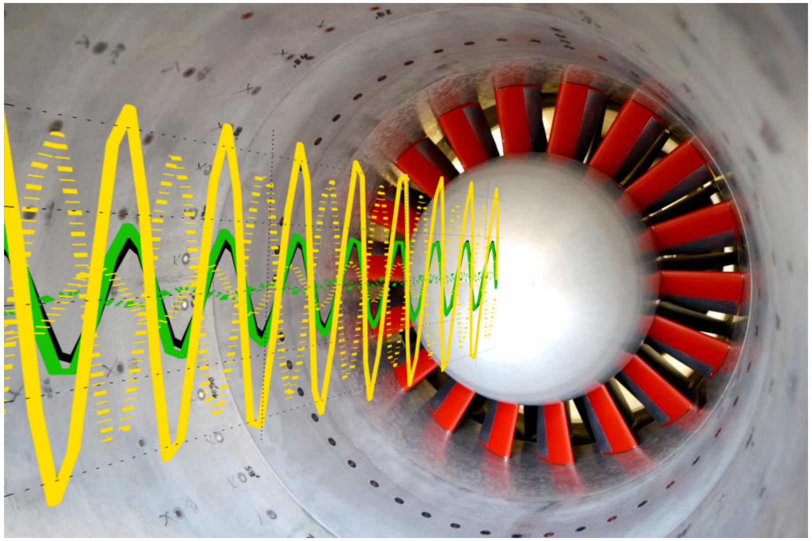
DLR

Motivation



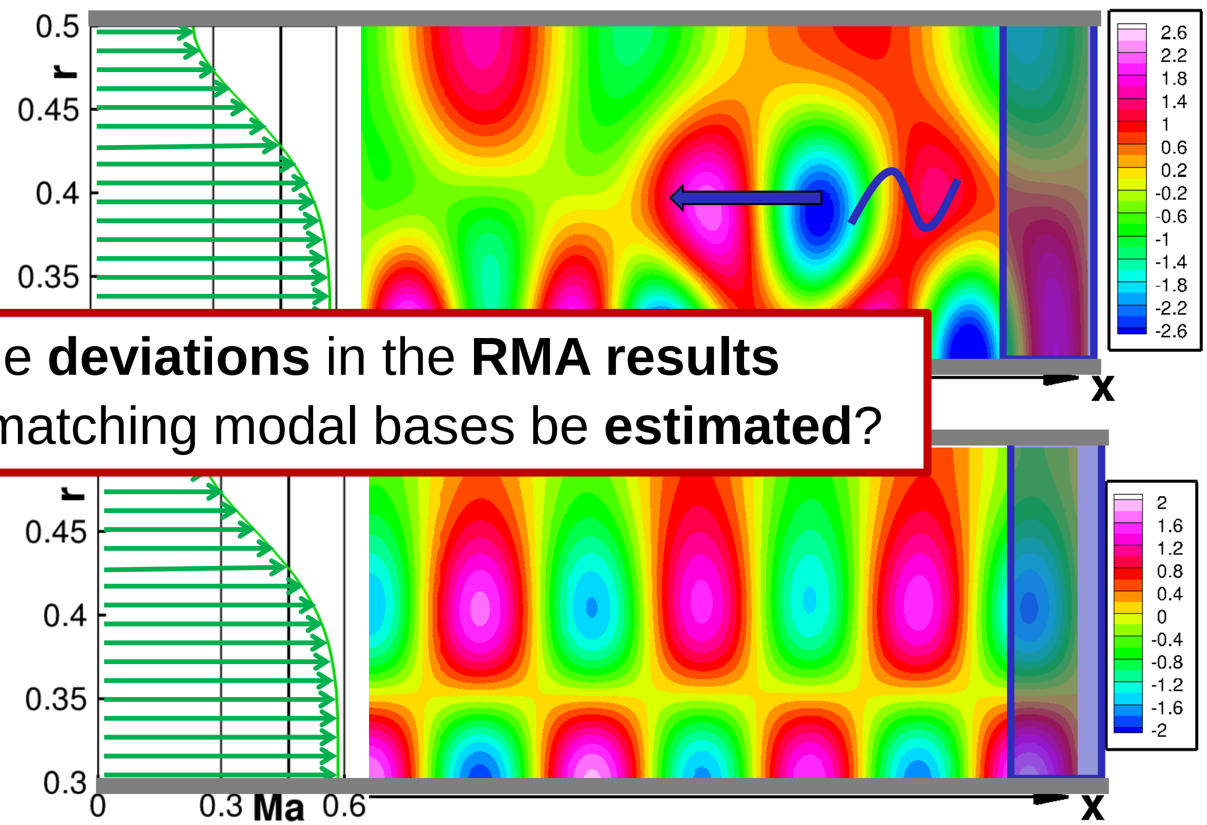
- **Radial Mode Analysis** (RMA) is used to assess **sound sources** in **turbo machines**
- The results can be used for **CAA validation**

Challenges:
- **Acoustic modal bases** for RMA depend on the **complexity** with which the **flow field** is considered
- **Flow profiles** are not always available from measurements
- Due to computational costs **simple flow profiles (e.g. Plug flow)** can be **preferred,** especially in **broad band analysis**
- **Experimental data** has **fewer data points** so mode matching is more challenging

Motivation: Sound pressure contours on a shear flow profile with different modal basis (Weckmüller et al. AIAA 2014)

Analytic modal basis

Numeric modal basis



How can the **deviations** in the **RMA results** due to mismatching modal bases be **estimated**?

