FWH integral in the frequency domain for arbitrary flow Mach numbers

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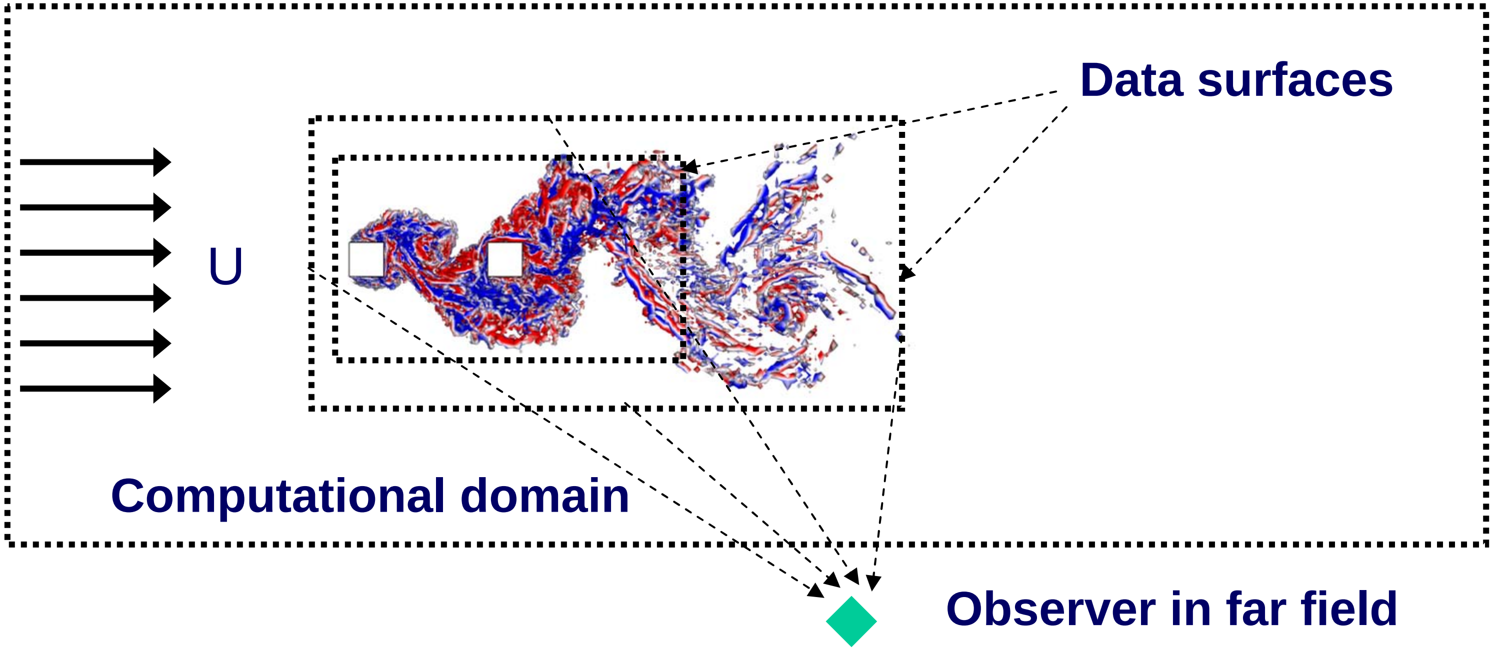
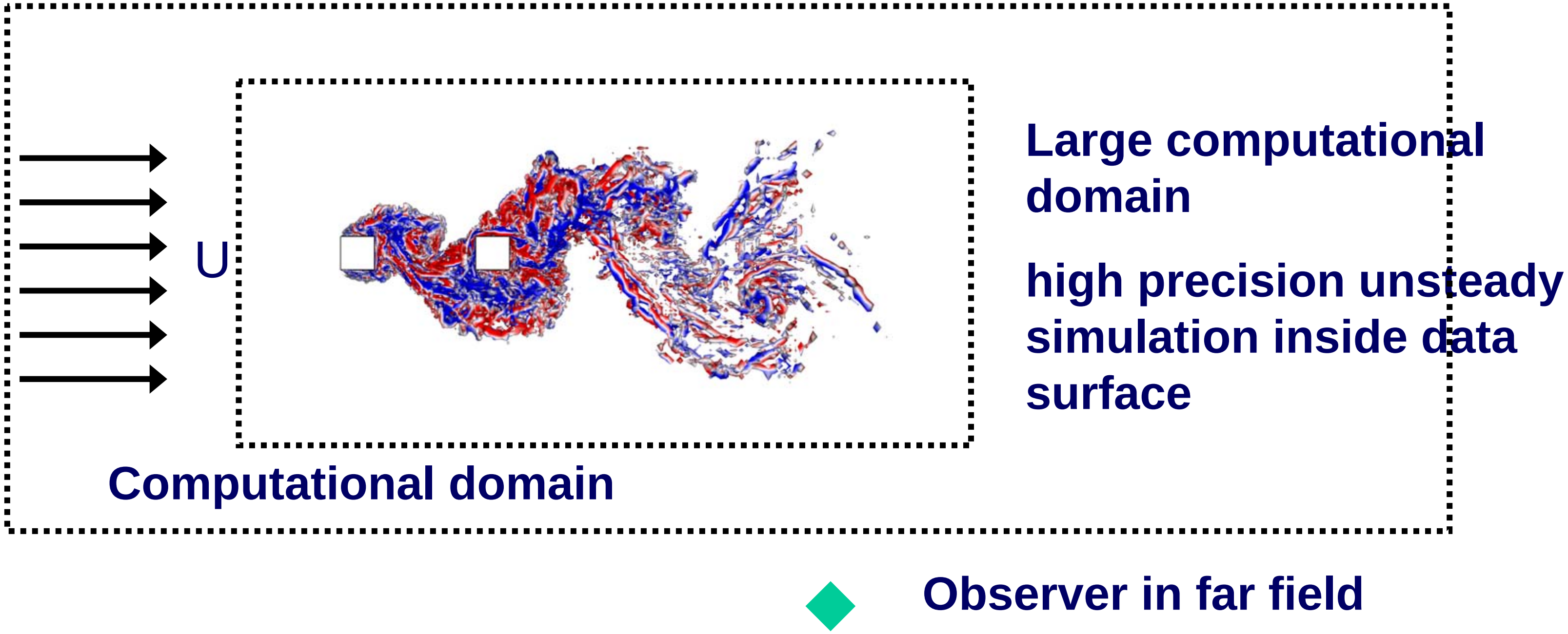
Introduction

- Numerical Experiments have to be limited to the smallest possible computational domain for cost reasons.
- The noise emission into positions in the far field is generally determined in a second step with integrals based on solutions on a data surface.



