Computational Experiment in AeroAcoustics - CEAA 2018

# Adjoint-based Sound Source Identification

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# Sound Source Identification

## As data assimilation problem

Experimental analysis ↓

Numerical analysis ↓

Adaption of a numerical model to experimental data

- Euler/Navier-Stokes-equations as numerical model
- Microphone measurements provide experimental data

# Sound Source Identification

As data assimilation problem

Experimental analysis

↓

Numerical analysis

↓

Adaption of a numerical model to experimental data

$$N(q,t) = f(t)$$

$$\min(J) = \min\left(\iint (q - q_{\exp})^2 \, d\Omega\right)$$

# Adjoint-based Data Assimilation

## Introductory example

### Objective function

$$J = \frac{1}{2} \iint \left(q - q_{\mathrm{exp}}\right)^2 \mathrm{d}\Omega$$

$$\delta J = \iint \underbrace{\left(q - q_{\mathrm{exp}}\right)}_{g} \delta q \, \mathrm{d}\Omega$$

# Adjoint-based Data Assimilation

## Introductory example

**Objective function**

$$J = \frac{1}{2}\iint \left(q - q_{\mathrm{exp}}\right)^2 \mathrm{d}\Omega$$

$$\delta J = \iint \underbrace{\left(q - q_{\mathrm{exp}}\right)}_{g} \delta q \, \mathrm{d}\Omega$$

Constrained optimisation

**Linearised Euler equations**

$$N\delta q = \delta f$$

# Adjoint-based Data Assimilation

## Introductory example

### Lagrangian approach

$$\delta J = g^T \delta q - (q^*)^T \underbrace{(N\delta q - \delta f)}_{=0}$$

# Adjoint-based Data Assimilation

## Introductory example

**Lagrangian approach**

$$\begin{aligned} \delta J &= g^T \delta q - (q^*)^T \underbrace{(N\delta q - \delta f)}_{=0} \\ &= \delta q^T \underbrace{(g - N^T q^*)}_{=0} + (q^*)^T \delta f \end{aligned}$$

# Adjoint-based Data Assimilation

## Introductory example

**Lagrangian approach**

$$\begin{aligned}\delta J &= g^T \delta q - (q^*)^T \underbrace{(N\delta q - \delta f)}_{=0} \\ &= \delta q^T \underbrace{(g - N^T q^*)}_{=0} + (q^*)^T \delta f\end{aligned}$$

Change of $J$ becomes simply

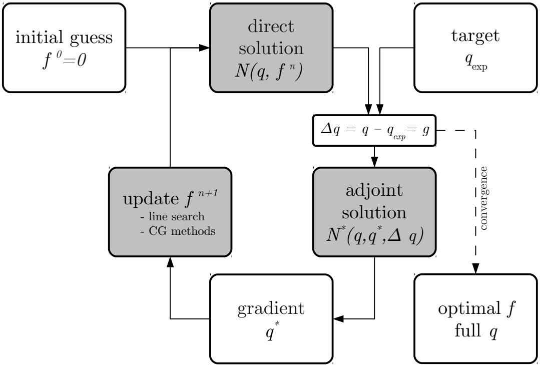
$$\delta J = {q^*}^T \delta f \rightarrow \frac{\delta J}{\delta f} = q^* \approx \nabla_f J$$

# Adjoint-based Data Assimilation

## Iterative procedure

### Optimal change of $f$

$$f^{n+1} = f^n + \nabla_f J$$



# Adjoint-based Data Assimilation

Source positions

## Adjoint sensitivity

$$\hat{q} = \sum_{t_{n=0}}^{t_{n=end}} |q^*|$$

- In case of $f^0 = 0$ the first adjoint solution $q^*$ provides information on spatial locations with maximum impact on the objective
- This locations correspond to optimal source positions

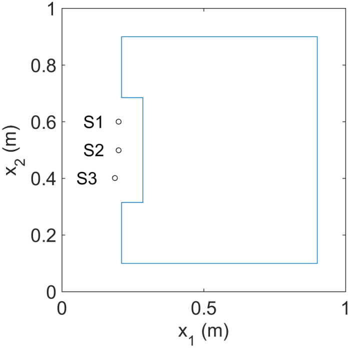
# Example I – Simplified Line-Array (I)