Outline

1. Radial mode analysis (RMA) method

2. Properties of analytical and numerical modal bases

3. Strategies for error estimation due to mismatching modal bases

4. Test cases

5. Summary

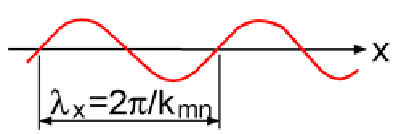
Radial Mode Analysis method

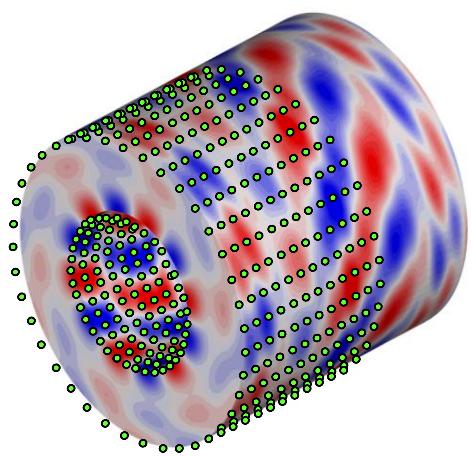
Modal basis

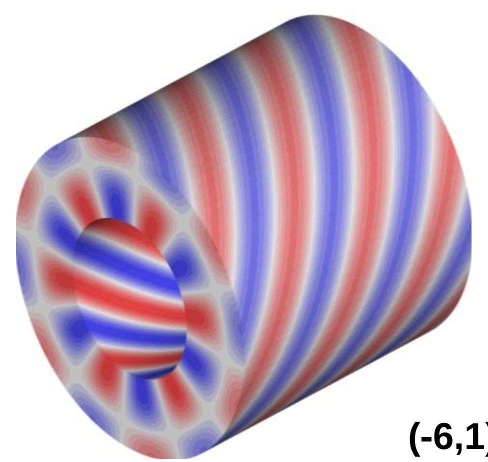
$$p(x,r,\varphi) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} a_{mn}^{\pm} \cdot e^{i(\boxed{k_{mn}^{\pm}}x+\boxed{m}\varphi)} \cdot \boxed{f_{mn}(r)}$$

$$p = W \cdot a$$

Linear system of equations







(-6,1)

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Modal Bases

	Analytic	Numeric
Closed mathematical description (no approximation/discretisation)	+	-
Fast computation	+	-
Proper flow approximation	-	+
Mathematic stability	+	o

- Numeric Modal Bases in Literature:
  - Extended Eigenvalue Problem (Kerrebrock, Kousen, Golubev and Atassi)
  - Greens Functions (Peak, Posson, Mathews)

  - Shooting method (Enghardt)

Numeric Modal Bases

- DLR tool Connect 3D is used to calculate numeric modal bases

- Numeric modal bases are calculated following the method by Kousen
  - Linearized Euler equation is solved
  - Formulation of an extended Eigenvalue problem which is solved on a Chebyshev grid using a spectral method

  - Isentropic mean flow
  - Homogeneous flow in axial and circumferential direction
  - No radial flow component
  - Hard Wall boundary conditions

Illustration (1/4) of differences in the modal bases for plug flow (PF) and a boundary layer (BL) profile