Introduction

- Classical solution for sound in far-field position: Kirchhoff integral
- Data surface must be in a region, in which the wave equation is valid. No nonlinear terms on (and outside) data surface permitted.



Kirchhoff integral

Kirchhoff integral for zero mean flow speed in the ambiance

$$p(x_i,t) = -\frac{1}{4\pi}\int\limits_S \left[\frac{1}{r}\frac{\partial p}{\partial n} + \frac{1}{r^2}\frac{\partial r}{\partial n}p + \frac{1}{c_0 r}\frac{\partial r}{\partial n}\frac{\partial p}{\partial t}\right] \mathrm{d}S(y_i)$$

Kirchhoff integral for uniform mean flow speed in the ambiance derived with the convective wave equation (Morino 1985)

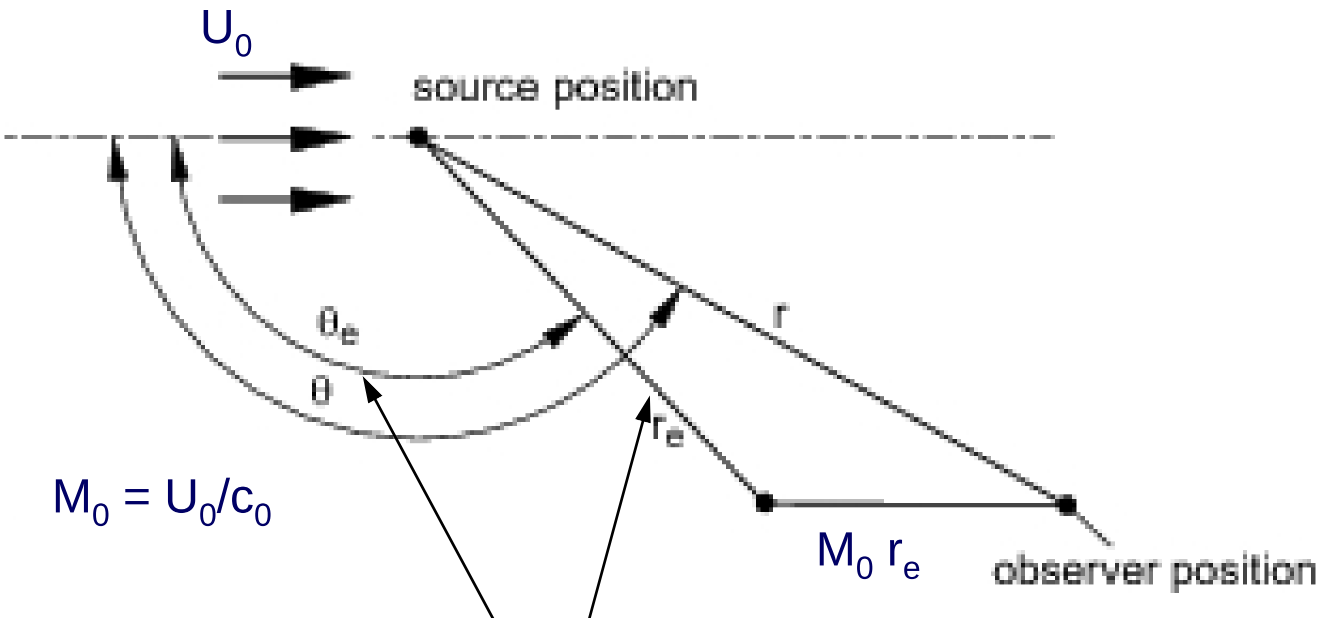
$$p(x_i,t) = -\frac{1}{4\pi}\int\limits_{S_g} \left[\frac{1}{r_g}\frac{\partial p}{\partial n_g} + \frac{1}{r_g^2}\frac{\partial r_g}{\partial n_g}p + \frac{1}{c_0 r_g \beta^2}\left(\frac{\partial r}{\partial n_g} - M\frac{\partial y_{g,1}}{\partial n_g}\right)\frac{\partial p}{\partial t}\right] \mathrm{d}S_g(y_i)$$

With Prandtl-Glauert coordinates

$$r_g = \sqrt{(x_1-y_1)^2 + \beta^2\left[(x_2-y_2)^2 + (x_3-y_3)^2\right]}$$
$$\beta^2 = 1 - M_0^2$$

Emission coordinates

Uniform external motion can be considered in a more compact form if emission coordinates (r_e,θ_e) are used (see Michalke & Michel 1979)



emission angle (wave-normal angle)

emission distance (wave-normal distance)

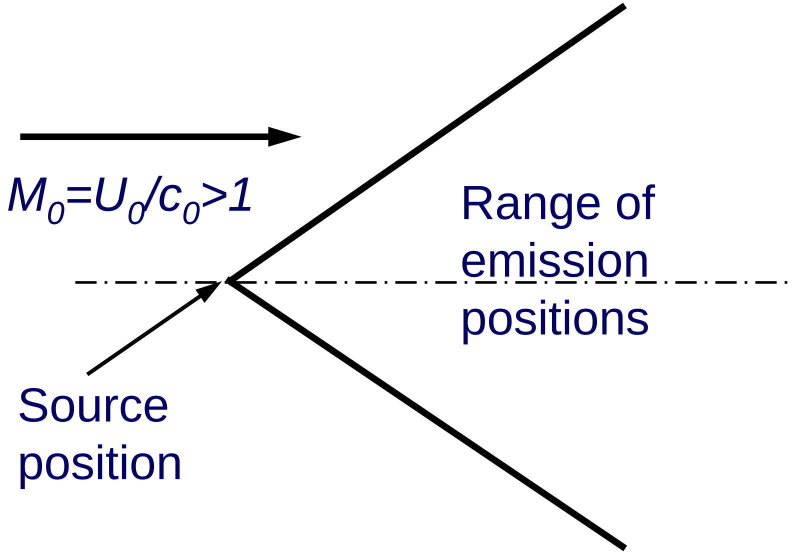
Emission coordinates

Emission coordinates in terms of geometric coordinates

$$r_e = \frac{r}{\sqrt{1 - M_0^2 \sin^2\theta} - M_0 \cos\theta}$$

$$\cos\theta_e = \cos\theta \left[\sqrt{1 - M_0^2 \sin^2\theta} - M_0 \cos\theta\right] + M_0$$