## Motivation


- Cooperation with Fachgebiet Audiokommunikation, TU Berlin (Prof. Weinzierl)

# Example I – Simplified Line-Array (I)

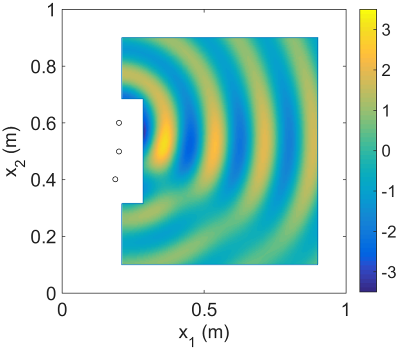
Setup




- 2D example
- 3× Speaker
  - 2 kHz sine
  - 4:2:1 Amplitude ratio
  - 2 samples phase delay ($f_s$ = 160 kHz)
- Quadratic measurement region

# Example I – Simplified Line-Array (I)

Reference sound field : $p - p_\infty$ in (Pa)



# Example I – Simplified Line-Array (I)

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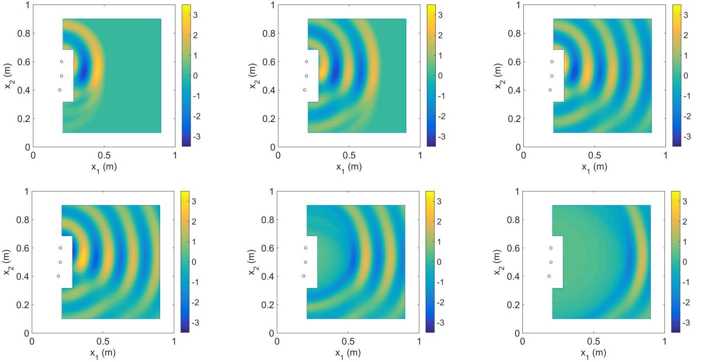
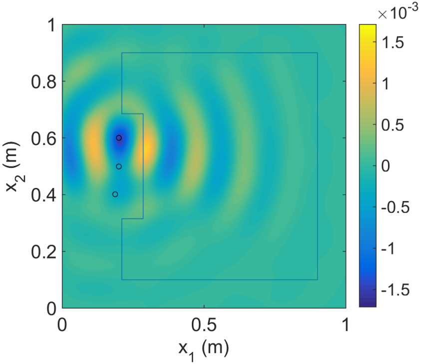


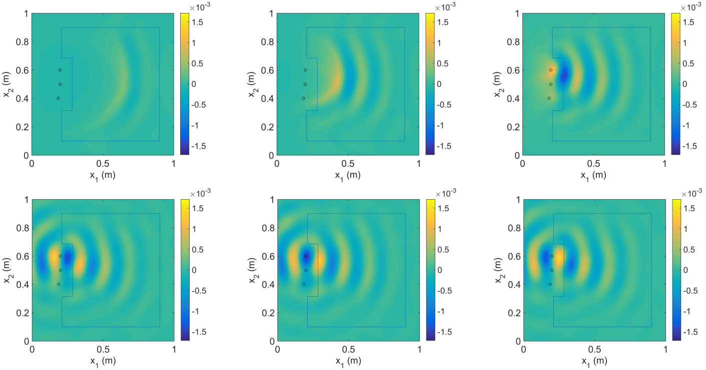
# Example I – Simplified Line-Array (I)

## Adjoint solution : $p^*$



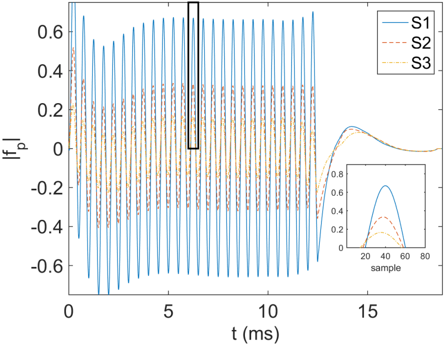
# Example I – Simplified Line-Array (I)

## Adjoint solution : p*



# Example I – Simplified Line-Array (I)

Optimal speaker output (30 iterations)