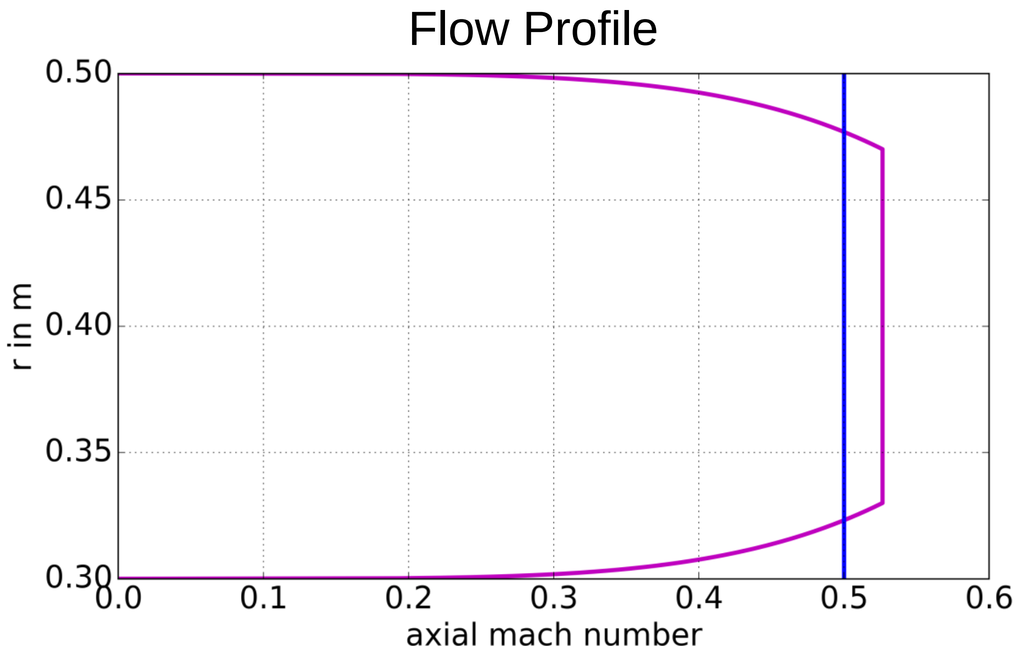
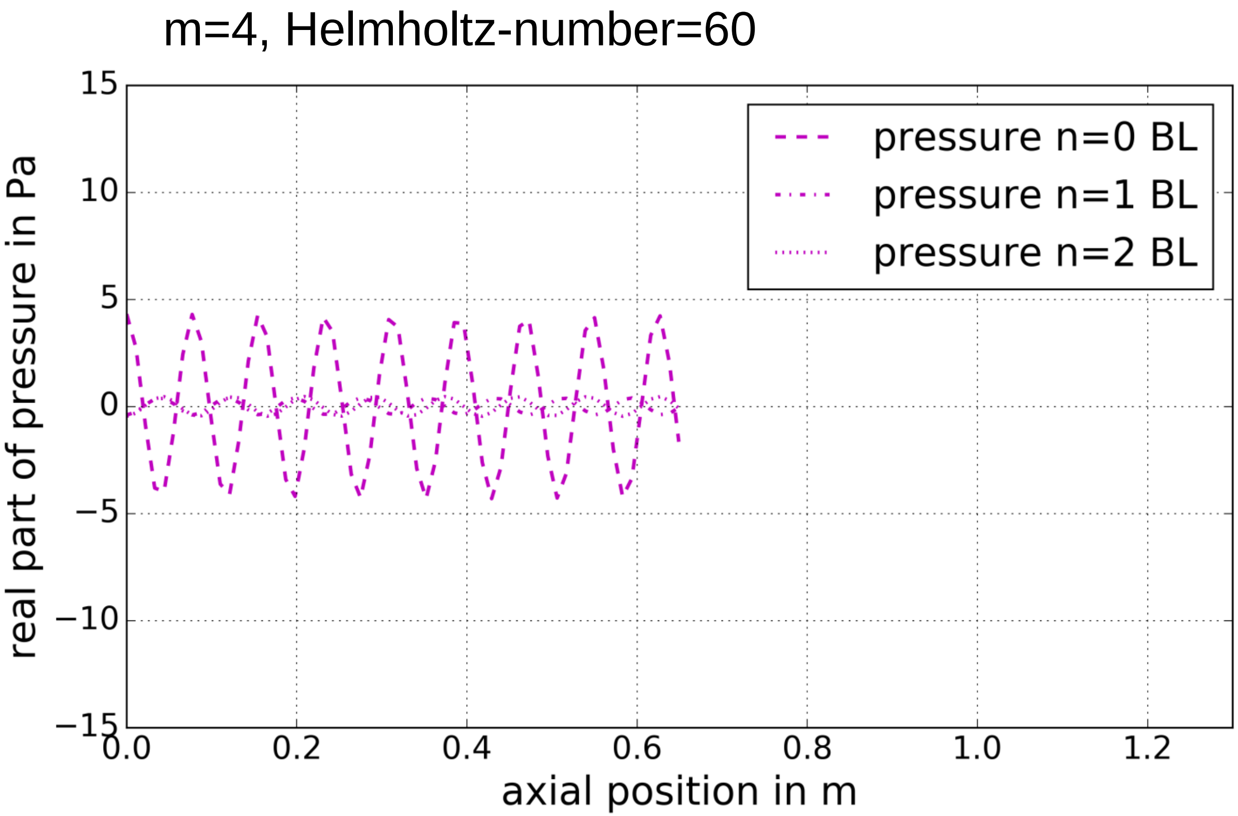
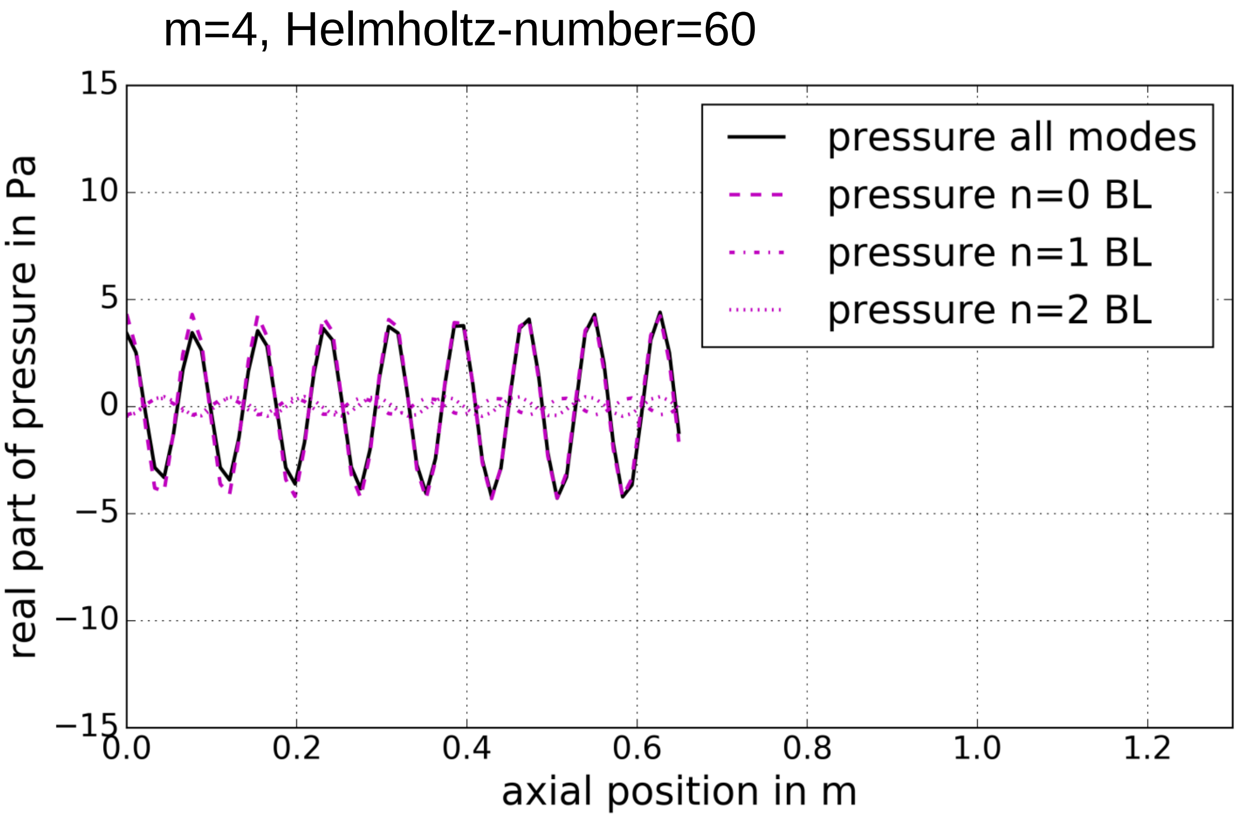