- Emission coordinates are unlimited in space
- Corresponding geometric coordinates are restricted to Mach cone for supersonic flight Mach numbers.
- Description in emission coordinates is valid for any Mach number



Green function

Green function in emission coordinates can be derived with convective wave equation

$$\frac{1}{c_0^2}\left[\frac{\partial}{\partial t}+U_i\frac{\partial}{\partial x_i}\right]^2 p+\frac{\partial^2 p}{\partial x_i^2}=q$$

Green function is solution for point source

$$q=f(t)\delta(x_i-y_i)$$

Green function for convective wave equation

$$p(x_i,t)=\frac{1}{4\pi r_e D_f}f(t-r_e/c_0)$$

Doppler factor D_f

$$D_f=1-M_0\cos\theta_e$$

Kirchhoff integral

- Kirchhoff integral in emission coordinates

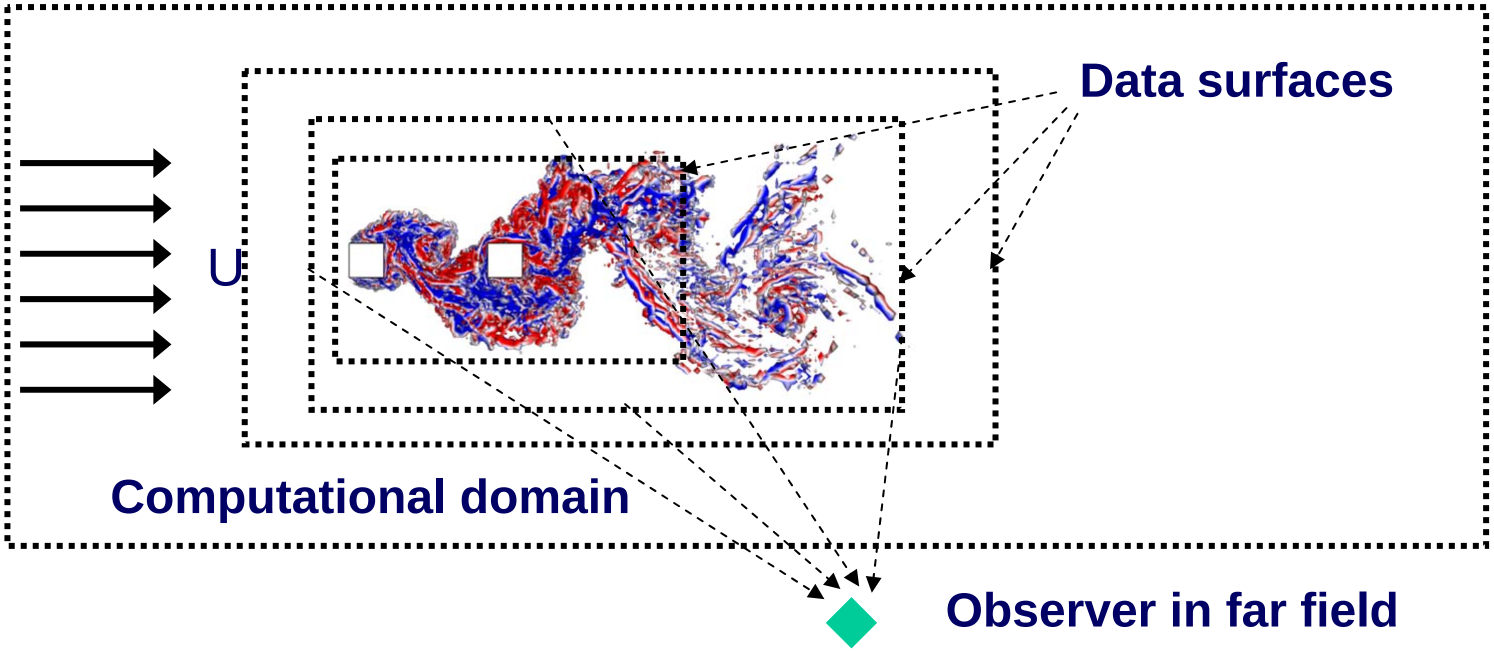
$$p(x_i,t) = \frac{1}{4\pi}\int\limits_S \left[\frac{\rho_0}{r_e D_f}\frac{\partial v_{e,n}}{\partial t} - \frac{1}{r_e^2 D_f^2}\frac{\partial r_e}{\partial n}p - \frac{1}{c_0 r_e D_f}\frac{\partial r_e}{\partial n}\frac{\partial p}{\partial t}\right] \mathrm{d}S(y_i)$$

- Classic solution for *M=0*

$$p(x_i,t) = \frac{1}{4\pi}\int\limits_S \left[\frac{\rho_0}{r}\frac{\partial v_n}{\partial t} - \frac{1}{r^2}\frac{\partial r}{\partial n}p - \frac{1}{c_0 r}\frac{\partial r}{\partial n}\frac{\partial p}{\partial t}\right]_{t_r} \mathrm{d}S(y_i)$$

Ffowcs-Williams & Hawkings integral

- Ffowcs-Williams & Hawkings integral preferred over Kirchhoff integral
- Data surface can be placed closer to sources, even on the surface of a body in the flow.
- Sound pressure in observer position = integral over data surface plus integral over volume with sources outside data surface.