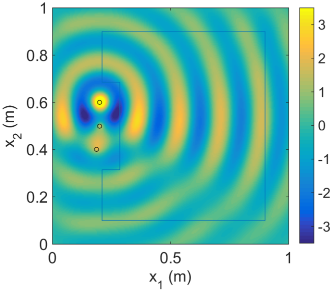


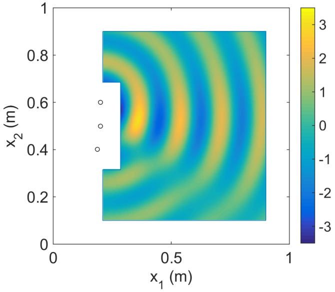

- 2 kHz sine
- 4:2:1 amplitude ratio
- 2 samples phase delay

✓ preset input signal and drive identified

# Example I – Simplified Line-Array (I)

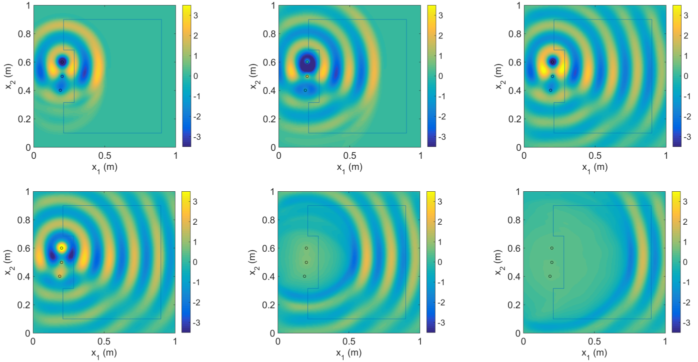
Resulting sound field (30 iterations) : $p - p_\infty$ in (Pa)



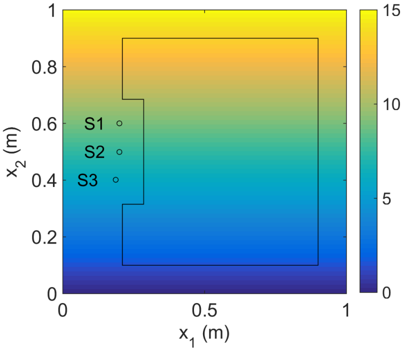
# Example I – Simplified Line-Array (I)

Resulting sound field (30 iterations) : $p - p_\infty$ in (Pa)



# Example I – Simplified Line-Array (II)

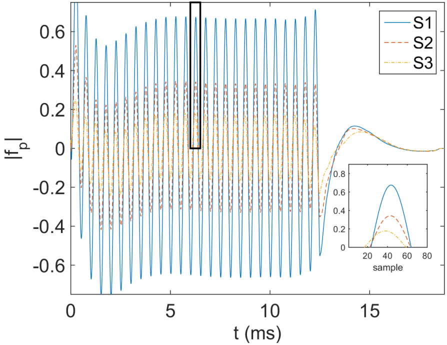
Setup with base-flow : $u_1$ in (m/s)




- same arrangement
- same target sound field
- **base flow with linear velocity profile**

# Example I – Simplified Line-Array (II)

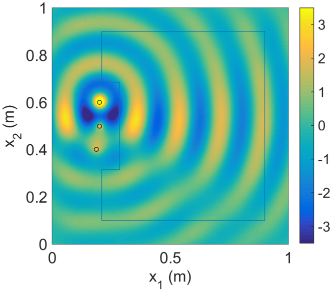
Optimal speaker output (30 iterations)

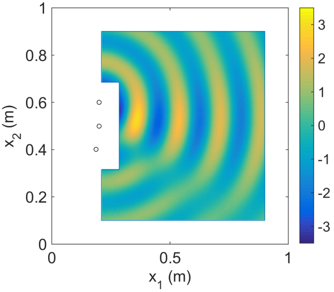



- 2 kHz sine
- similar 4:2:1 amplitude ratio
- changed phase delay (1; 5 samples)

