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NUMERICAL SIMULATION OF INTERACTION BETWEEN SHOCK WAVES AND ACOUSTIC DISTURBANCES

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INTERACTION OF ISOTROPIC TURBULENCE WITH SHOCK WAVES

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Background and Objective

Several studies on turbulence/shock interaction have done:

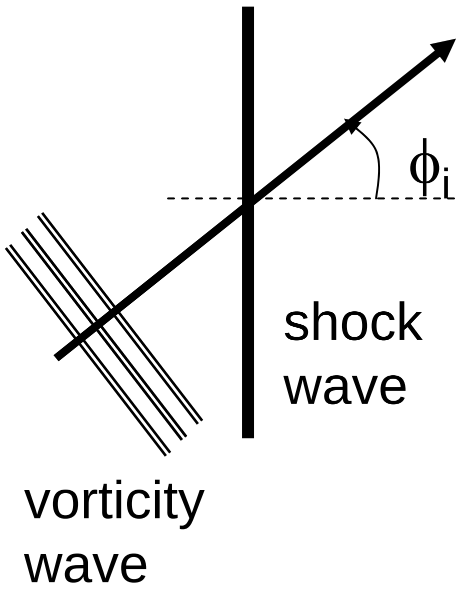
i) Experiments by Debieve and Lacharme(1986) and Keller and Merzkirch(1990) – shock wave and grid-generated turbulence interaction – have shown **considerable amplification of turbulence fluctuations and increase of Taylor microscales.**

ii) Experiments by Honkan & Andreopoulos (1992) - normal shock/grid-turbulence interaction – **the amplification factor is not the same for different lengthscales.**

iii) Computations by Lee, Lele and Moin (1993) – interaction of isotropic quasi-incompressible turbulence with a weak ($1.05<M_1<1.2$) shock wave.

Background and Objective

Present study addresses **the interaction of isotropic quasi-incompressible turbulence with a strong shock wave (M_1>2).**



At a strong shock wave, the amplification of a vorticity wave depends on the incidence angle ϕ_i.

The quasi-incompressible turbulence is mainly contributed by numerous vorticity waves with different wavevectors.

Therefore, isotropicity of the turbulence might be lost, and one can expect the formation of large ordered structures (of preferable orientation) in the turbulence behind shock wave.

Outline

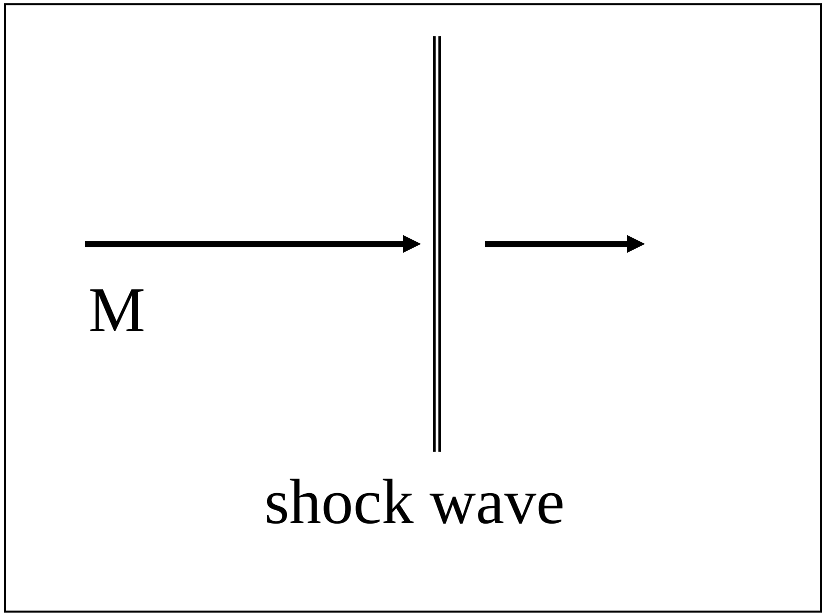
The presentation is outlined as follows.

I) Review results of **<u>the linear analysis</u>** for the interaction between a plane small-amplitude vorticity wave and a normal shock wave:
- refraction law and critical incidence;
- transmission amplification factors vs incidence angles.

II) **<u>Numerical simulation of isotropic turbulence and a shock wave interaction</u>**:
- modeling of the inflow turbulence;
- effect of the turbulence lengthscale;
- effect of the turbulence intensity;

I. Linear analysis

Basic flow notations



Upstream parameters:

ρ, p = density, pressure

u = velocity;

a = speed of sound;

M = Mach number;

Downstream parameters:

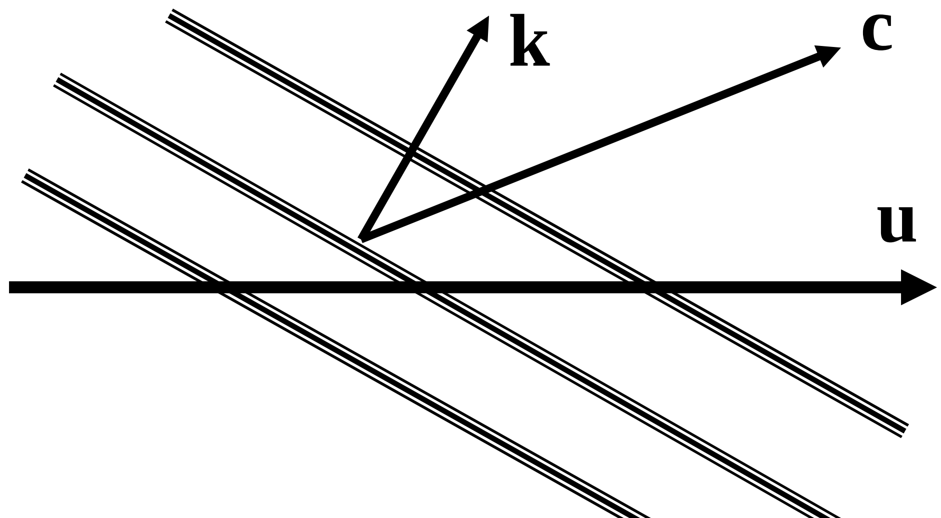
ρ_s, p_s = density, pressure

u_s = velocity;

a_s = speed of sound;

M_s = Mach number;

Disturbance wave notations



$$\delta \mathbf{z} = \mathbf{z}' exp\{ i [(\mathbf{k}, \mathbf{x}) - \omega t] \}$$

$\mathbf{z}'$ = amplitude

ω = circular frequency, $\omega > 0$

$\mathbf{k}$ = wave vector

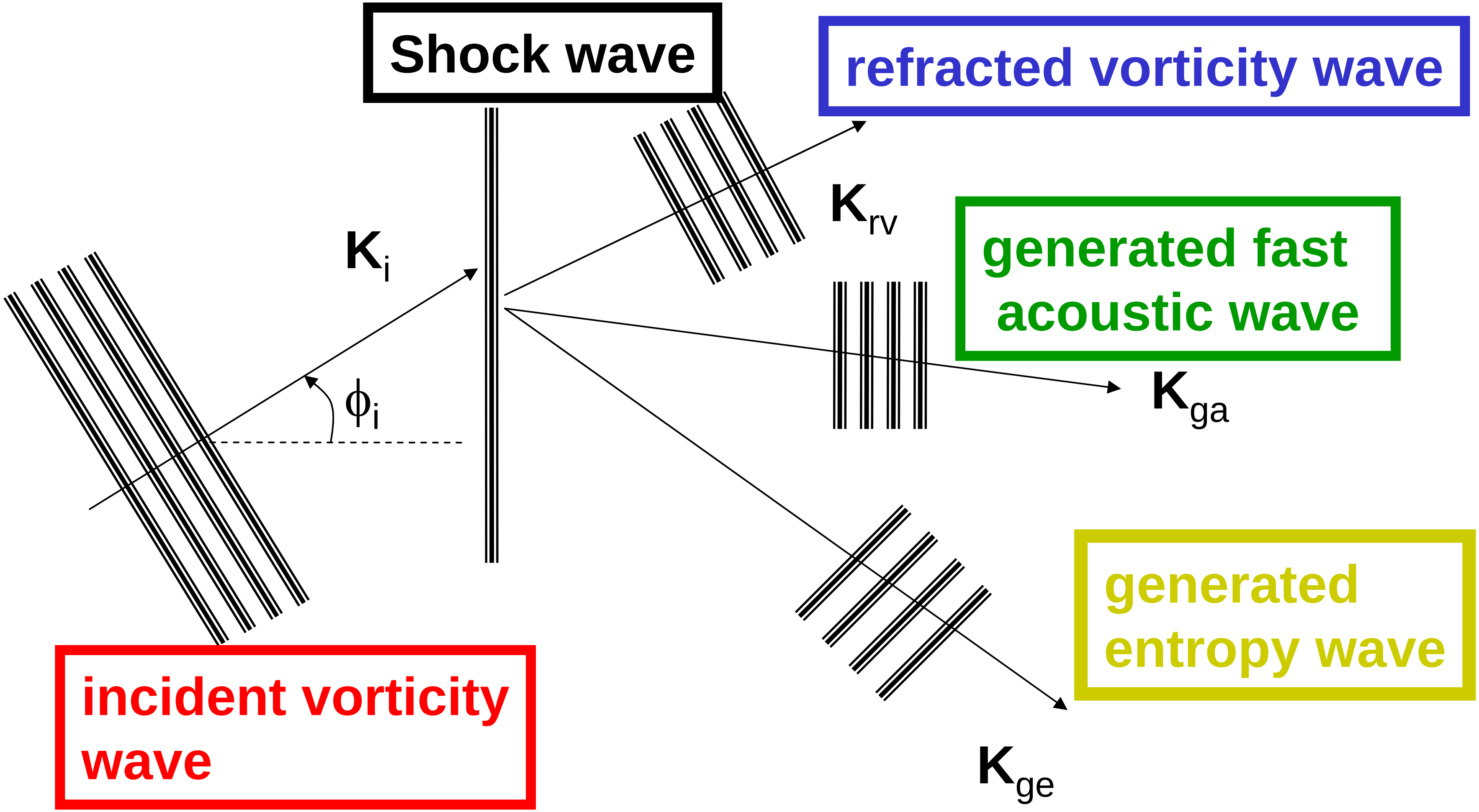
$\mathbf{c}$ = group velocity, $(\mathbf{k}, \mathbf{c}) = \omega$