Ffowcs-Williams & Hawkings integral

- Ffowcs-Williams & Hawkings integral for acoustic far field.

$$4\pi p'(x_i,t) = \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\int\limits_{V(t)}\left[\frac{q_q}{r\,|1-M_r|}\right]\mathrm{d}V(y_i)$$
$$+\frac{1}{c_0}\frac{\partial}{\partial t}\int\limits_{S(t)}\left[\frac{F_r}{r\,|1-M_r|}\right]\mathrm{d}S(y_i)+\frac{\partial}{\partial t}\int\limits_{S(t)}\left[\frac{\rho_0 U_n}{r\,|1-M_r|}\right]\mathrm{d}S(y_i)$$

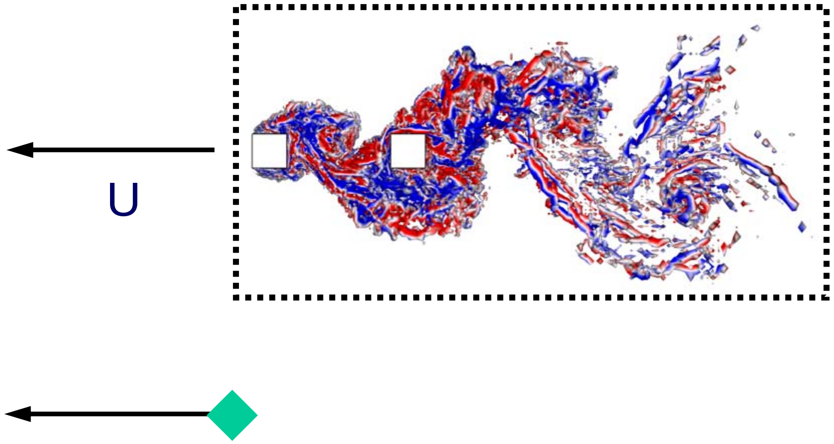
- Quadrupole source term, v_r is velocity component toward observer

$$q_q(y_i,\theta_e,t) = \rho_0 v_r^2\left(1+\frac{p'}{\rho_0 c_0^2}\right)-\left(1-\frac{\rho_0}{\rho}\right)p'$$

- F_r is component of force per unit area in the direction toward observer
- U_n is component of velocity normal to surface element
- M_r is component of velocity of volume or surface element toward observer

Ffowcs-Williams & Hawkings integral

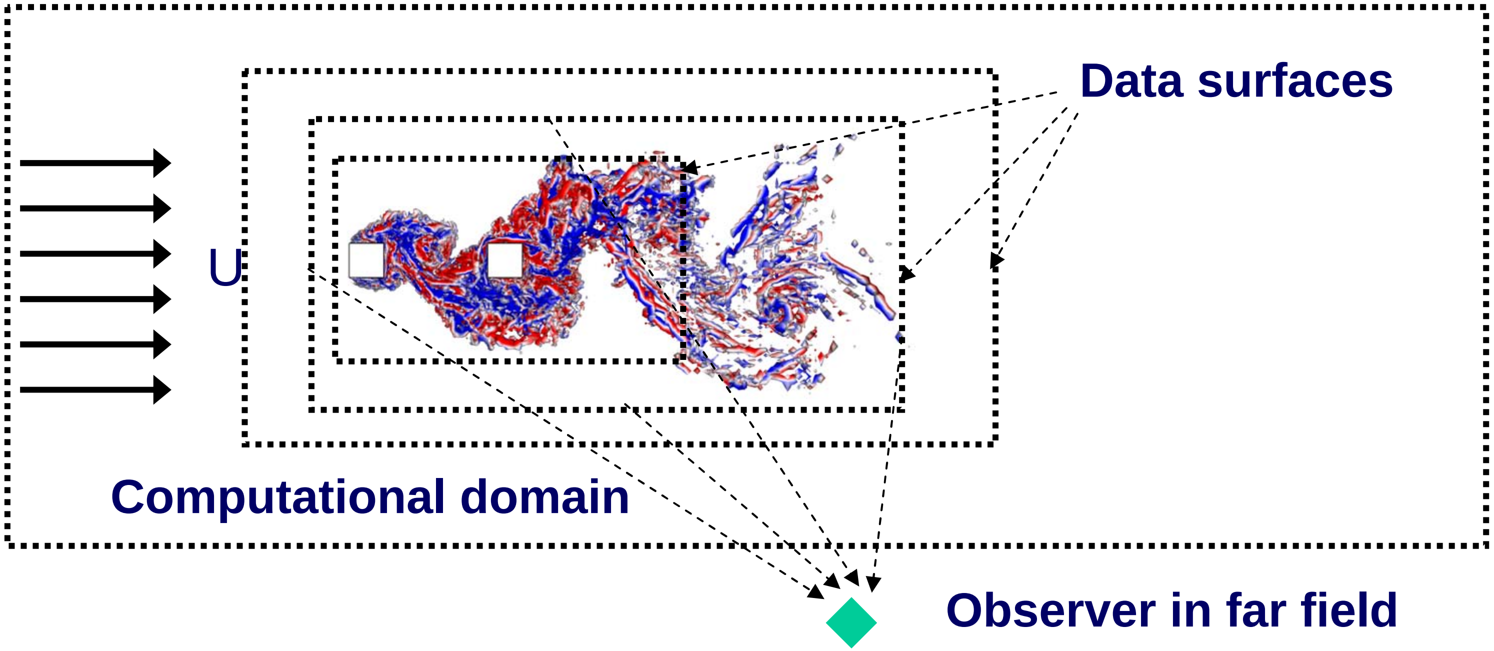
- Ffowcs-Williams & Hawkings integral valid for fixed observer position.
- Uniform motion must be simulated by moving data surface with uniform velocity in a quiet ambiance.
- Since observer stationary, there is a Doppler frequency shift between sources on data surface and observer.
- No transformation into frequency domain possible.



- Numerical workaround:
- Observer motion numerically considered by changing observer position for each time step.

FW-H vs. Kirchhoff

- Ffowcs-Williams & Hawkings integral only exact if volume integral included
  Volume integral also considers refraction effects in nonuniform external flow.
- FWH and Kirchhoff identical if condition for Kirchhoff satisfied
- FWH volume integrals generally neglected, resulting in errors.
- Errors of FWH integrals apparently smaller than those of Kirchhoff integrals if data surface too close to sources.