# Example I – Simplified Line-Array (II)

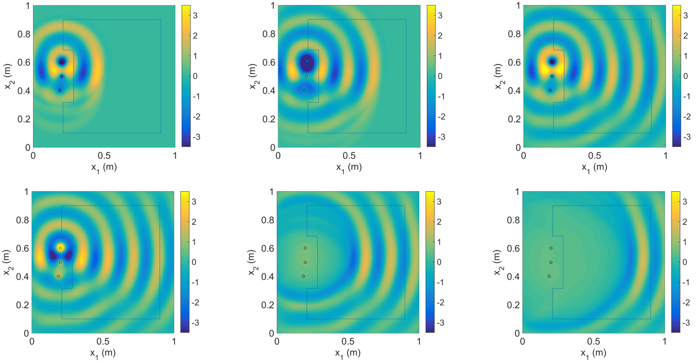
Resulting sound field (30 iterations) : $p - p_\infty$ in (Pa)



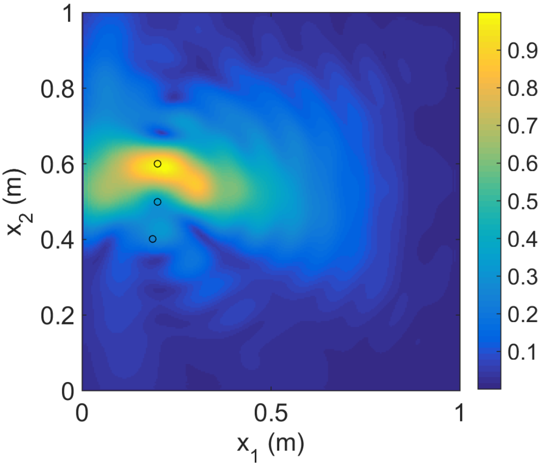
# Example I – Simplified Line-Array (II)

Resulting sound field (30 iterations) : $p - p_\infty$ in (Pa)



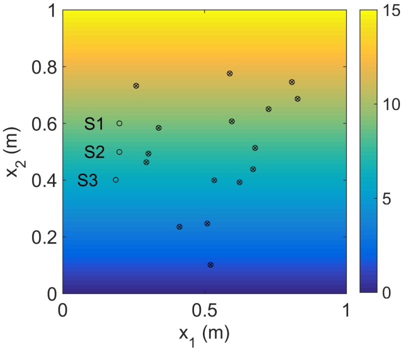
# Example I – Simplified Line-Array (II)

## Source arrangement



# Example I – Simplified Line-Array (III)

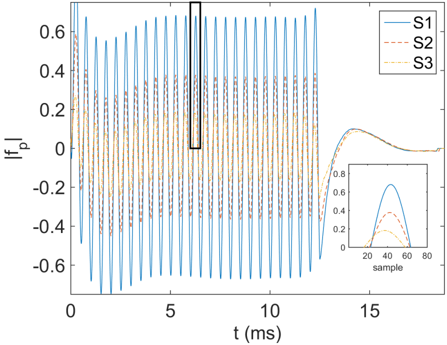
Setup with base-flow : $u_1$ in (m/s) and discrete measurement positions




- same arrangement
- same target sound field
- base flow with linear velocity profile
- **discrete measurement positions**

# Example I – Simplified Line-Array (III)

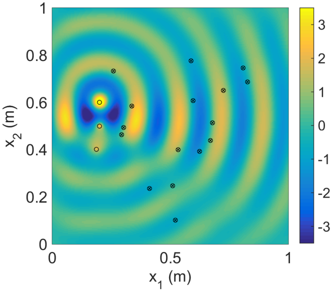
Optimal speaker output (30 iterations)

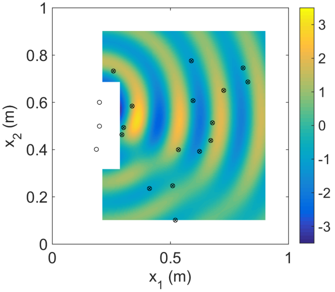



- 2 kHz sine
- similar 4:2:1 amplitude ratio
- changed phase delay (1; 5 samples)

# Example I – Simplified Line-Array (III)

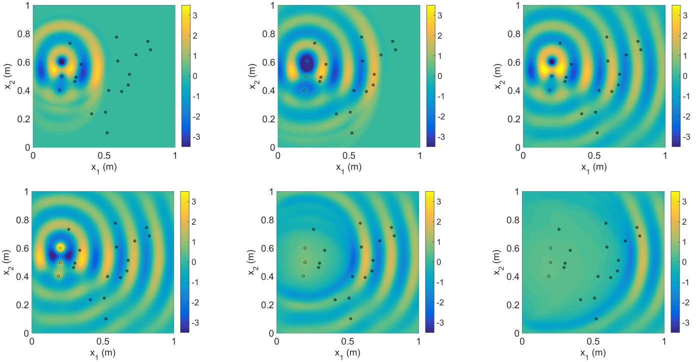
Resulting sound field (30 iterations) : $p - p_\infty$ in (Pa)