Vorticity waves:

$\omega = (\mathbf{u}, \mathbf{k})$; $\mathbf{c} = \mathbf{u}$

$p' = 0$; $\rho' = 0$; $\mathbf{u}' = \varepsilon_v \mathbf{m}$

$(\mathbf{k}, \mathbf{m}) = 0$



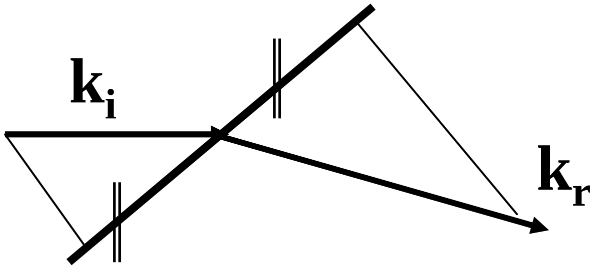
Wave structure downstream the shock wave. The problem is to 3 wavevectors Krv, Kga, and Kge and the amplitudes

Determination of wave vectors:

Wave vector $\mathbf{k}_{\mathbf{rv}}$, $\mathbf{k}_{\mathbf{ga}}$ and $\mathbf{k}_{\mathbf{ge}}$ are defined by:

i) **continuity of the tangent components:**

$$[(\mathbf{k}, \tau)] = 0$$



ii) **continuity of frequencies:**

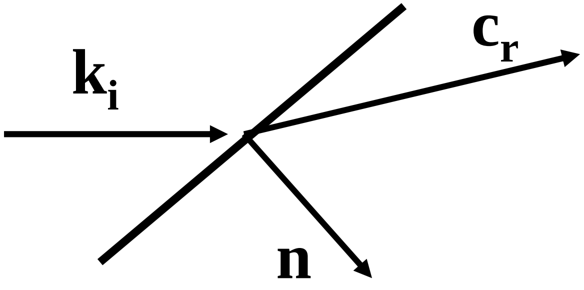
$$[\omega] = 0; \quad \omega = (\mathbf{u}_s, \mathbf{k}_r) - \varepsilon_r a_r k_r$$

$\varepsilon_r = -1$ for acoustic wave

$\varepsilon_r = 0$ for entropy/vorticity wave

iii) **run away:**

$$(\mathbf{c}_r, \mathbf{n}) > 0$$



Solutions for r-wave vectors:

i) For the ***entropy/vorticity wave***, solution exists at any incidence vectors $\mathbf{K_i}$:

$$\mathbf{k_r} = \mathbf{k_i} + \frac{\omega - (\mathbf{u_s}, \mathbf{n_i})}{u_{\mathbf{sn}}}\mathbf{n}$$

ii) For the ***acoustic wave***, there exists ***a limit for the incidence angle***,

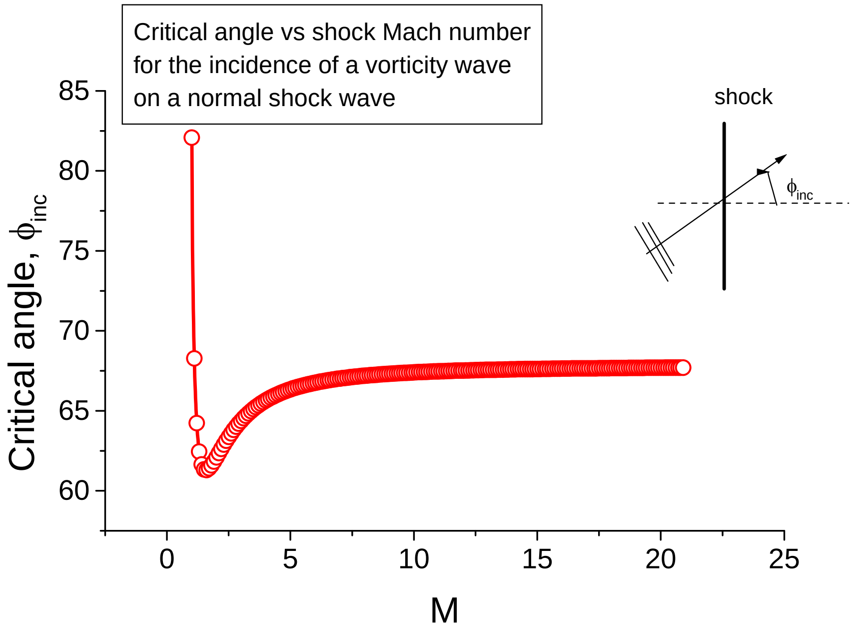
$$z = (\mathbf{k_i}, \mathbf{n}) = z_*$$

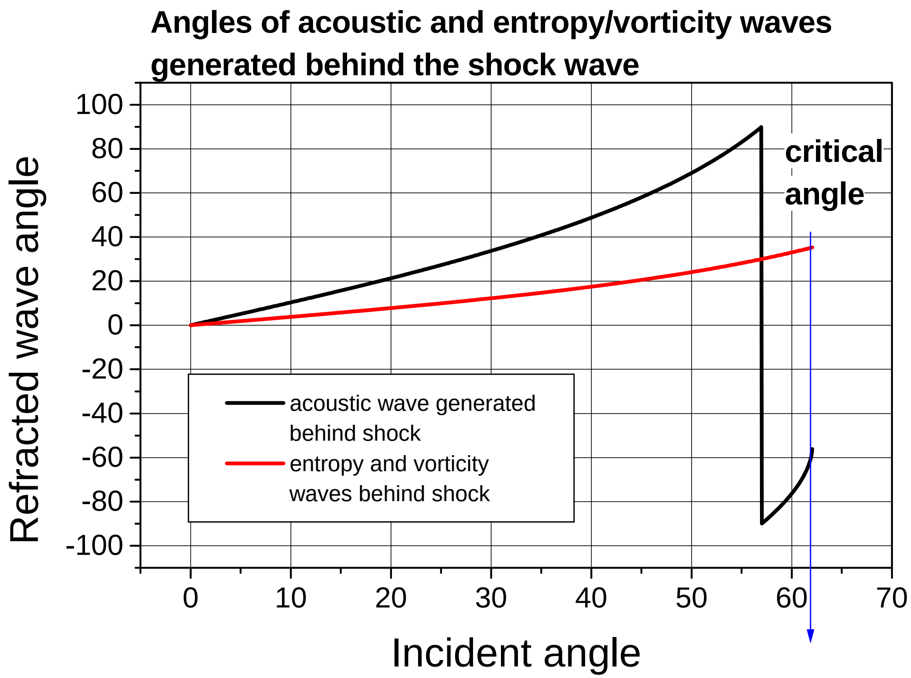
<u>a critical angle</u> so that

the solution exists, if $z_* < z < 1$

and doesn't exist, if $z < z_*$

Critical angle vs. Mach number





Wave amplitudes:

Upstream the shock

$$\boldsymbol{q}' = \boldsymbol{q}_i'$$

Downstream the shock

$$\mathbf{q}_{s,r}' = q_\rho \mathbf{e}_{en} + q_p \mathbf{e}_{ac} + q_u \mathbf{e}_{vrt}$$

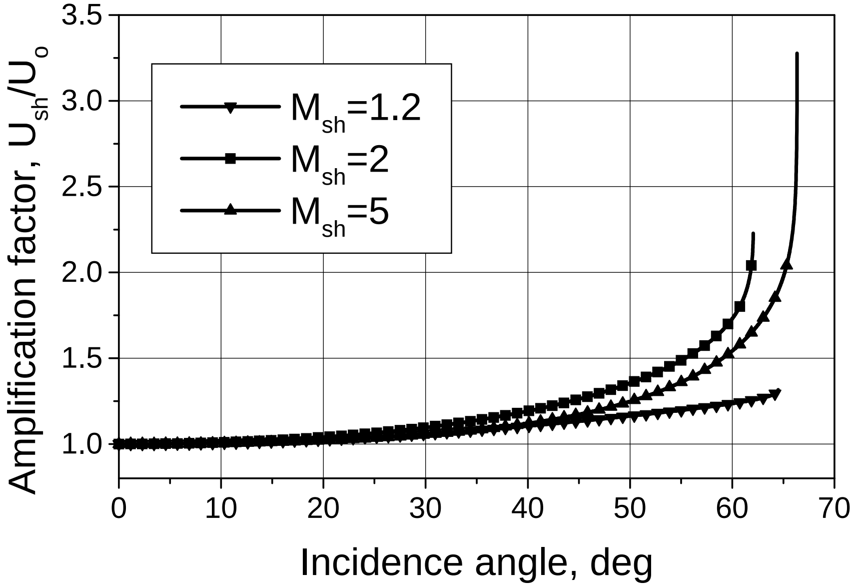
$q_{\rho,p,u}$ = wave characteristic amplitudes to be defined

$\mathbf{e}_{en,ac,vrt} = \mathbf{e}_{en,ac,vrt}(\mathbf{q}) =$ normalized amplitude vectors

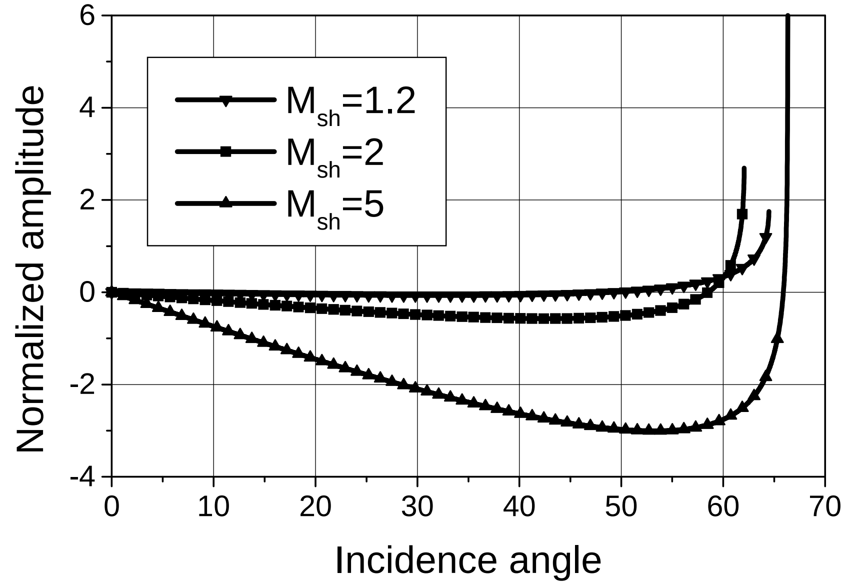
The amplitudes of 3 waves (entropy, vorticity and fast acoustic) and the velocity of the disturbed shock wave are determined from the linearized Rankine-Hugoniot relations