Illustration (2/4) of differences in the modal bases for plug flow (PF) and a boundary layer (BL) profile



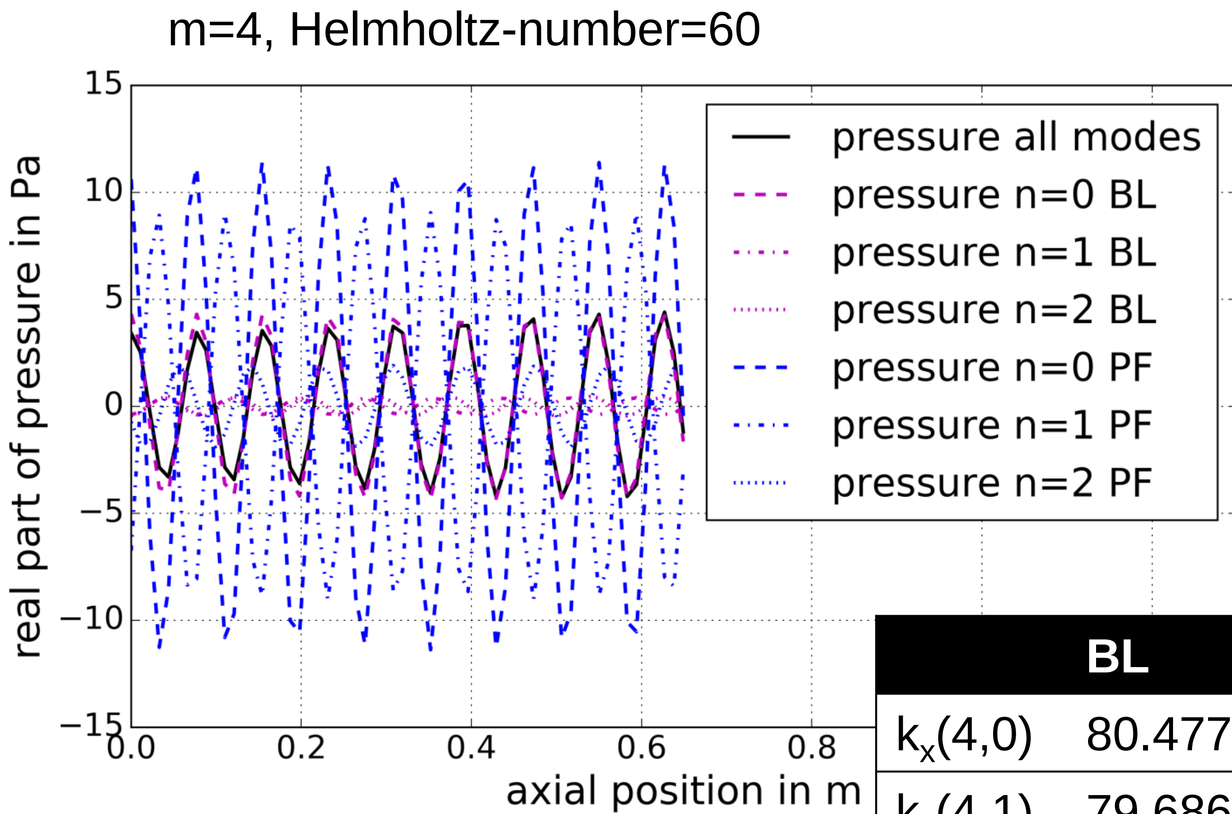
Three modes with numeric modal bases considering a BL profile are synthesized.