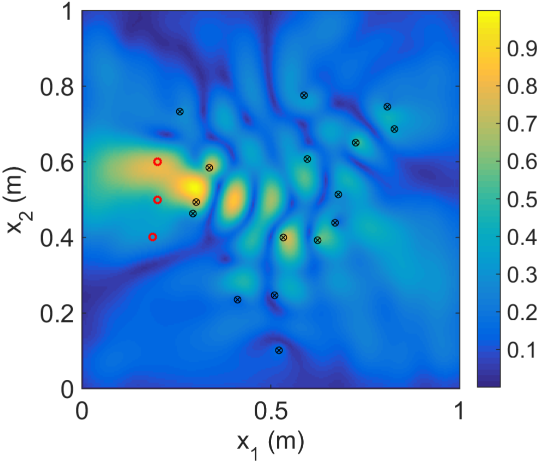
# Example I – Simplified Line-Array (III)

Resulting sound field (30 iterations) : $p - p_\infty$ in (Pa)



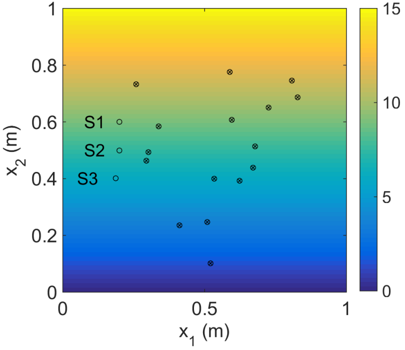
# Example I – Simplified Line-Array (III)

Source arrangement



# Example I – Simplified Line-Array (IV)

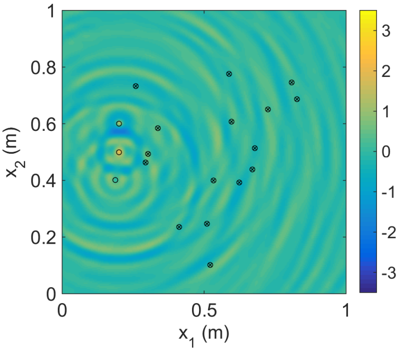
Setup with base-flow : $u_1$ in (m/s), discrete measurement positions and non-coherent reference drives




- same arrangement
- **target sound field based on white noise**
- base flow with linear velocity profile
- same discrete measurement positions

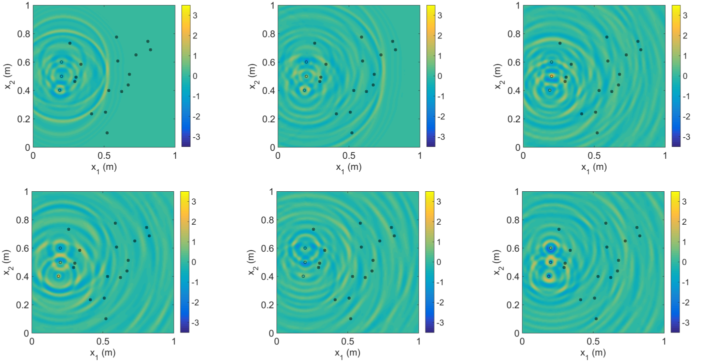
# Example I – Simplified Line-Array (IV)

Reference sound field : $p - p_\infty$ in (Pa)



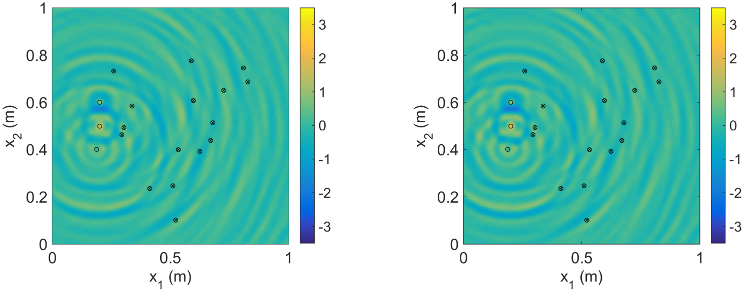
# Example I – Simplified Line-Array (IV)