FW-H integral for convective wave eq.

- Ffowcs-Williams & Hawkings integral for convective wave equation (Wellner 2009)
- Solution for acoustic far field, only surface integral shown

$$4\pi p'(x_i,t) = \frac{\partial}{\partial t}\int\limits_S \left[\frac{\rho_0 U_i u_j' \Gamma_{1j} + (\rho_0 u_i' + \rho' U_i)(c_0 - U_j \Gamma_{1j}) + p'\Gamma_{1i} - \tau_{ij}\Gamma_{1j}}{c_0 r_e D_f} n_i\right] \mathrm{d}S(y_i)$$

$$\Gamma_{1i} = \begin{pmatrix} \partial r_e/\partial x_1 \\ \partial r_e/\partial x_2 \\ \partial r_e/\partial x_3 \end{pmatrix}$$

- Observer position remains stationary with respect to data surface
- Frequency of observer equal to source frequency
- Fourier transform to frequency domain possible

FW-H integral for convective wave eq.

- Solution in emission coordinates much more compact

$$4\pi p'(x_i,t) = \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\int\limits_V \left[\frac{q_{qe}}{r_e D_f^2}\right] \mathrm{d}V(y_i) + \frac{1}{c_0}\frac{\partial}{\partial t}\int\limits_S \left[\frac{F_{re}}{r_e D_f}\right] \mathrm{d}S(y_i) + \frac{\partial}{\partial t}\int\limits_S \left[\frac{\rho_0 U_{ne}}{r_e D_f}\right] \mathrm{d}S(y_i)$$

- q_{qe} is quadrupole source term based on relative velocity component into the direction of emission angle.
- F_{re} is component of force per unit area in the direction of emission angle
- U_{ne} is component of relative velocity normal to surface element
- D_f is Doppler factor

Advantages of frequency domain

Advantages of computations in frequency domain:

- Retarded time differences can be described exactly by a phase shift. No interpolation needed.
- Time derivatives can be described exactly by a complex multiplication.
- It is possible to propagate single frequencies. Speed-up for problems with a few frequencies (e.g. open rotors)

Further advantages:

- Directivity of each frequency different due to source interference
- Interference effects can be studied.
- Interference effects play a large role for directivity of open rotors and also for jet noise
- Analysis in the frequency domain allows physical insight into noise mechanisms