Amplification factor and amplitudes of the generated waves

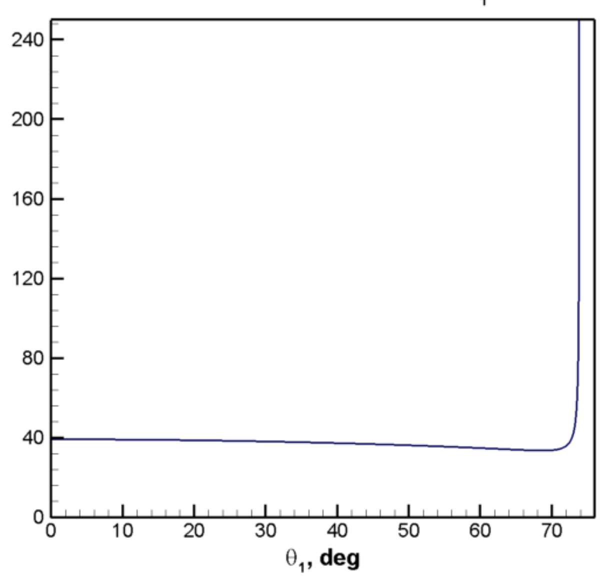
Vorticity wave incidence: amplification factor for the vorticity wave



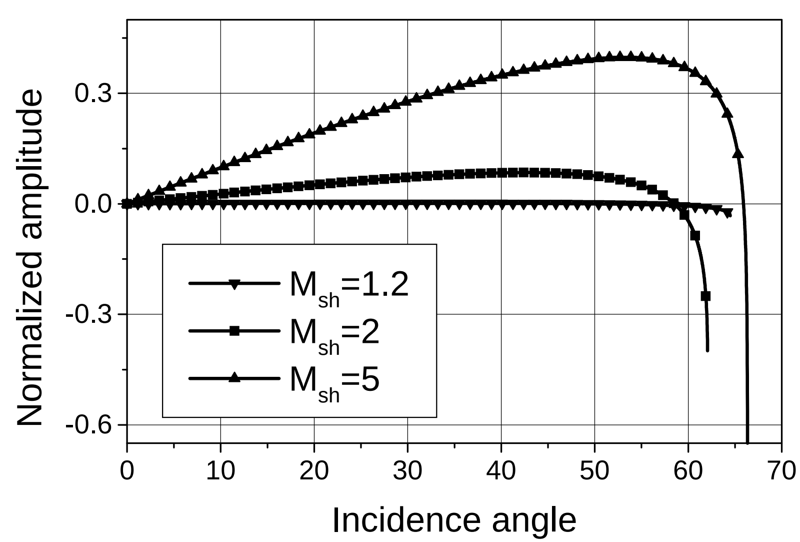
Vorticity wave incidence: amplitude of the generated acoustic wave



Coefficient of sound refraction $M_1=8.0$



Vorticity wave incidence: amplitude of the generated entropy wave



Conclusions from the linear analysis

1) **Amplification** of the wave that runs at a shock wave from upstream **depends on the angle of incidence.**

2) **At the critical incident angle,** the amplification **is drastically increases: the amplitude growth – $O(M^3)$.**

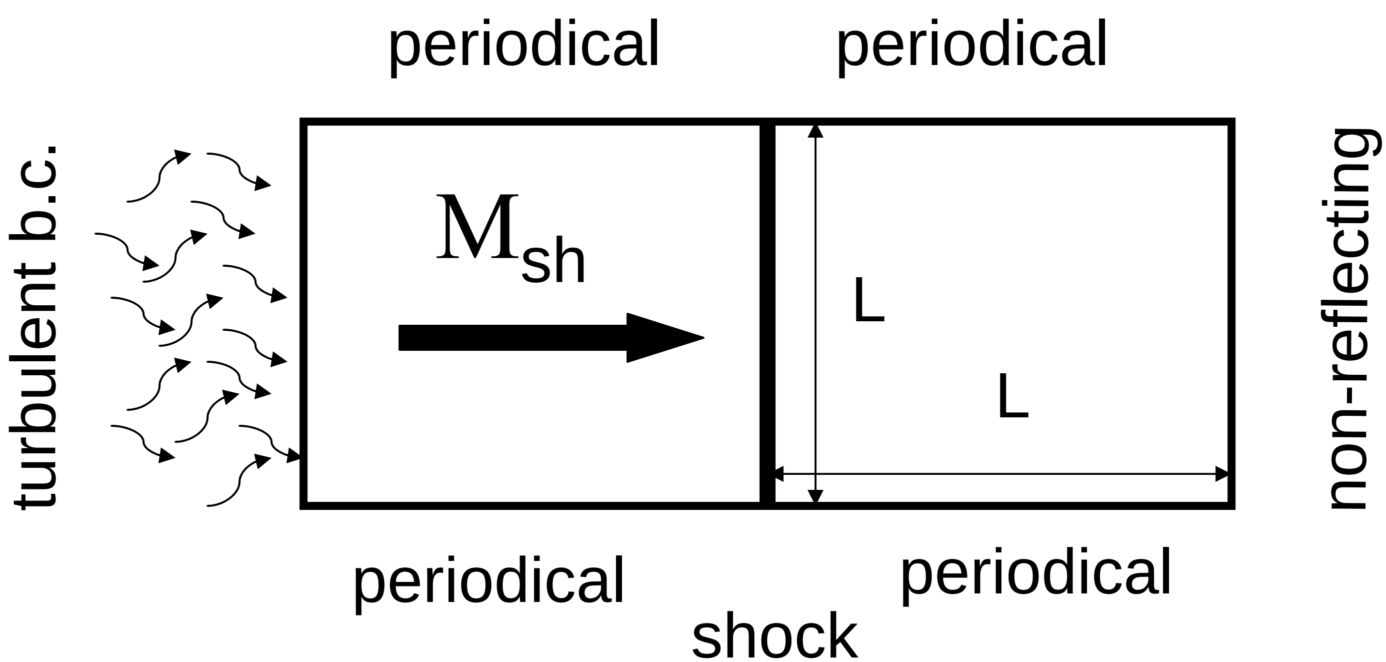
The modes of the turbulence incident the shock **will be amplified differently;** one may expect modes of **preferable amplification.**

The shock must **modify the inflow turbulence** and lead to:

- **loss of isotropicity** (if the turbulence is so in the upstream);
- appearance of **large ordered structures** in the field after the shock.

II. Numerical simulation of turbulence interacting with a shock wave

Diagram of the computational domain



Grid: 512x256

Model: disturbance field Navier-Stokes equations

Method: Godunov-type based on the variational Riemann problem solution with MUSCL interpolation

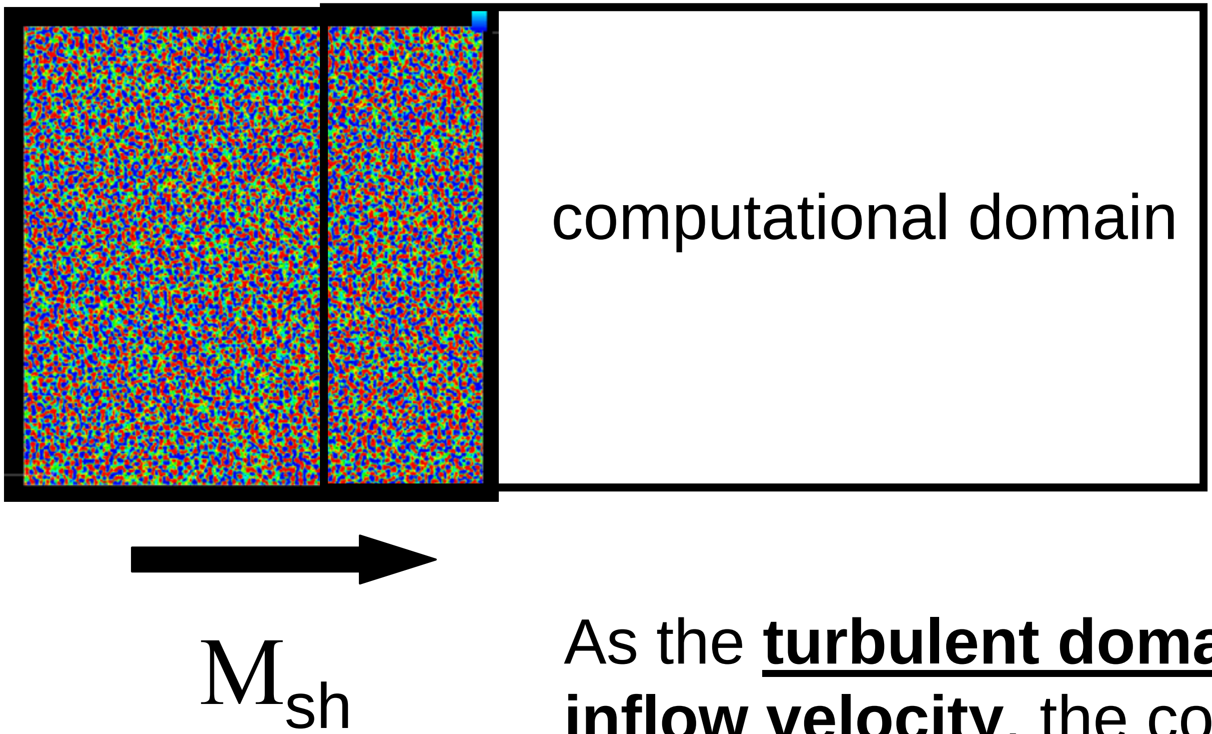
“Turbulent” condition at the inflow boundary

At the front of the inflow boundary a turbulent flow field is generated with following properties:

- ✓ isotropic;
- ✓ dilatation free;
- ✓ no fluctuation in pressure and density;
- ✓ energy spectrum is given by

$$E(k) = \frac{32}{3}\sqrt{\frac{2}{\pi}}\frac{u_0^2}{k_0}\left(\frac{k}{k_0}\right)^4 \exp\left[-2\left(\frac{k}{k_0}\right)^2\right]$$