## Reference sound field : $p - p_\infty$ in (Pa)



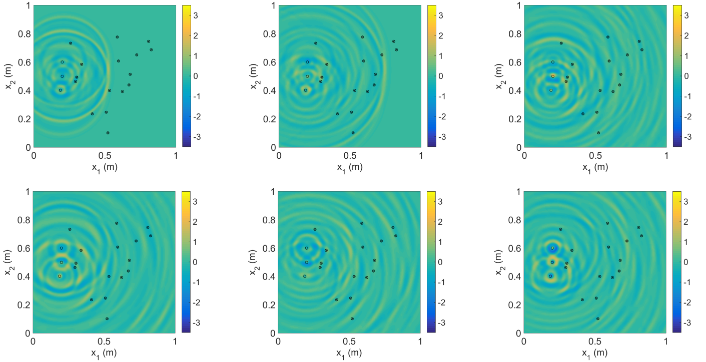
# Example I – Simplified Line-Array (IV)

Resulting sound field (30 iterations) : $p - p_\infty$ in (Pa)



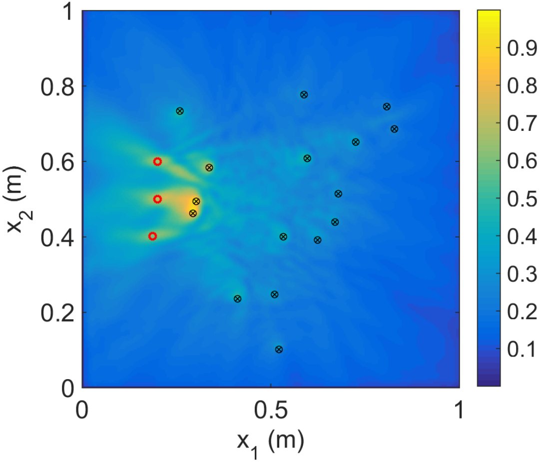
# Example I – Simplified Line-Array (IV)

Resulting sound field (30 iterations) : $p - p_\infty$ in (Pa)



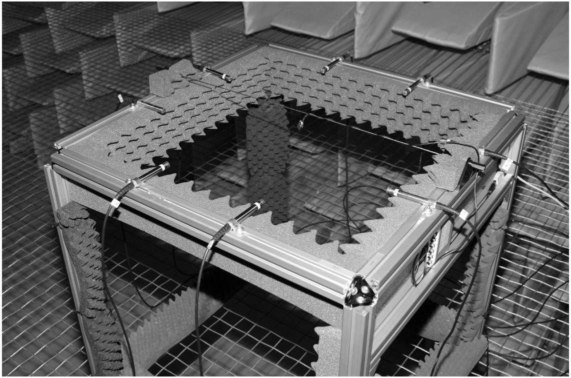
# Example I – Simplified Line-Array (IV)

## Source arrangement



# Example II – Experimental Test

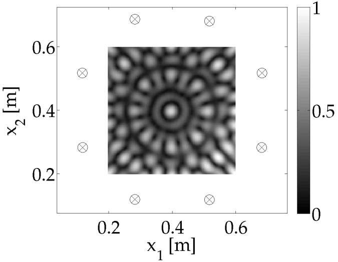
## Setup

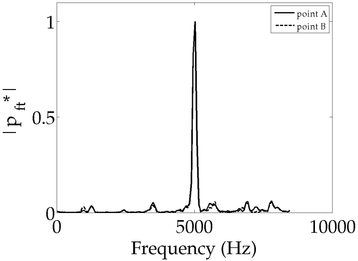


Microphones: 1 2 3 4 5 6 7 8

# Example II – Experimental Test

## Optimal speaker output




The speaker is no monopole. Momentum source required.