FW-H in frequency domain

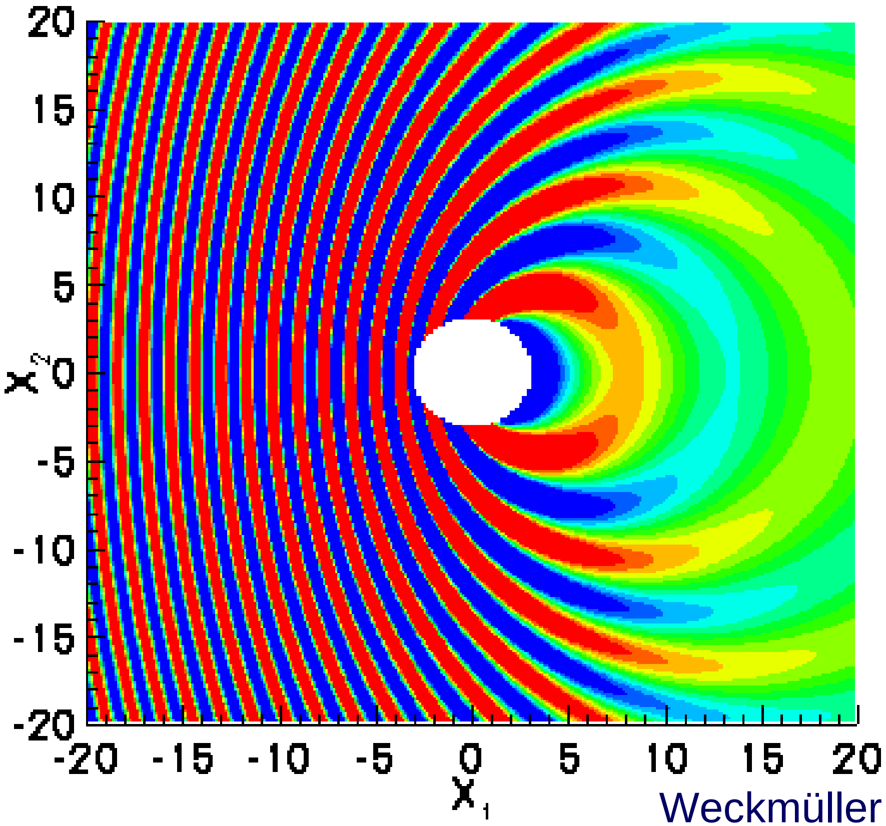
- FW-H in time domain

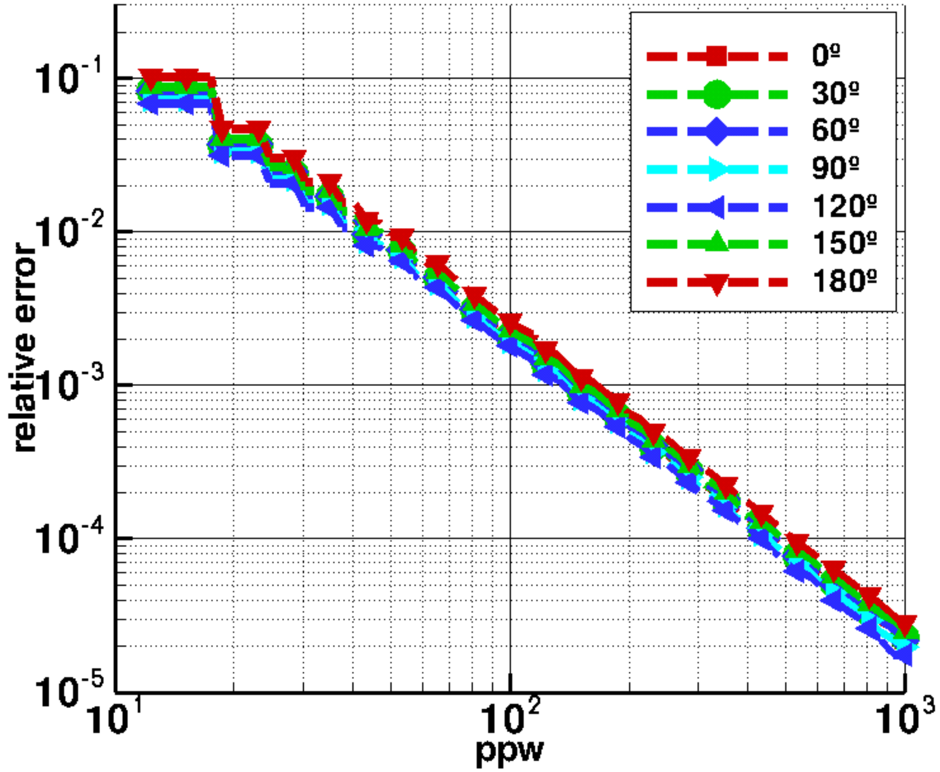
$$4\pi p'(x_i,t) = \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\int\limits_V \left[\frac{q_{qe}}{r_e D_f^2}\right] \mathrm{d}V(y_i) + \frac{1}{c_0}\frac{\partial}{\partial t}\int\limits_S \left[\frac{F_{re}}{r_e D_f}\right] \mathrm{d}S(y_i) + \frac{\partial}{\partial t}\int\limits_S \left[\frac{\rho_0 U_{ne}}{r_e D_f}\right] \mathrm{d}S(y_i)$$

- FW-H in frequency domain

$$\hat{p}(x_i,\omega) = -\frac{\omega^2}{4\pi c_0^2}\int\limits_V \frac{\hat{q}_{qe}}{r_e D_f^2} \mathrm{e}^{ikr_e} \mathrm{d}V(y_i) - \frac{\omega}{4\pi c_0}\int\limits_S \frac{\hat{F}_{re}}{r_e D_f} \mathrm{e}^{ikr_e} \mathrm{d}S(y_i) - \frac{\omega}{4\pi}\int\limits_S \frac{\rho_0 \hat{U}_{ne}}{r_e D_f} \mathrm{e}^{ikr_e} \mathrm{d}S(y_i)$$

Validation with analytical Monopole





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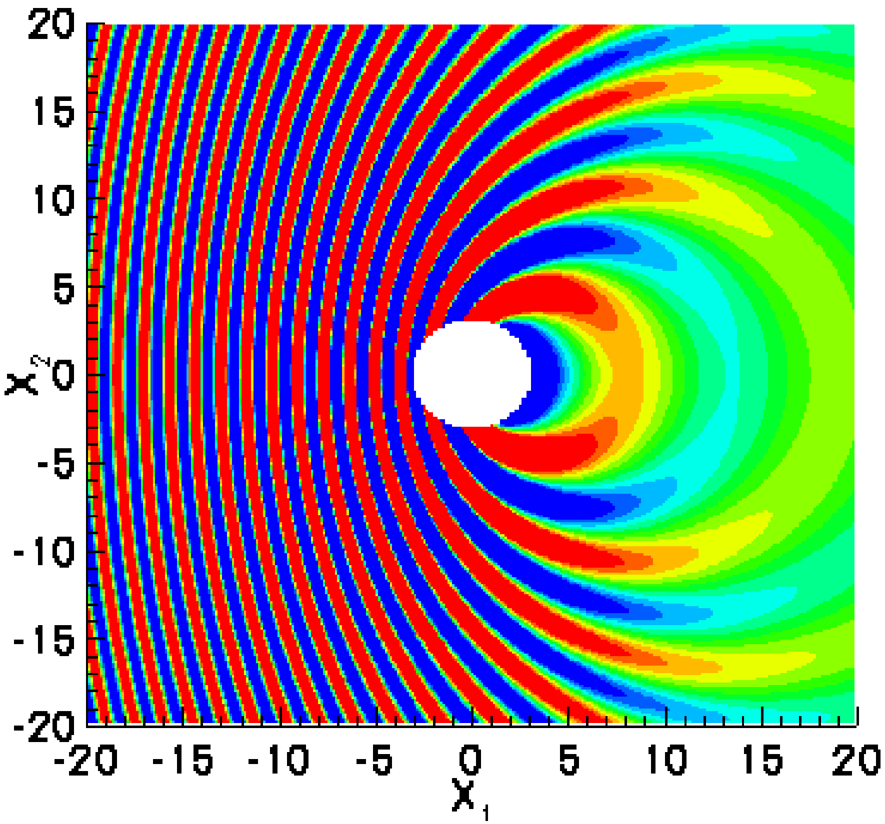
analytical solution Levine 1980

$M_0 = 0.78$

- Required number of points per wavelength increases rapidly with Mach number
- Cause is compression of waves in flight direction

Validation with analytical Monopole (Levine 1980)



analytical solution

$M_0 = 0.78$

Conclusions for grid requirements on data surface:

- Grid density depends on emission angle

$\Delta x_1 = 1/D_f = 1/(1-M_0 \cos \theta_e)$

- Grid must be fine in the forward arc.
- Grid may be coarse in the rear arc.