Illustration (3/4) of differences in the modal bases for plug flow (PF) and a boundary layer (BL) profile



In a subsequent step the resulting pressure distribution is analyzed using a plug flow (PF) modal basis.

Illustration (4/4) of differences in the modal bases for plug flow (PF) and a boundary layer (BL) profile



	BL	PF
$k_x(4,0)$	80.477	79.358
$k_x(4,1)$	79.686	78.232
$k_x(4,2)$	75.319	75.116

Downstream, axial wave numbers

Mode amplitudes are heavily overestimated by analysis with plug flow modes.

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Approaches to determine the amplification of modal amplitudes due to mismatching modal bases

1. Synthesis of Pressure Field with boundary layer (BL) profile

$$p_{syn} = W_{BL} a_{syn}$$

2. Analysis of synthesized pressure field with plug flow (PF) profile

$$a_{ana} = W_{PF}^{\dagger} p_{syn}$$

3. Evaluation of combined mode transfer matrix

$$a_{ana} = W_{PF}^{\dagger} W_{BL} a_{syn}$$

$$W_{comb} = W_{PF}^{\dagger} W_{BL}$$

Method 1:
Monte-Carlo study

Method 2:
Spectral norm of W_{comb}

Method1: Monte-Carlo based error estimation with realistic mode distribution

Relation between input and output modal amplitudes

$$a_{ana} = W_{PF}^{\dagger} W_{BL} a_{syn}$$

- Monte Carlo Study can use **random** input **mode amplitudes** for synthesis **or** contain information about the **dominance** of certain **modes.**
- The mode amplitude vector is **scaled** to a **2-norm of 1** for comparison with the output vector.
- **Statistic Analysis** gives a **probability** for the **mode amplitude deviation** due to mismatching modal bases.

Method 2: Spectral norm of combined mode transfer matrix

Relation between input and output **modal amplitudes**

$$a_{ana} = W_{PF}^{\dagger} W_{BL} a_{syn}$$

Matrix W_{comb} includes the **differences** in the **sound field models** and the **sampling positions** of the sound pressure

$$W_{comb} = W_{PF}^{\dagger} W_{BL}$$

Lambda is related to the spectral norm of the matrix A and gives **maximum** and **minimum amplification** due to mismatching modal bases

$$\lambda_{min} \leq x \left(W_{comb}^{H} W_{comb} \right) x \leq \lambda_{max}$$