“Turbulent” condition at the inflow boundary

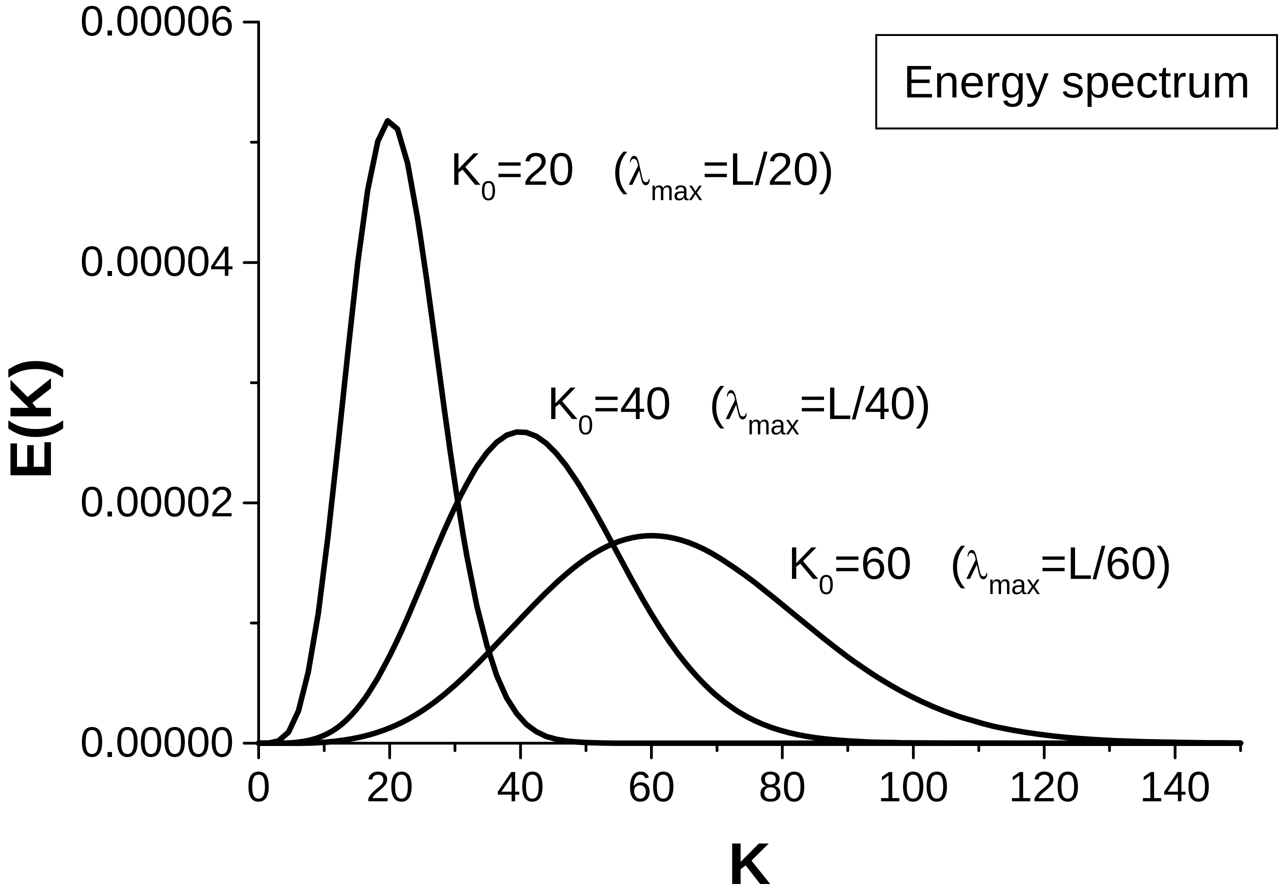


As the **turbulent domain moves with the inflow velocity**, the corresponding data is pick out from the field as the boundary conditions.

The **turbulent field is updated** once in a given time interval T_{ud} **at a random instance**.

Parameters of the inflow turbulence

K_0 = most energetic wavenumber
$M_{tr} = U_0/a$ = turbulent Mach number (turbulent intensity)



Parameters to be analyzed in the calculation

Turbulent energy: $\varepsilon' = \frac{1}{2}\left\langle (\vec{u}')^2 \right\rangle / u_0^2$

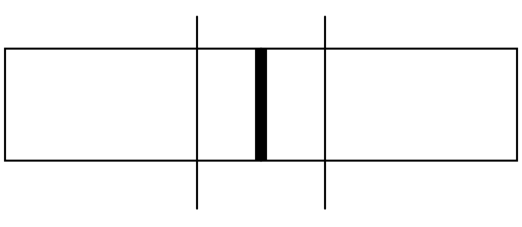
Enstrophy: $\omega' = \left\langle (\vec{\omega}')^2 \right\rangle / (u_0^2 k_0^2)$

Longitudinal Taylor microscale: $\lambda'_x = 2\varepsilon' / \left\langle \left(\partial u'_x / \partial x\right)^2 \right\rangle (u_0^2 k_0^2)$

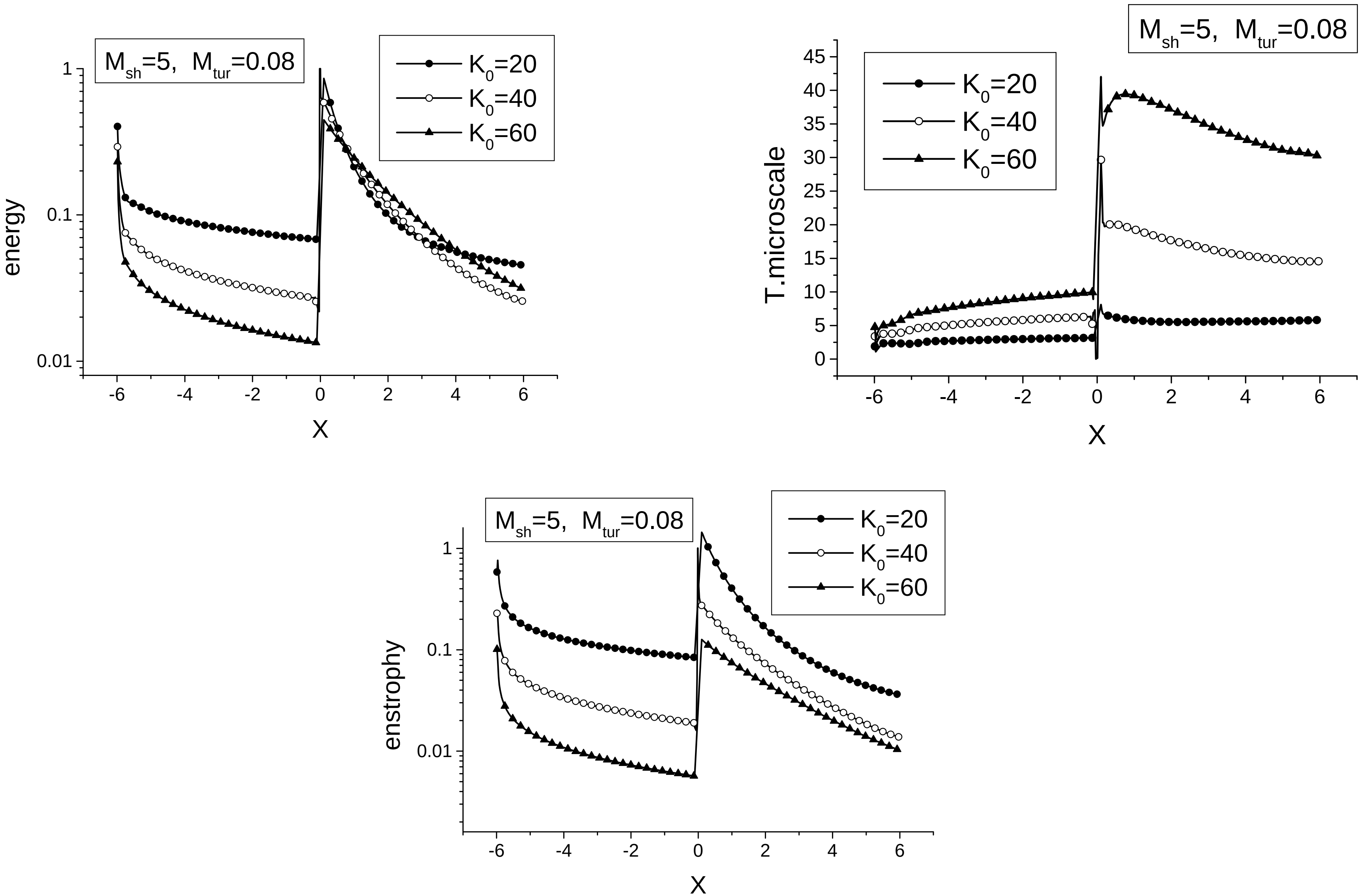
Probability density function for the turbulent velocity at 2 stations upstream and downstream the shock

$$x = x_{sh} \pm 0.25L$$

$$PdF_1(\boldsymbol{u}')\ \ and\ \ PdF_2(\boldsymbol{u}')$$



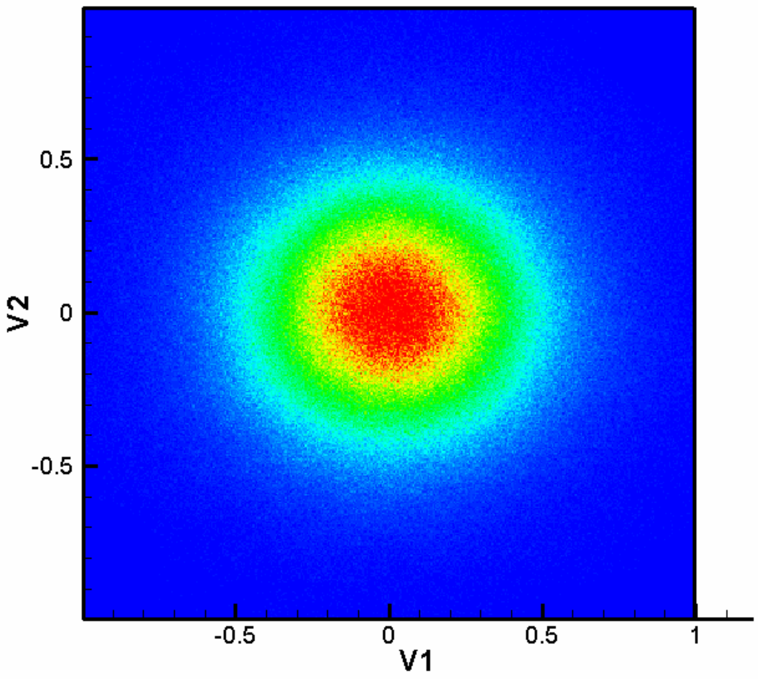
II.1.
Effect of the turbulence lengthscale (K_0)

