# Prediction of turbulence-cascade interaction noise using modal approach

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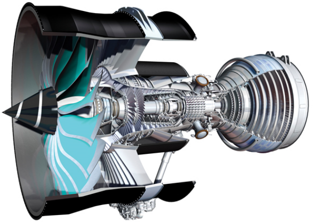
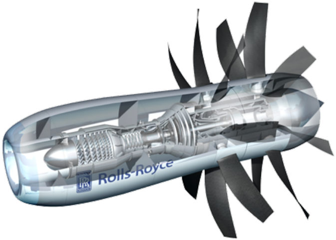
# Overview

# Turbulence-cascade interaction noise

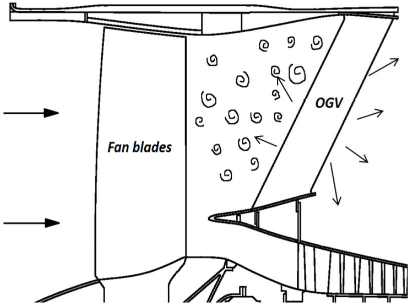
- Fan is one of the dominant noise sources of turbofan engine
- Rotor-OGV interaction is the dominant fan broadband (BB) mechanism
- Ultra high bypass ratio engine
    - Low speed fan
    - More loading on the rotor blades
    - Increase in turbulence
    - Shorter nacelle





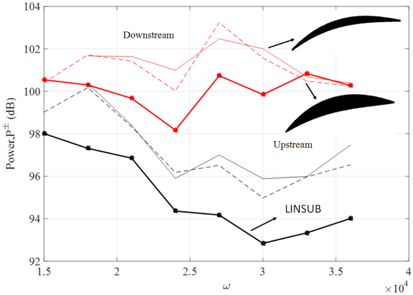
# Aim & Objective

- To synthesize rotor wake turbulence as input for linearised Navier-Stokes solver and develop a prediction method for Rotor-OGV interaction noise



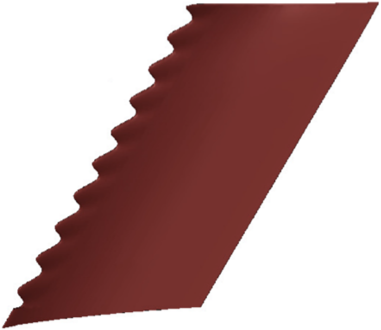
# Turbulence synthesis

- Rolls-Royce *plc.* LNS solver HYDRA is used
- Frequency domain approach
- Turbulence synthesised by sum of Fourier modes in 2D has been demonstrated
- Geometry effects on cascade interaction noise has been performed



# Nature of the flow

- Homogeneous in $\theta$ direction
- Inhomogeneous in radial direction
- Anisotropic turbulence
- Varying length scales
- Span-wise correlation effects need to be incorporated
- Correct two-point statistics are important for noise



Need to develop a generalised global approach to establish correct two-point statistics

# 3D Turbulence synthesis

Fan broadband noise is completely determined by the turbulence velocity cross-spectrum at the leading edge of OGV.

$$C(r,\theta,r',\theta',\omega) = \lim_{T\to\infty}\left\{\frac{1}{T}E\big[u(r,\theta,\omega)u^*(r',\theta',\omega)\big]\right\}$$

The main objective is to reconstruct the cross-spectral matrix by the modal summation:

$$u(r,\theta,\omega) = \sum_{m}^{\infty} u_m(r,\omega)\mathrm{e}^{\mathrm{i}m\theta}$$

Each azimuthal modal component $u_m(r,\omega)$ comprises of the sum of radial components $n$ with amplitudes $\hat{u}_{mn}(r,\omega)$, whose radial variation is described by non-dimensional functions (mode shape) $\phi_{mn}(r,\omega)$.

$$u_m(r,\omega) = \sum_{n}^{\infty} \hat{u}_{mn}(\omega)\phi_{mn}(r,\omega)$$

# 3D Turbulence synthesis

The cross-spectrum can be expressed in the theta-direction as only a function of $\Delta\theta = \theta - \theta'$, as

$$C(r, r', \Delta\theta, \omega) = \sum_{m=0}^{\infty} C_m(r, r', \omega)\mathrm{e}^{\mathrm{i}m\Delta\theta}$$

$$C_m(r, r', \omega) = \lim_{T\to\infty} \left\{ \frac{1}{T} E\left[ u_m(r, \omega) u_m^*(r', \omega) \right] \right\}$$