Azimuthal Fourier expansion

- Open rotors generate acoustic field with azimuthal structure.
- FWH surface integral in short form

$$\hat{p}(x_i,\omega) = \int_S \hat{A}(y_i, x_i, \omega)\mathrm{d}S(y_i)$$

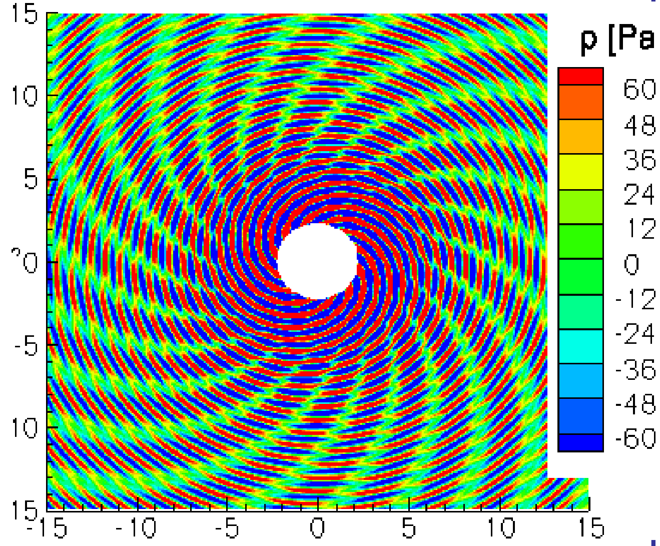
- Fourier series expansion into azimuthal direction for axisymmetric data surface

$$\hat{A}(y_1,\theta) = \sum_m \hat{A}_m(y_1)\mathrm{e}^{im\psi}$$

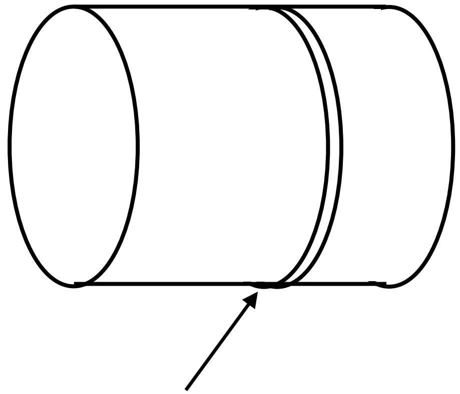
- Far-field pressure in frequency domain = sum of azimuthal components

$$\hat{p}(x_i,\omega) = \sum_m \hat{p}_m(x_i,\omega)$$

$$\hat{p}_m(x_i,\omega) = \int_S \hat{A}_m(y_1, x_i, \omega)\mathrm{d}s$$



m=12 and 22 visible



One ring of data surface

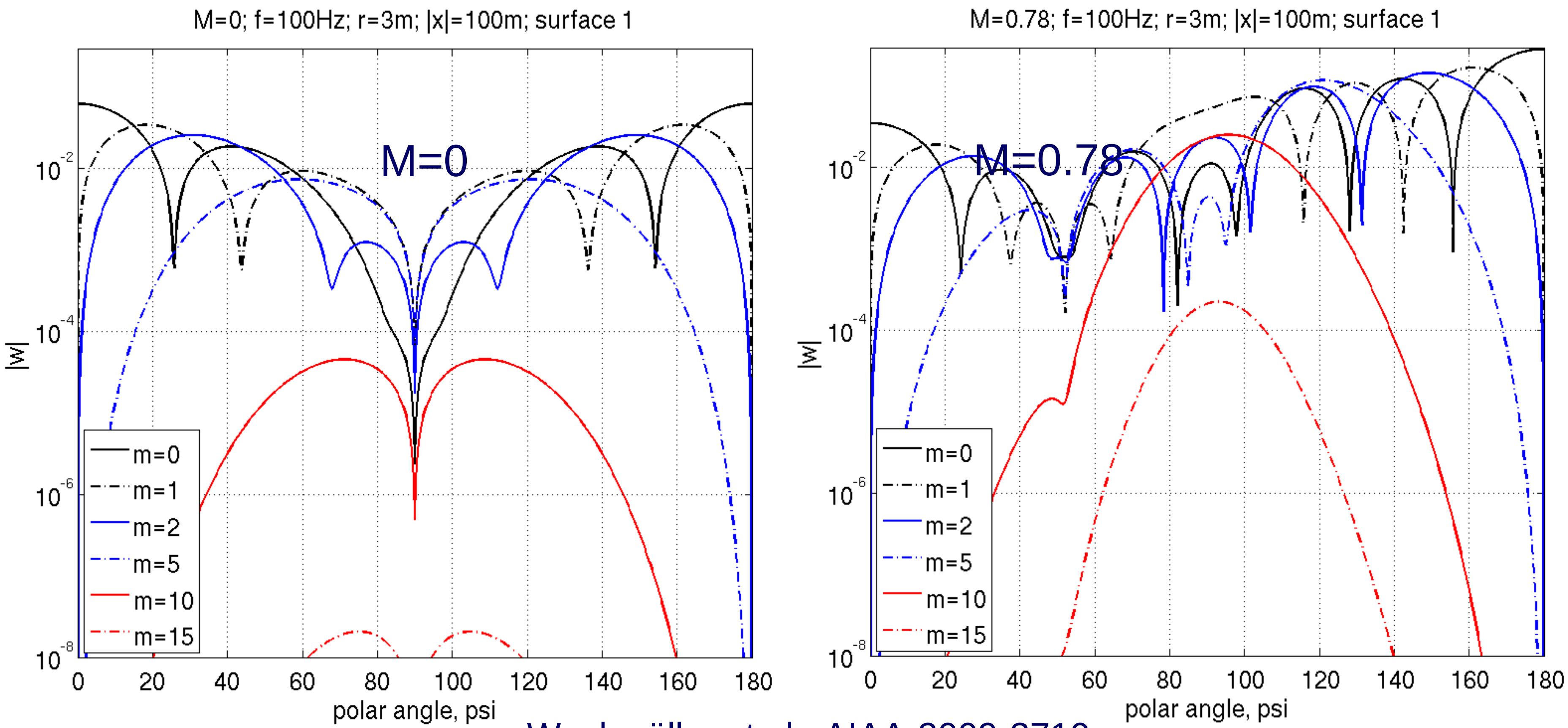
Azimuthal Fourier expansion

- Benefits of Fourier series expansion of the acoustic field on the data surface:
  - Improvement on accuracy:
    Field data on surface are described by Fourier series, yielding an integral of a continuous function rather than of a step function.
  - Decreasing need of computer resources when the series contains only a few azimuthal components.
- Each azimuthal component can be studied separately
  - Insights to physical source mechanism
  - Source interference effects can be studied.
  - Radiation efficiency can be studied as function of component 0rder m and Mach number M_0

Demonstration of interference effect

- Source interference has large influence on radiation efficiency.