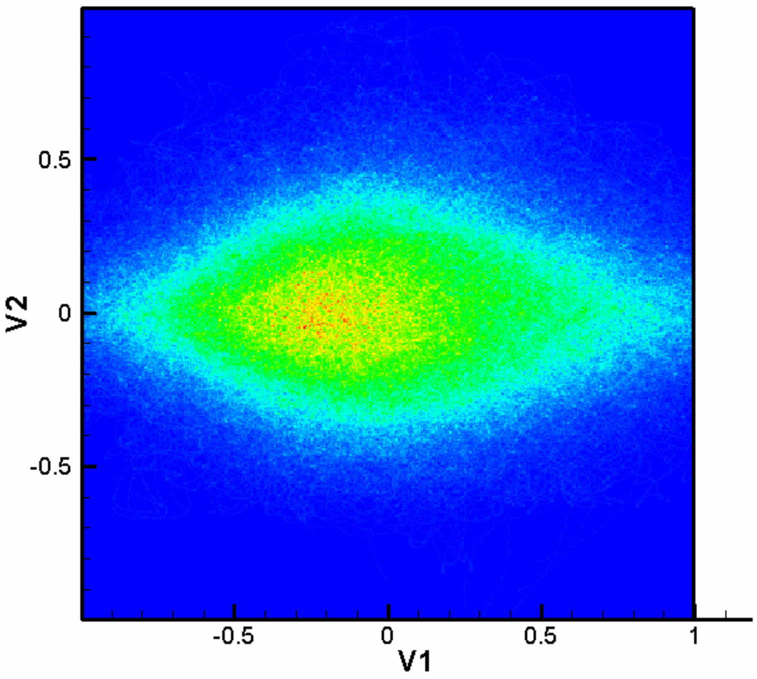


M_{sh}=5, M_{tur}=0.08
K_0=20
K_0=40
K_0=60
energy
X
T.microscale
enstrophy

Probab. density function (M_{sh}=5, M_{tur}=0.08)

K_0=20

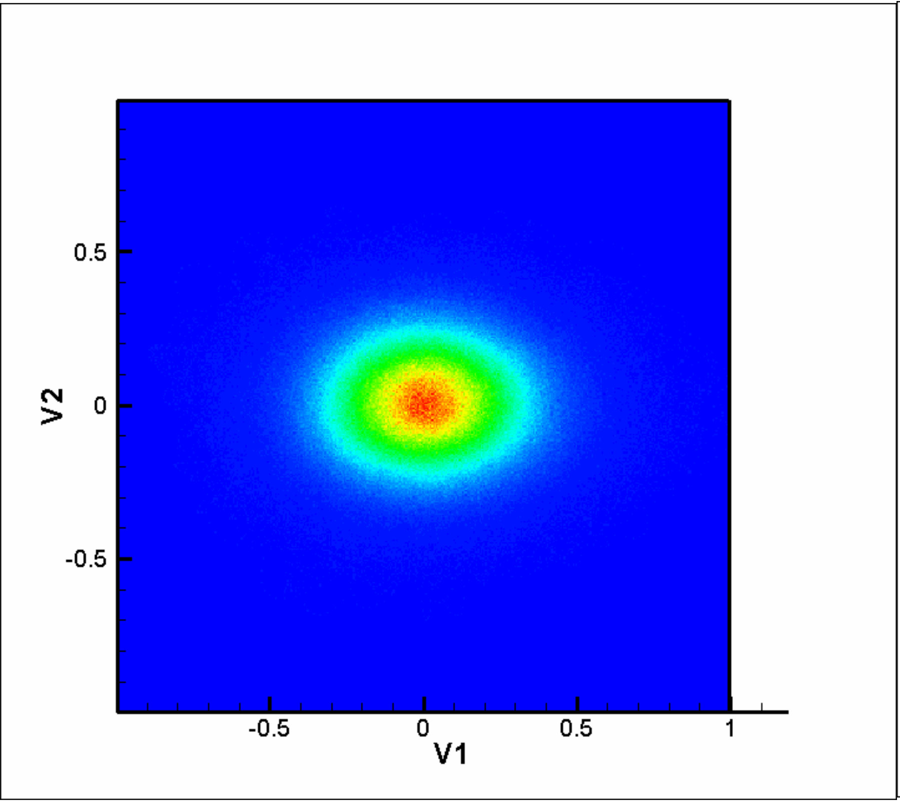


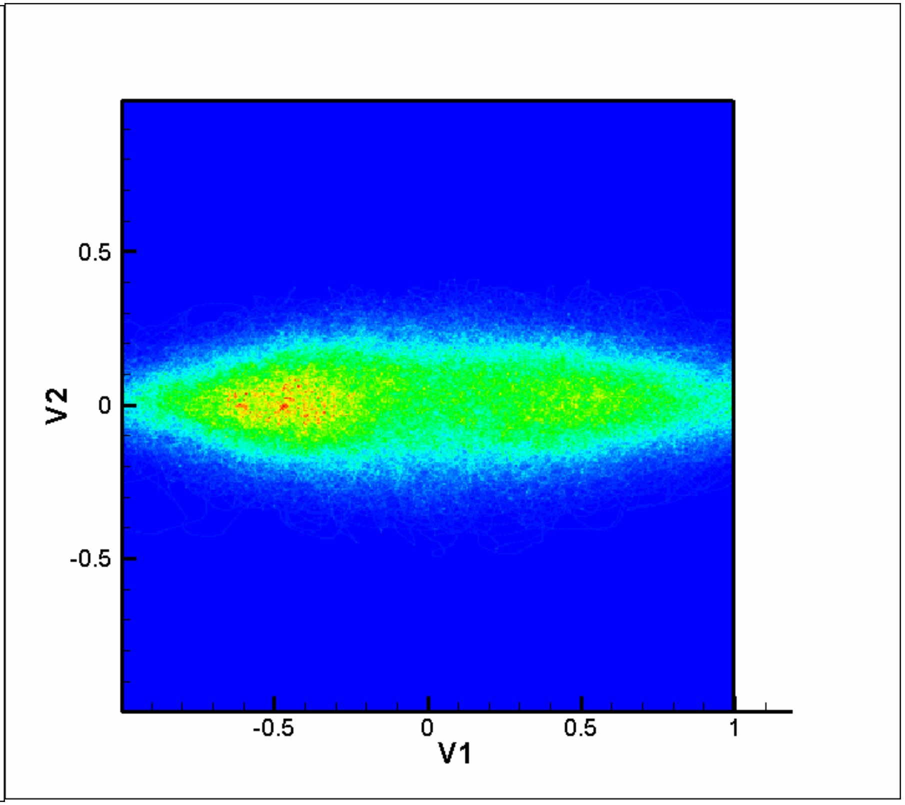


PdF_1 PdF_2

Probab. density function (M_{sh}=5, M_{tur}=0.08)