# Conclusion

## Summary

- ✓ Source identification as optimization/data assimilation problem
- ✓ Consideration of base-flow (thermal-stratification, walls, ...)
- ✓ First numerical and experimental validation
- Quality of source localization depends of sound field and number of microphones

# Conclusion

## Summary

- ✓ Source identification as optimization/data assimilation problem
- ✓ Consideration of base-flow (thermal-stratification, walls, ...)
- ✓ First numerical and experimental validation
- Quality of source localization depends of sound field and number of microphones

## Next steps

- Experimental analysis with base-flow
- Reconstruction of base-flow properties
- Looking for cooperation

Thank you.

Appendix

(1) Adjoint equations - detailed derivation

(2) Inclusion of walls and rectangular arrangement of speakers

# Derivation of the adjoint equations

**Euler equations**

$$\partial_t \begin{pmatrix} \varrho \\ \varrho u_j \\ \frac{p}{\gamma-1} \end{pmatrix} + \partial_{x_i} \begin{pmatrix} \varrho u_i \\ \varrho u_i u_j + p\delta_{ij} \\ \frac{u_i p \gamma}{\gamma-1} \end{pmatrix} - u_i \partial_{x_i} \begin{pmatrix} 0 \\ 0 \\ p \end{pmatrix} = f$$

$$\partial_t a + \partial_{x_i} b^i + C^i \partial_{x_i} c = f.$$

# Derivation of the adjoint equations

## Linerisation

$$\partial_t \underbrace{\frac{\partial a_\alpha}{\partial q_\beta}}_{A} \delta q_\beta + \partial_{x_i} \underbrace{\frac{\partial b_\alpha}{\partial q_\beta}}_{B^i} \delta q_\beta + C^i \partial_{x_i} \delta q_\beta + \delta C^i \partial_{x_i} c_\beta = \delta f$$

$q = [\varrho, u_j, p]$

# Derivation of the adjoint equations

## Resulting matrices 1/2

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ u_1 & \varrho & 0 & 0 & 0 \\ u_2 & 0 & \varrho & 0 & 0 \\ u_3 & 0 & 0 & \varrho & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\gamma-1} \end{bmatrix} \qquad B^1 = \begin{bmatrix} u_1 & \varrho & 0 & 0 & 0 \\ u_1^2 & 2\varrho u_1 & 0 & 0 & 1 \\ u_1 u_2 & \varrho u_2 & \varrho u_1 & 0 & 0 \\ u_1 u_3 & \varrho u_3 & 0 & \varrho u_1 & 0 \\ 0 & \frac{\gamma p}{\gamma-1} & 0 & 0 & \frac{\gamma u_1}{\gamma-1} \end{bmatrix}$$

$$B^2 = \begin{bmatrix} u_2 & 0 & \varrho & 0 & 0 \\ u_1 u_2 & \varrho u_2 & \varrho u_1 & 0 & 0 \\ u_2^2 & 0 & 2\varrho u_2 & 0 & 1 \\ u_2 u_3 & 0 & \varrho u_3 & \varrho u_2 & 0 \\ 0 & 0 & \frac{\gamma p}{\gamma-1} & 0 & \frac{\gamma u_2}{\gamma-1} \end{bmatrix} \qquad B^3 = \begin{bmatrix} u_3 & 0 & 0 & \varrho & 0 \\ u_1 u_3 & \varrho u_3 & 0 & \varrho u_1 & 0 \\ u_2 u_3 & 0 & \varrho u_3 & \varrho u_2 & 0 \\ u_3^2 & 0 & 0 & 2\varrho u_3 & 1 \\ 0 & 0 & 0 & \frac{\gamma p}{\gamma-1} & \frac{\gamma u_3}{\gamma-1} \end{bmatrix}$$

# Derivation of the adjoint equations

## Resulting matrices 2/2

$$C^i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -u_i \end{bmatrix} \quad \delta C^i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\delta u_i \end{bmatrix}$$

# Derivation of the adjoint equations

## Lagrangian approach

$$\iint \delta J \mathrm{d}\Omega = \iint g^T \delta q \mathrm{d}\Omega$$

$$- \iint {q^*}^T \underbrace{\left(\partial_t A \delta q + \partial_{x_i} B^i \delta q + C^i \partial_{x_i} \delta q + \delta C^i \partial_{x_i} c - \delta f\right)}_{=0} \mathrm{d}\Omega$$