The cross-spectra $C_m$ in the above series expansion may be obtained by

$$C_m(r, r', \omega) = \frac{1}{2\pi} \int_0^{2\pi} C(r, r', \Delta\theta, \omega)\mathrm{e}^{-\mathrm{i}m\Delta\theta} d\Delta\theta$$

# 3D Turbulence synthesis

The modal sum is taken over a finite number of radial modes N and then can be expressed in the matrix form

$$\mathbf{u_m} = \mathbf{\Phi_m \hat{u}_m}$$

The cross-spectral matrix $C_m$, at a number of discrete radial locations for a fixed azimuthal mode number $m$ may be expressed as:

$$\mathbf{C_m} = \mathbf{\Phi_m \hat{C}_m \Phi^H}$$

$$\mathbf{\hat{C}_m} = \lim_{T\to\infty} \left\{ \frac{1}{T} E\left[ \mathbf{\hat{u}_m \hat{u}_m^H} \right] \right\}$$

# 3D Turbulence synthesis ($\mathbf{C_m} = \mathbf{\Phi_m \hat{C}_m \Phi^H}$)

The objective now is to estimate the optimum least-square cross-spectral amplitude matrix $\mathbf{\hat{C}_m}$ for a fixed azimuthal mode $m$ that provides the least squares fit to cross-spectral matrix $\mathbf{C_m}$.

The optimum cross-spectral $E\left[\mathbf{\hat{u}_m \hat{u}_m^H}\right]$ of vortical mode amplitudes may therefore be written as

$$\mathbf{\hat{C}_m} = \mathbf{\Phi}^{+}\mathbf{C_m}\mathbf{\Phi}^{+\mathbf{H}}$$

The upstream and downstream modal acoustic sound power can be expressed in terms of $\mathbf{\hat{C}_m}$ and scattering matrix $\mathbf{S_m^{\pm}}$ (from LNS solver).

# Modelling of Cross-spectral matrix

# Sample cross-spectra matrix $C(r, r', \Delta\theta, \omega)$

For simplicity we first assume the cross-spectrum follows Gaussian distribution and turbulence is assumed to be locally homogeneous.

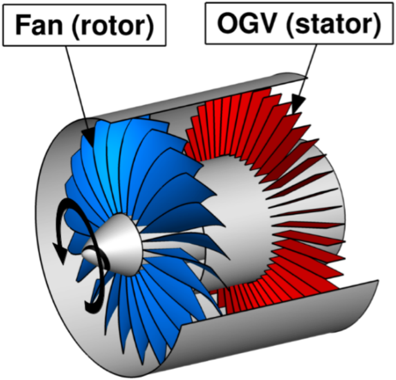
$$C(r, r', \Delta\theta, \omega) = \Big(S(r, \omega) S(r', \omega)\Big)^{0.5} \gamma\Big(\Lambda_\theta, \Lambda_r, r - r', r(\Delta\theta)\Big)$$

$$\gamma\Big(\Lambda_\theta, \Lambda_r, r - r', r(\Delta\theta)\Big) = e^{-(r-r')/\Lambda_r^2(\omega)} e^{-r(\Delta\theta)/\Lambda_\theta^2(\omega)}$$

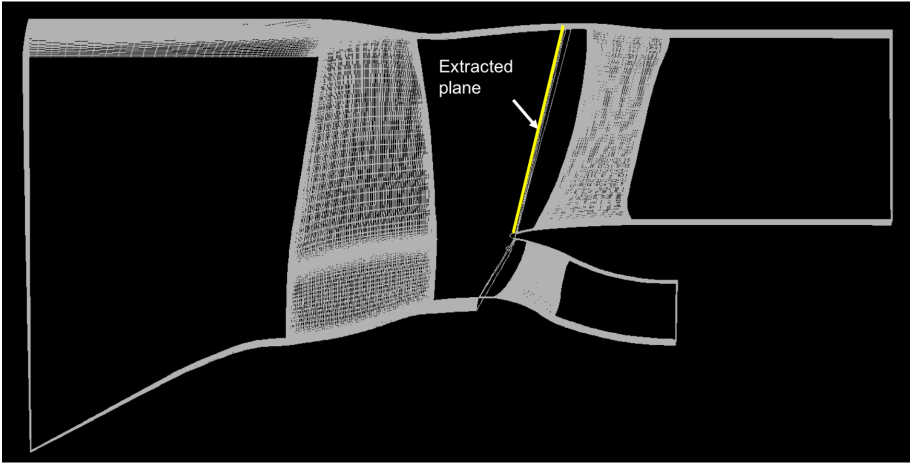
Length scales and intensity variation along the radius are extracted from RANS