K_0=40

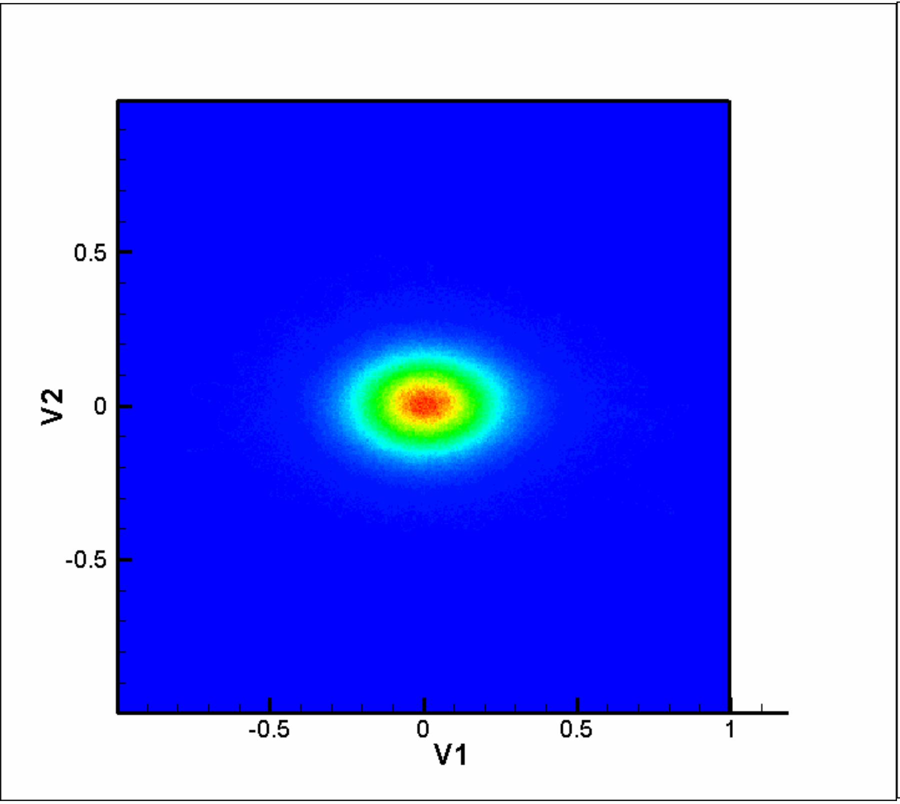


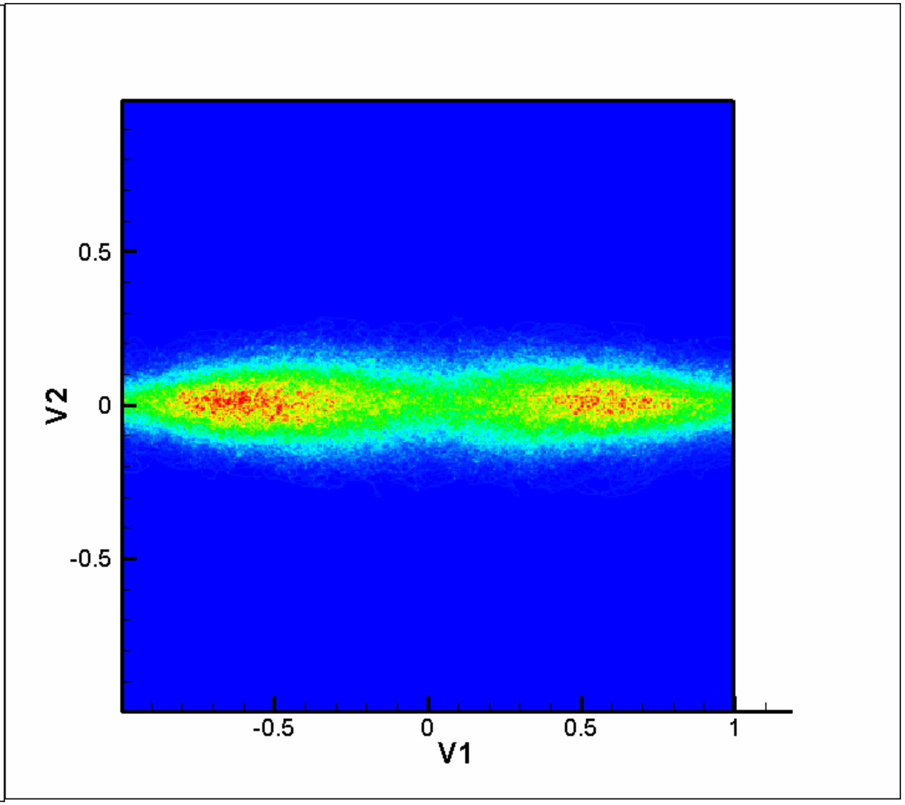


PdF_1 PdF_2

Probab. density function (M_{sh}=5, M_{tur}=0.08)

K_0=60

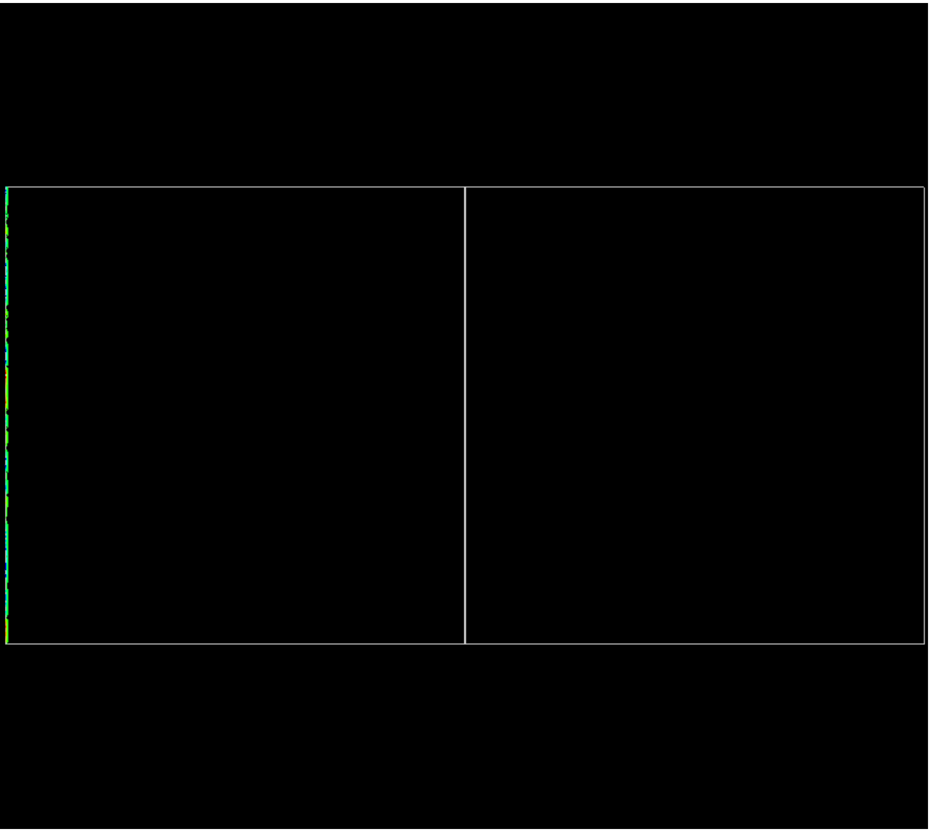


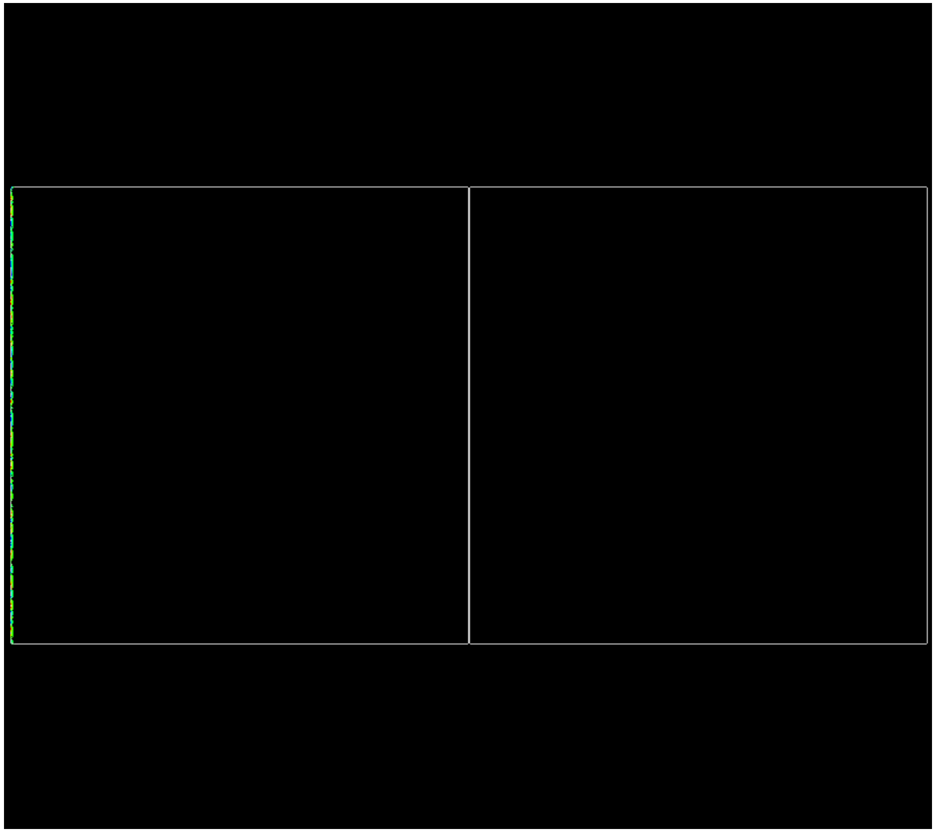


PdF_1 PdF_2

Visualization of transversal velocity

(M_{sh}=5, M_{tur}=0.08) :