$$\|x\| = 1$$

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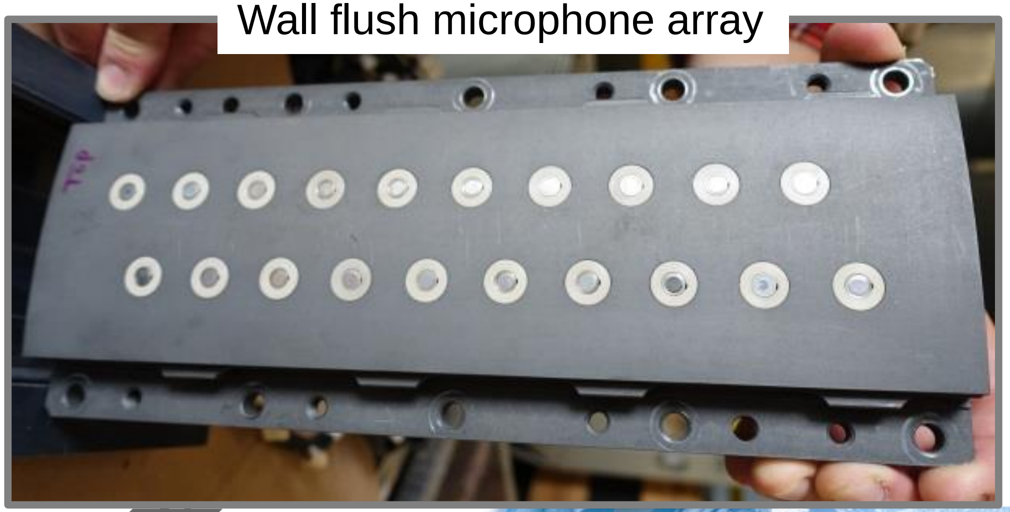
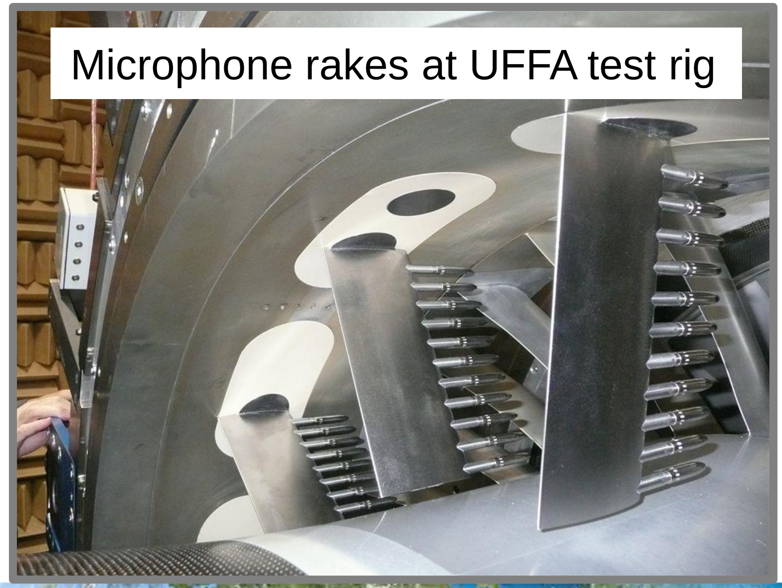
Parameters for the study three typical microphone arrays

Grid	Nx	Nr	Nphi
A (Tip only)	60	1	180
B (Hub and tip)	20	2	180
C (Rake)	3	10	180



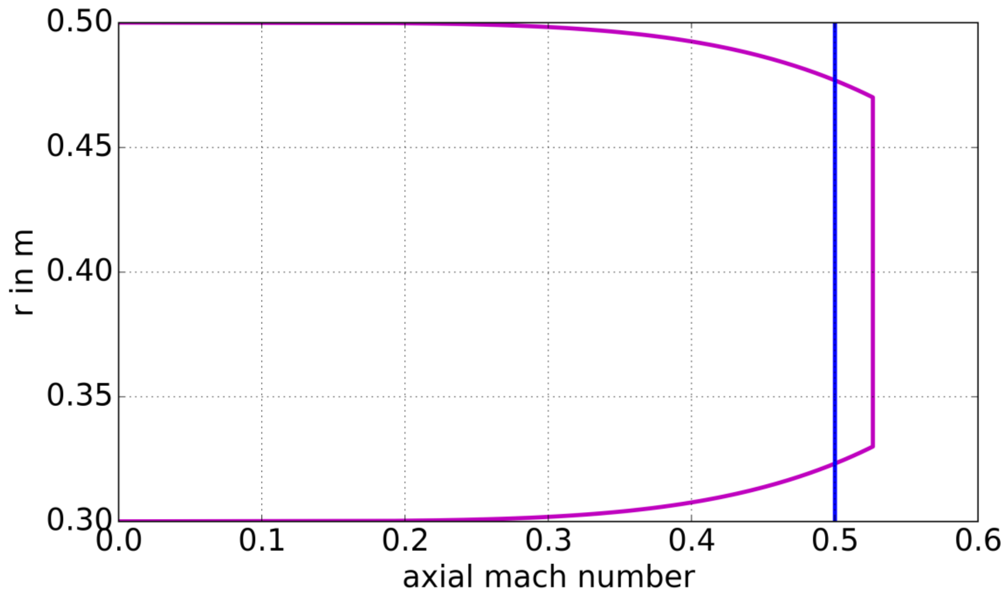
Wall flush microphone array



Microphone rakes at UFFA test rig

Parameters for the study

- Hub to tip ratio = 0.6
- Plug flow Mach number = 0.5
- Symmetric 1/5th power law boundary layer profile with a thickness of 15%
- Frequency range Helmholtz numbers 1-90