# RANS mean flow

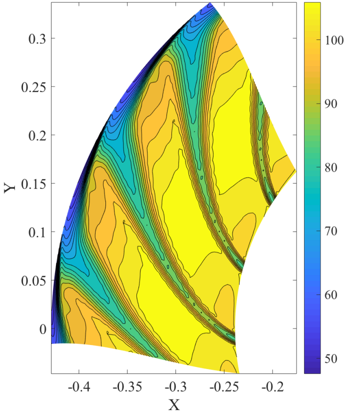
- Mesh size: $\approx$ 6.5 million cells
- Fully resolved boundary layer ($y^+ < 1$) on blades
- RANS with mixing-plane between rotor and stator blocks
- k-omega turbulence model used
- AP: AP geometry w/tip clearance of 0.78 mm
- CB, SL: hot geometry w/tip clearance of 0.2 mm



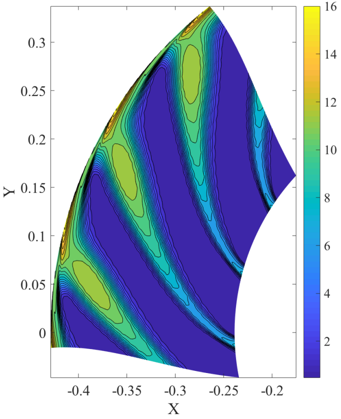
# Extraction plane



# Axial velocity and turbulent intensity contours

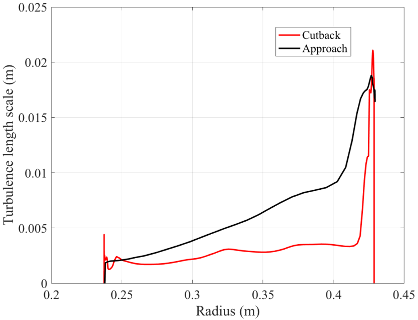



Axial velocity ($m/s$)

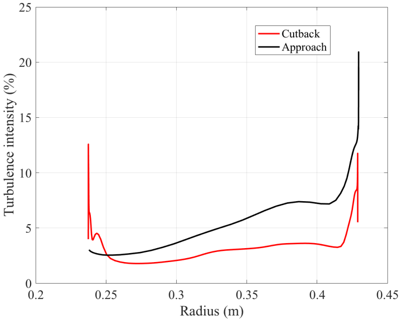



Turbulence intensity (%)

# Circumferentially averaged turbulence intensity and length scales




Turbulent length scale




Turbulence intensity

# Vortical mode shapes

# Choice of vortical mode shapes

Our work has so far considered a total three vortical modal solutions.

- Normal modes of LEE, $\Phi_{LEE}$
- Fourier modes, $\Phi_F$
- POD modes, $\Phi_{POD}$

Important requirements:

- Divergence free
- Efficient representation of cross-spectral matrix

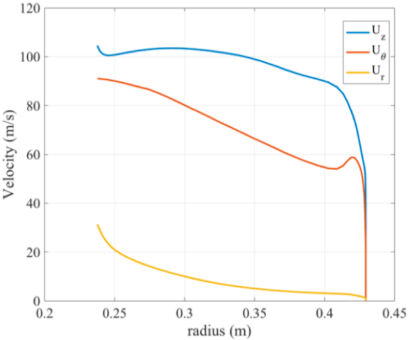
# Normal modes of LEE ($\Phi_{LEE}$)

Linearized Euler equations can be expressed in matrix form as

$$A\frac{\partial}{\partial z}\begin{bmatrix} u_r \\ u_\theta \\ u_z \\ p \end{bmatrix} = B\begin{bmatrix} u_r \\ u_\theta \\ u_z \\ p \end{bmatrix}$$

$$A = \begin{bmatrix} U_z & 0 & 0 & 0 \\ 0 & U_z & 0 & 0 \\ 0 & 0 & U_z & 1 \\ 0 & 0 & 1 & U_z \end{bmatrix}$$

$$B = \begin{bmatrix} -i\omega + \frac{imU_\theta}{r} & \frac{U_\theta}{r} & 0 & -D_r \\ -\frac{U_\theta}{r} - (D_r U_\theta) & -i\omega + \frac{imU_\theta}{r} & 0 & \frac{im}{r} \\ -(D_r U_z) & 0 & -i\omega + \frac{imU_\theta}{r} & 0 \\ -\frac{1}{r} - D_r & \frac{im}{r} & 0 & -i\omega + \frac{imU_\theta}{r} \end{bmatrix}$$