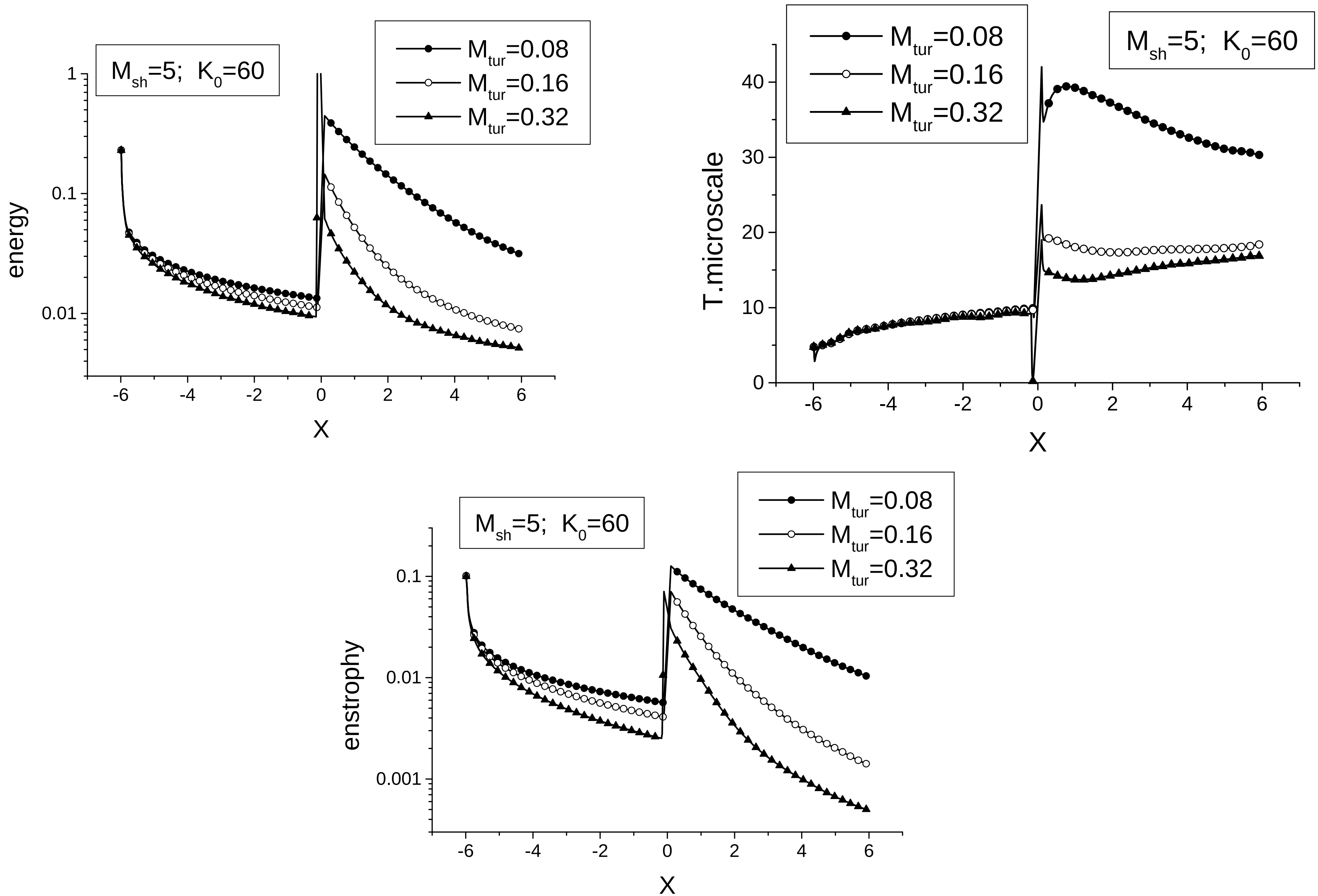




K_0=20

K_0=60

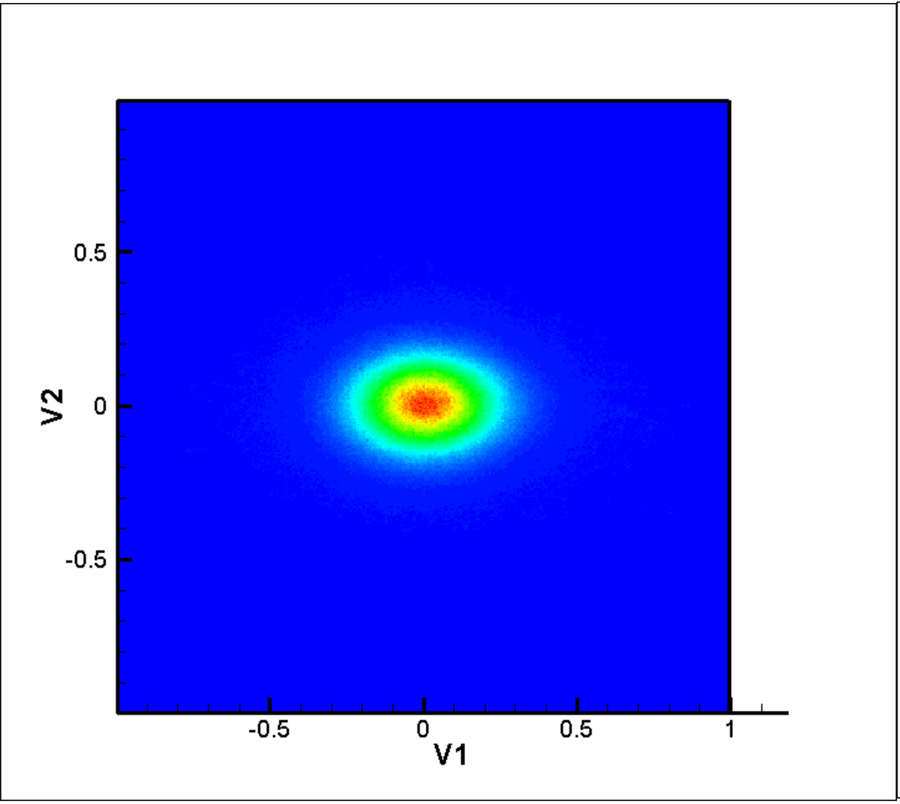
II.2.
Effect of the turbulence intensity (M_{tur})

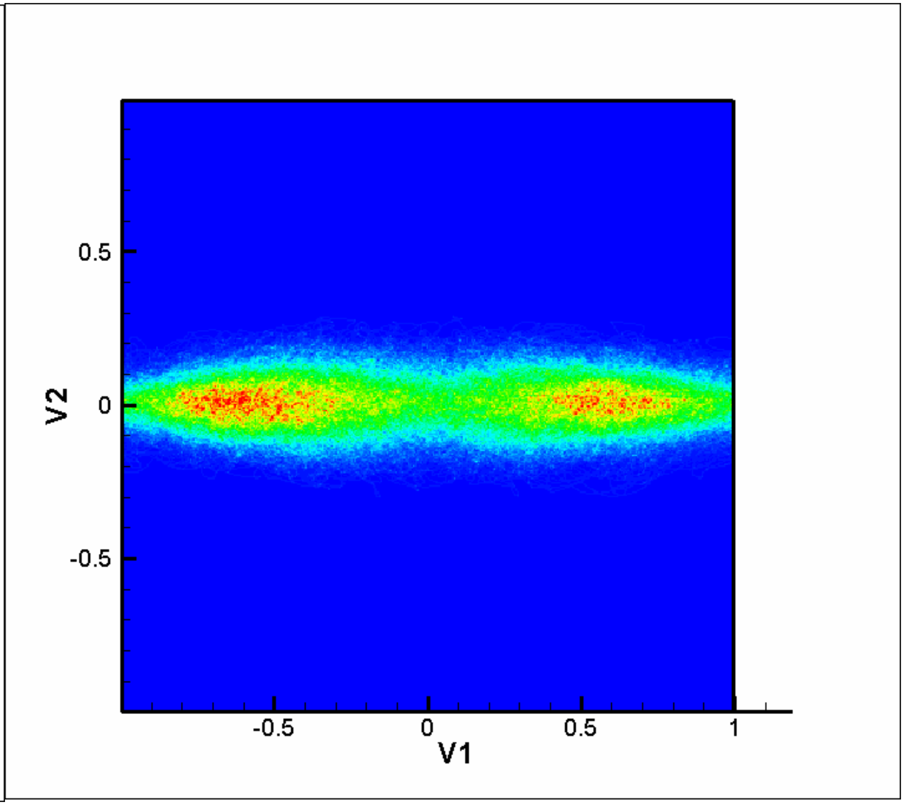


M_sh=5; K_0=60
M_tur=0.08
M_tur=0.16
M_tur=0.32
energy
X
T.microscale
enstrophy

Probab. density function (M_{sh}=5, K_0=60)

M_{tur}=0.08

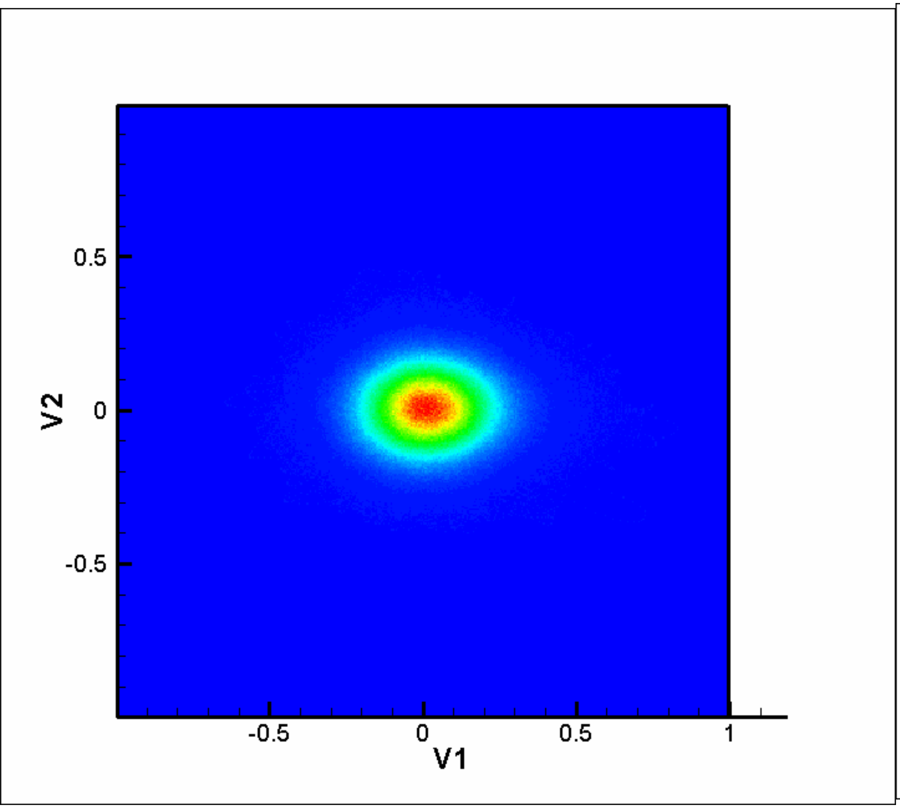


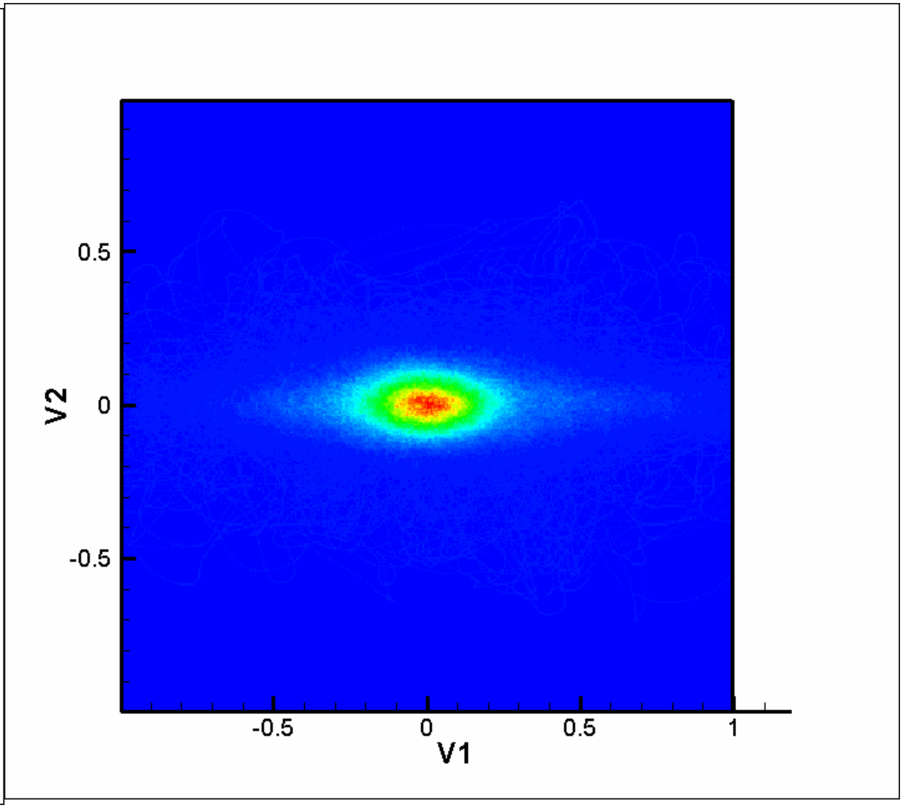


PdF_1 PdF_2

Probab. density function (M_{sh}=5, K_0=60)