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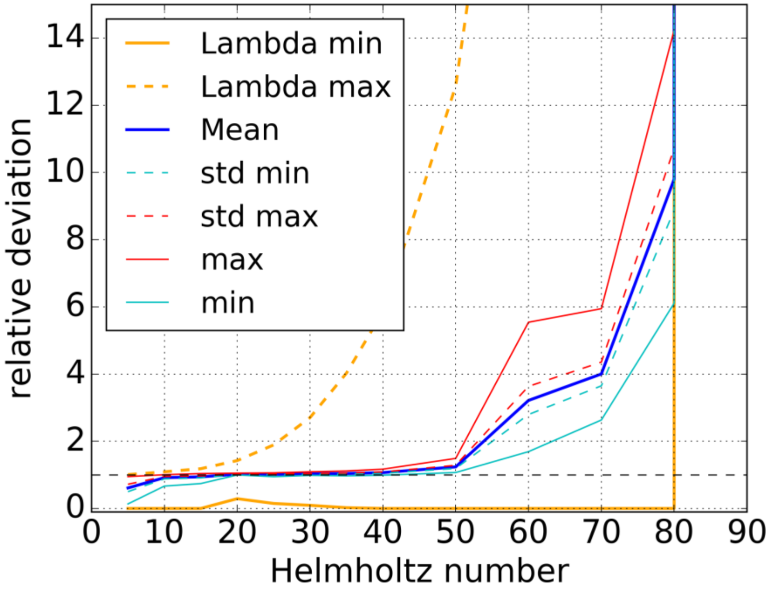
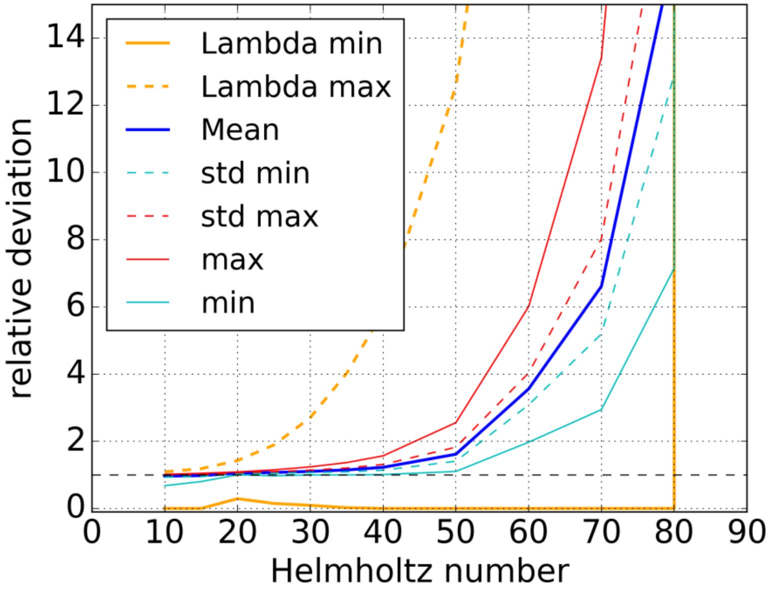
Distribution of probabilities of amplification factors (Grid A – Tip only)

- Downstream modes 10dB above upstream



- Downstream modes 10dB above upstream
- Dominant modes are present ((-6,0); (-4,0); (4,0); (8,0))



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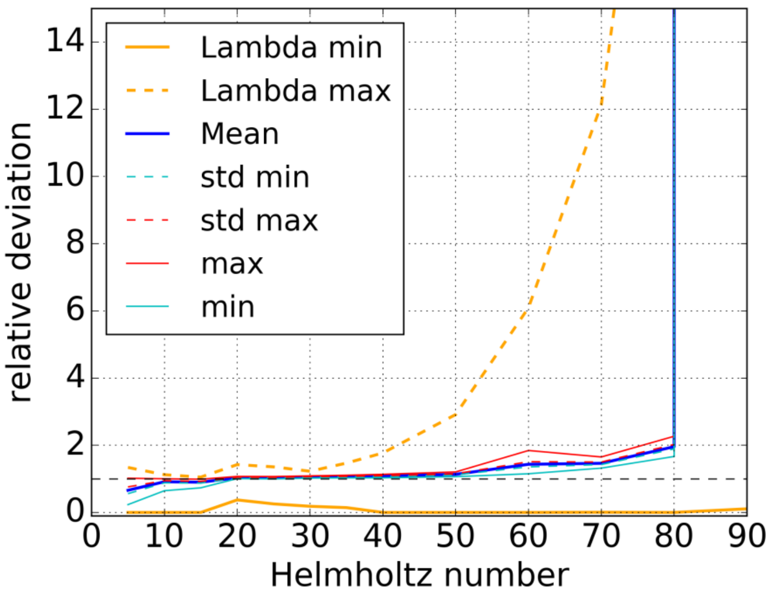
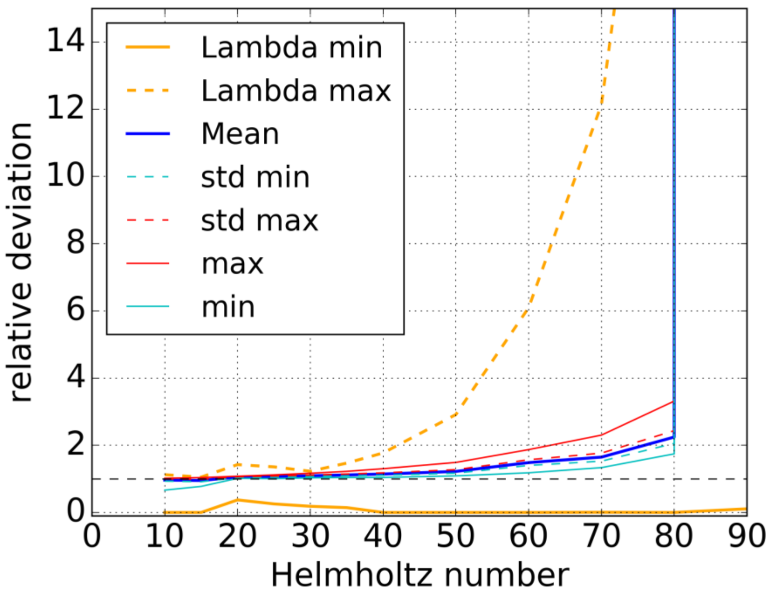
Distribution of probabilities of amplification factors (Grid B – Tip and hub)