Variation of azimuthal order m



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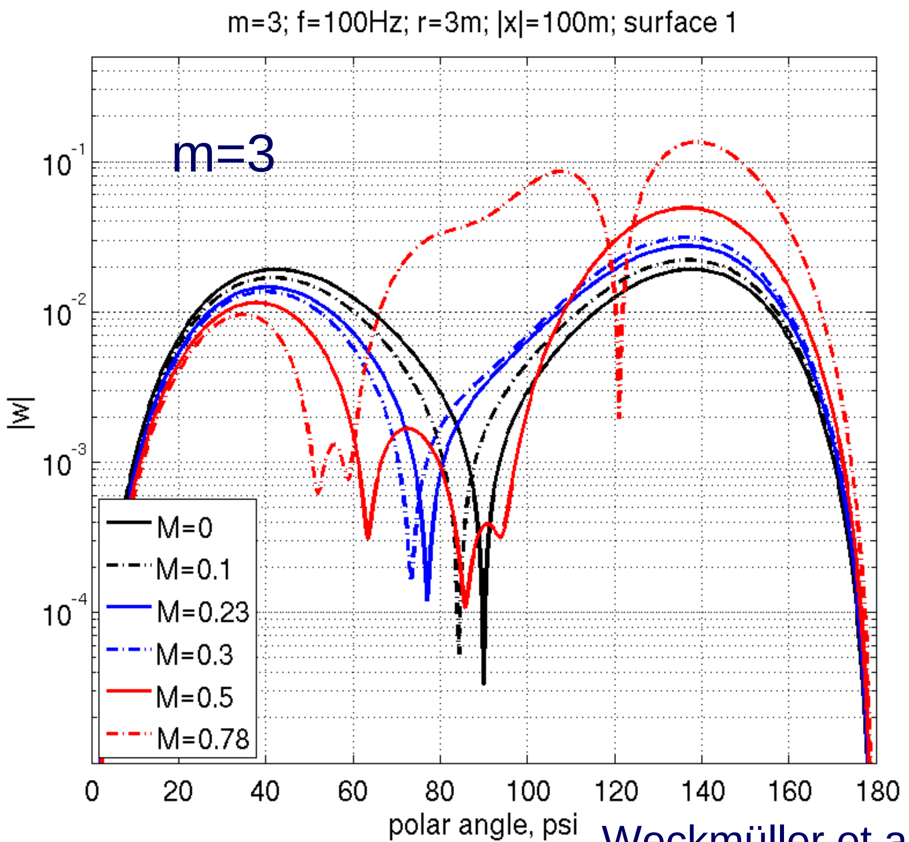
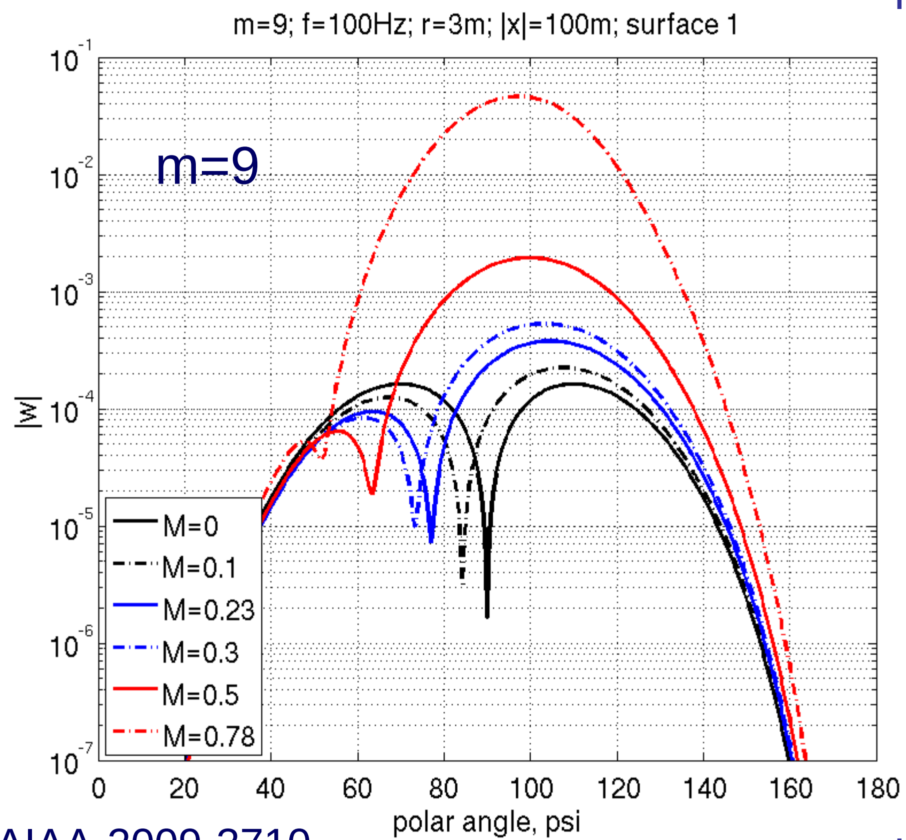
- Radiation efficiency decreases with rising mode order
- Only m=0 contributes to SPL on the axis

Variation of Mach number M





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- Radiation efficiency increases with rising Mach number
- Sound radiation is directed more to the upstream direction, due to the convective amplification

Results

- Radiation efficiency of equivalent sources on data surface depends on azimuthal mode order and Mach number
- Only m=0 contributes to SPL on the axis
- Radiation efficiency increases with rising Mach number
- Sound radiation is directed more to the upstream direction due to convective amplification

Conclusions

- A convective FW-H formulation for permeable surfaces is derived
- Compact description in emission coordinates presented
- Solution allows transform into frequency domain
- Frequency domain solution more precise (no interpolation errors for retarded time, exact time derivatives)
- Validation with a monopole
- Grid density on upstream part of data surface upstream needs to be much higher
- Axisymmetric data surfaces allow description of far field as sum of line integrals (2.5D solution)
  - Increase of accuracy
  - Reduction of computing time
  - Reduction of data storage
  - Direct coupling with 2.5D CAA possible

Outlook

- Development of theory with open rotors in mind
- Application to jet noise also possible
  - Investigation of the azimuthal content of jet noise based on DES data.
  - Jet noise radiation as function of azimuthal component order