M_{tur}=0.16

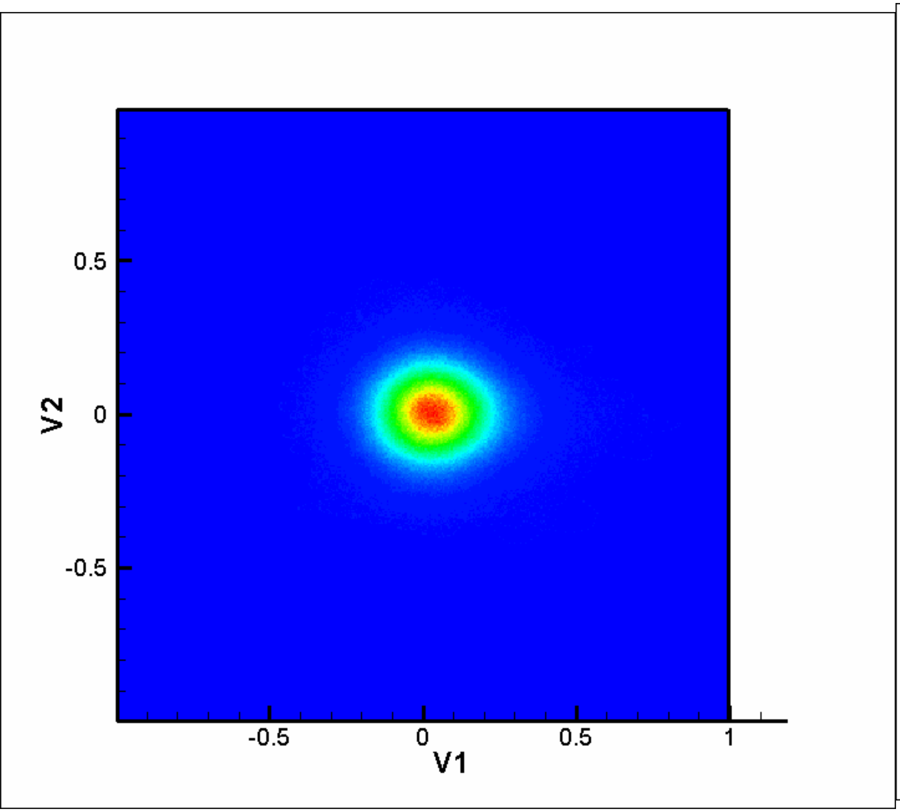


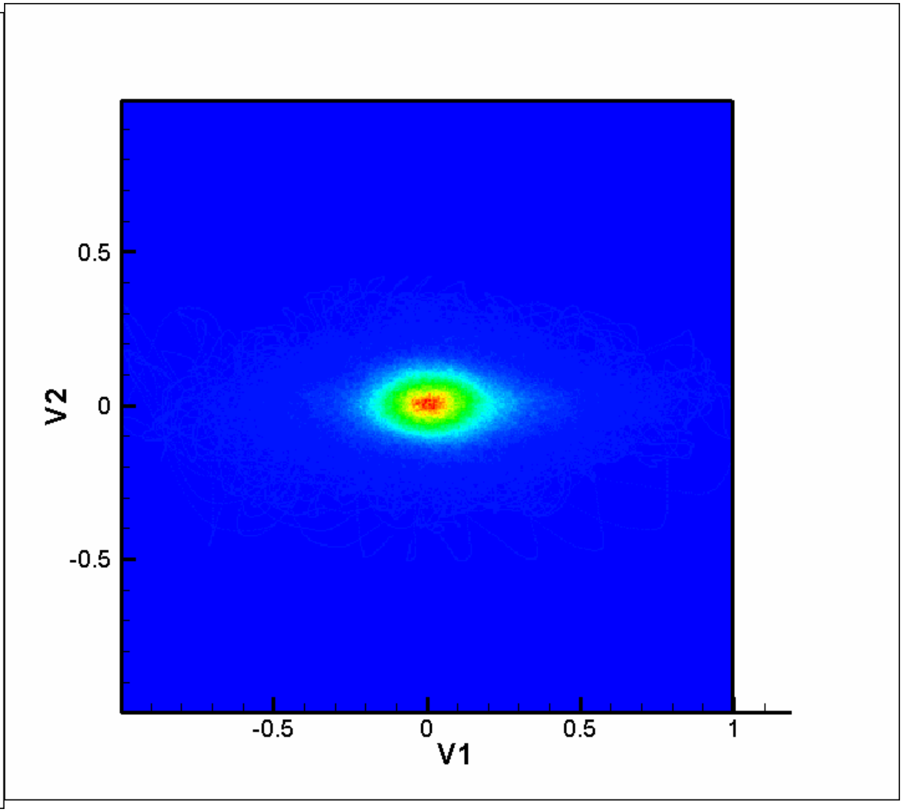


PdF_1 PdF_2

Probab. density function (M_{sh}=5, K_0=60)

M_{tur}=0.32

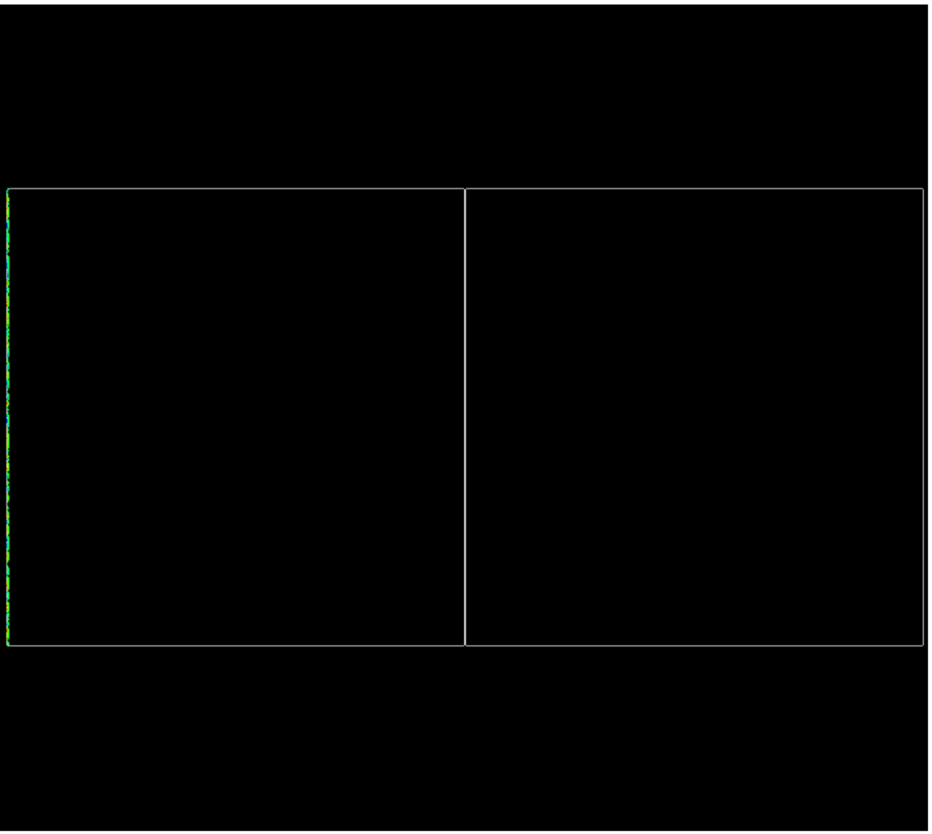




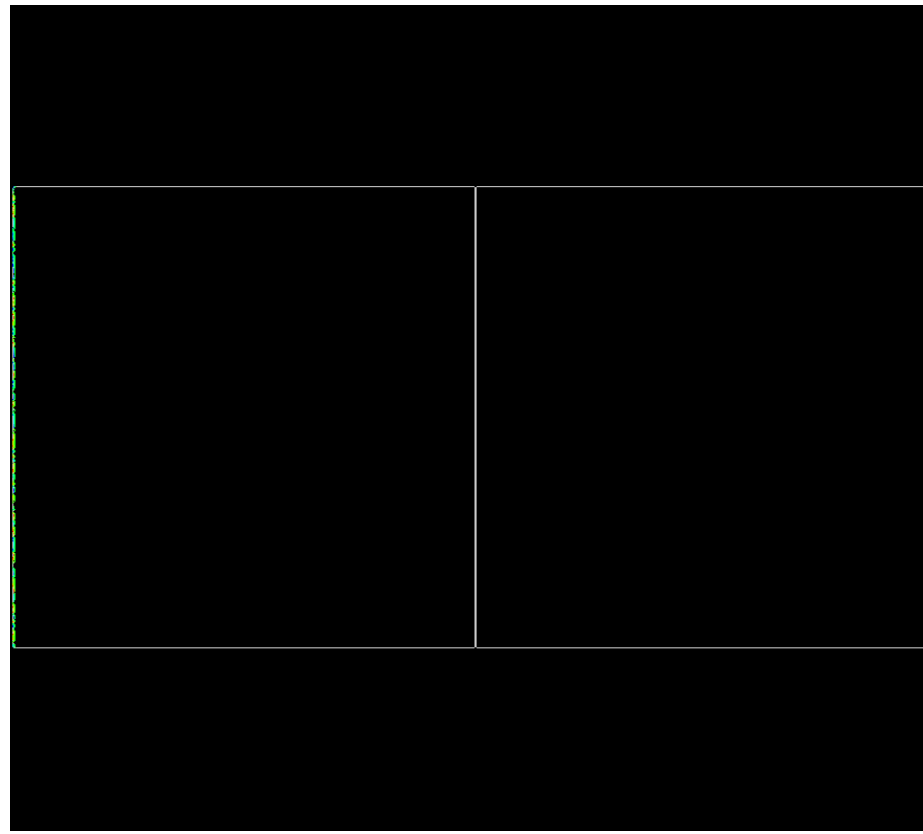
PdF_1 PdF_2

Visualization of transversal velocity

(M_{sh}=5, K_0=60) :



M_{tur}=0.08



M_{tur}=0.32

Conclusions

i) The **interaction of isotropic turbulence with a strong shock wave has been studied numerically** to investigate the statistics of the downstream turbulent flow. The data was obtained regarding the behavior of the turbulent energy, enstrophy, lengthscale, and other parameters at the shock and downstream from it.

ii) The basic finding concerns **the fact that the turbulence looses the property of isotropicity when crossing the shock**; this is evidenced in the change of the velocity PDF that acquires an asymmetry; also the visualization of the flow reveals appearance of large coherent structures in the flow behind the shock.