- Downstream modes 10dB above upstream

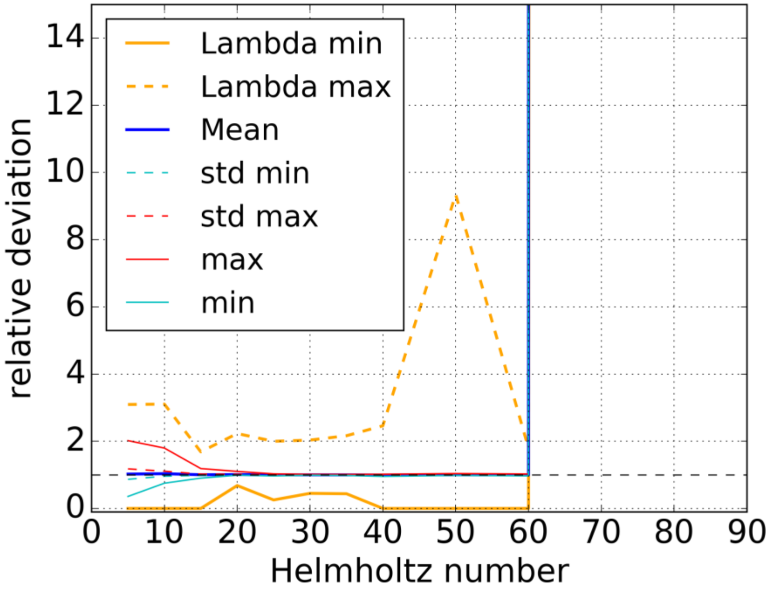


- Downstream modes 10dB above upstream
- Dominant modes are present ((-6,0); (-4,0); (4,0); (8,0))

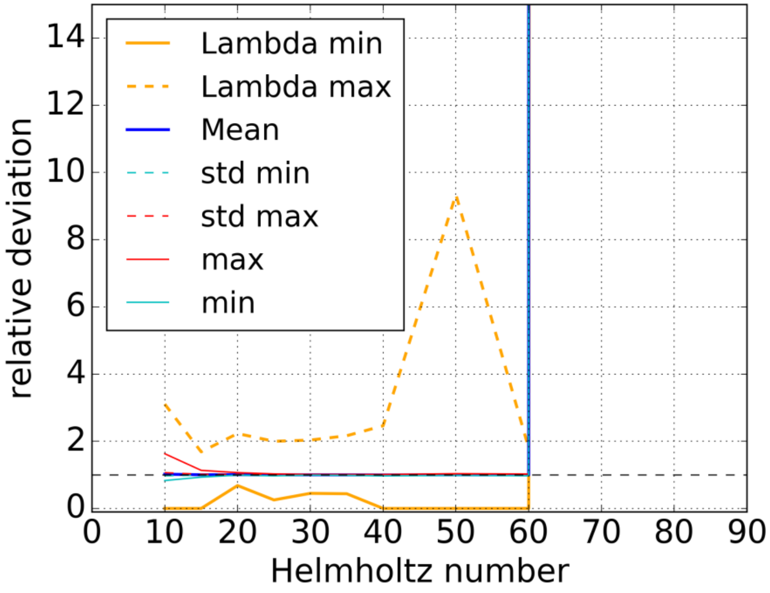


Distribution of probabilities of amplification factors (Grid C - Rake)

- Downstream modes 10dB above upstream



- Downstream modes 10dB above upstream
- Dominant modes are present ((-6,0); (-4,0); (4,0); (8,0))



Summary

- **Two methods** for the estimation of amplification of modal amplitudes due to mismatching modal bases have been presented
  - **Monte-Carlo study** is computationally expensive but gives a good estimation for realistic mode distribution
  - **Spectral norm** gives **absolute errors** and is **not dependent** on assumptions regarding the **mode distribution**.

- Methods can be used:
  - In **the design of microphone arrays** prior to tests
  - To **calculate a trust interval** for the RMA results**,** also when **measurement data** is used for **CAA validation**

- Already **Small variations** of the **axial wave numbers** can lead to **large deviations** of the **modal amplitudes**

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The work was supported within the Luftfahrtforschungsprogramm V by the German Federal Ministry for Economic Affairs and Energy.

Thank you for your interest!

Time for questions

End of presentation