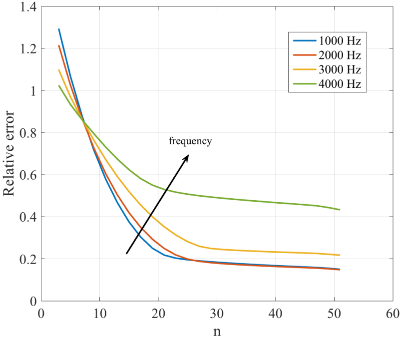

where

Solution to the Eigenvalue equation are the normal modes of LEE ($\Phi_{LEE}$)

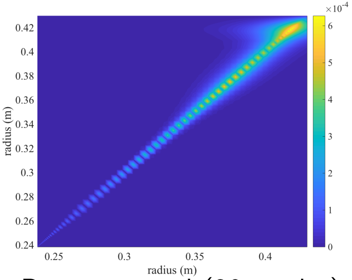
# Fourier modes ($\Phi_F$)

$$\phi_{F,mn}(r,\omega) = e^{\mathrm{i}k_r r} \;\rightarrow\; \mathbf{\hat{C}_m} = \mathbf{\Phi^{+} C_m \Phi^{+H}}$$

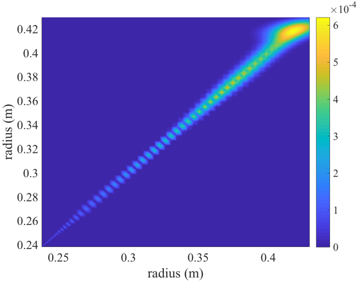
$$error = \frac{\| C_m - C_{m,new} \|}{\| C_{m,new} \|}$$




Original




Reconstructed (20 modes)

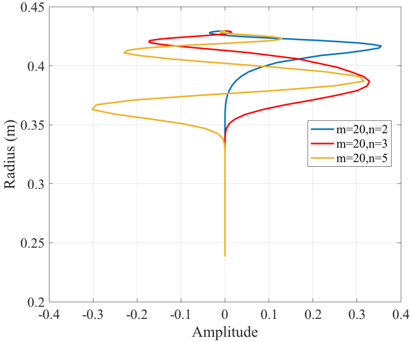


# POD modes ($\Phi_{POD}$)

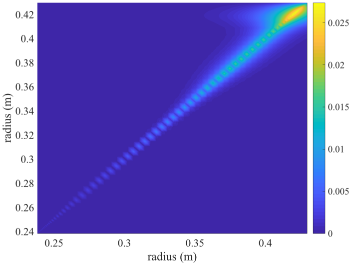
The POD of the two-point velocity cross-spectrum is of the form,

$$C_m(r, r', \omega) = \sum_{n=1}^{N} \lambda_{mn}(\omega)\phi_{mn}(r, \omega)\phi^*_{mn}(r', \omega)$$

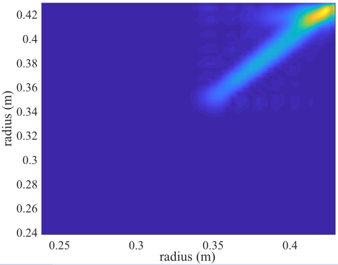
$$\hat{\mathbf{C}}_\mathbf{m} = diag(\lambda_{m1}, \lambda_{m2}, \lambda_{m3}, \cdots, \lambda_{mn})$$




Original




Reconstructed (8 modes)



# Comparison

| Approach | Advantages ☺ | Disadvantages ☹ |
|---|---|---|
| $\Phi_{LEE}$ | Naturally divergence-free | Sharp modes<br>Badly conditioned |
| $\Phi_{Fourier}$ | Good reconstruction of wake<br><br>Easy to impose divergence-free | More radial modes are required<br>Expensive to capture large scale structures |
| $\Phi_{POD}$ | Good reconstruction of large scale structures with limited modes | Expensive for wake turbulence<br><br>Few modes are badly discretized |

Different combinations can be used for different operating conditions

# Conclusions & Future work

Conclusions

- Presented a generalised framework for globally reconstructing the cross-spectral matrix
- Three different vortical modes are investigated to reconstruct the exact cross-spectral matrix: 1) Fourier modes, 2) POD modes 3) Normal modes of LEE.

Future work

- Development of time domain approach as frequency domain is expensive for Broadband noise calculations.

# Thank you!

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