$\mathrm{d}\Omega = \mathrm{d}x_i \mathrm{d}t$ is the space-time measure

# Derivation of the adjoint equations

## Integration by parts

$$\iint \delta J \mathrm{d}\Omega = \iint \delta q^T g \mathrm{d}\Omega$$

$$+ \iint \delta q^T A^T \partial_t q^* \mathrm{d}\Omega \quad - \left[ \int \delta q^T A^T q^* \mathrm{d}x_i \right]_{t=t_0}^{t=t_{\mathrm{end}}}$$

$$+ \iint \delta q^T {B^i}^T \partial_{x_i} q^* \mathrm{d}\Omega \quad - \left[ \int \delta q^T {B^i}^T q^* \mathrm{d}t \right]_{x_i=x_{i,0}}^{x_i=L_i}$$

$$+ \iint \delta q^T \partial_{x_i} {C^i}^T q^* \mathrm{d}\Omega \quad - \left[ \int \delta q^T {C^i}^T q^* \mathrm{d}t \right]_{x_i=x_{i,0}}^{x_i=L_i}$$

$$- \iint \delta q^T \tilde{C}^i \partial_{x_i} c \qquad q^*_\alpha \delta C^i_{\alpha\beta} \partial_{x_i} c_\beta = q^*_\alpha \delta q_\kappa \frac{\partial C^i_{\alpha\beta}}{\partial q_\kappa} \partial_{x_i} c_\beta$$

$$+ \iint {q^*}^T \delta f \mathrm{d}\Omega \qquad \delta q_\kappa \tilde{C}^i_{\kappa\beta} \partial_{x_i} c_\beta$$

# Derivation of the adjoint equations

## Factor out different variations

$$
\begin{aligned}
\iint \delta J \mathrm{d}\Omega = \iint & {q^*}^T \delta f \mathrm{d}\Omega \\
& + \iint \delta q^T \underbrace{\left( g + A^T \partial_t q^* + {B^i}^T \partial_{x_i} q^* + \partial_{x_i} {C^i}^T q^* - \tilde{C}^i \partial_{x_i} c \right)}_{I} \mathrm{d}\Omega \\
& \underbrace{- \left[ \int \delta q^T A^T q^* \mathrm{d}x_i \right]_{t=t_0}^{t=t_{\text{end}}}}_{II} \\
& \underbrace{- \left[ \int \delta q^T {B^i}^T q^* \mathrm{d}t \right]_{x_i=x_{i,0}}^{x_i=L_i} - \left[ \int \delta q^T {C^i}^T q^* \mathrm{d}t \right]_{x_i=x_{i,0}}^{x_i=L_i}}_{III}
\end{aligned}
$$

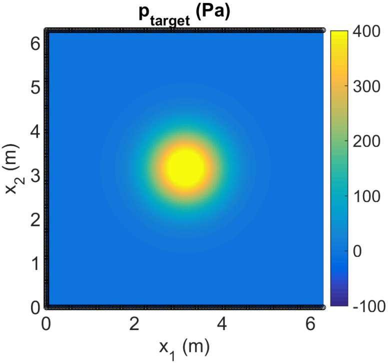
# Derivation of the adjoint equations

**Adjoint equations – Term $I$**

$$\partial_t q^* = -{A^T}^{-1} {B^i}^T \partial_{x_i} q^* - {A^T}^{-1} \partial_{x_i} {C^i}^T q^* + {A^T}^{-1} \tilde{C}^i \partial_{x_i} c - {A^T}^{-1} g.$$

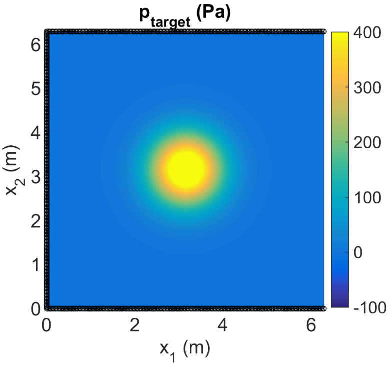
*II* and *III* define the adjoint initial- and boundary conditions

# Wall and rectangular speaker arrangement




- 2D example
- Boundaries represent speaker (585)
- Formation of a pressure pulse

# Wall and rectangular speaker arrangement




▷

# Wall and rectangular speaker arrangement