Numerucal Investigation of Resonators for Acoustic Liners

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Outline

- Motivation
- Scheme of Computational Experiment
- In-house Code in Use
- 2D Numerical Results
- 3D Numerical Results
- Conclusion

Motivation

Basic way of numerical investigation of real engine configuration with liners –

acoustics and flow simulation with **impedance boundary conditions** at the walls

⇓

impedance $Z(\omega)$ is needed

⇑

impedance is taken from the **physical experiment** (in channels)

⇑

the resulting impact
of the **collectivity effect** and **basic properties of a liner cell**

What is the impact of a liner cell?
Which factors influence its properties?
Are there options for the optimization?

Physical Experiment

Industrial
lined channels

Difficult to investigate
a single resonator

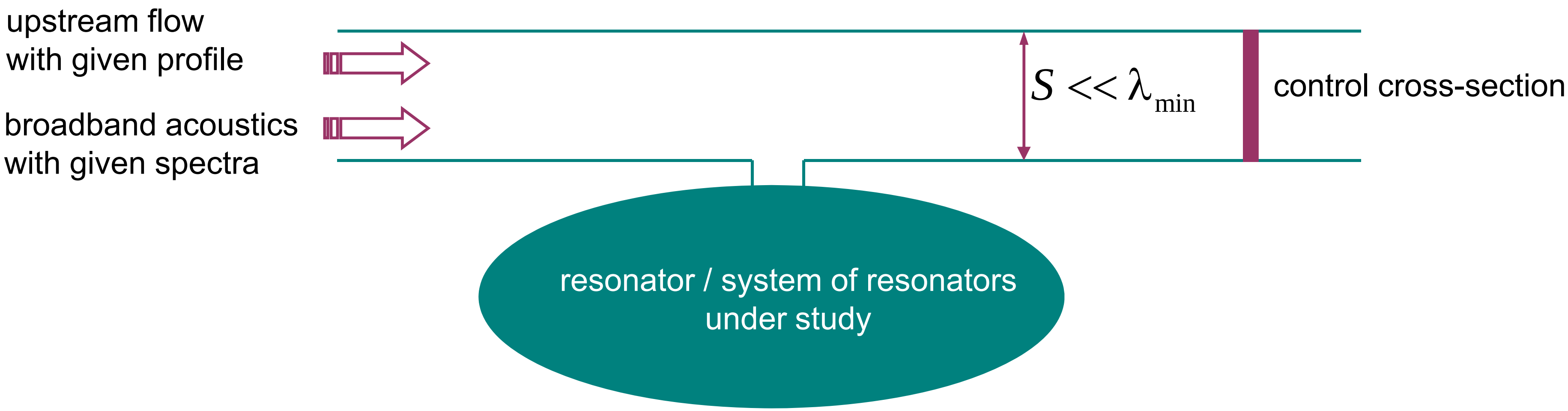
Academic
impedance tubes

Impossible to investigate
a resonator at real conditions
(upstream flow, boundary layers, etc)

Computational experiment

in configuration “resonator/system of resonators” in a wave guide

Scheme of Computational Experiment



Basing on the incoming and outgoing data

one can find the impedance of resonators under study

In-house code built on the vertex-centered edge-based methods

NOISEtte

in-house code in FORTRAN95
for solving CAA and CFD problems

Mathematical Models

- Euler Equations, Navier-Stokes Equations
- Non-Linear Disturbance Equations (NLDE) – different forms
- Linearized Euler Equations
- RANS with Spalart-Allmares turbulence model
- DES, DDES (are currently tested)

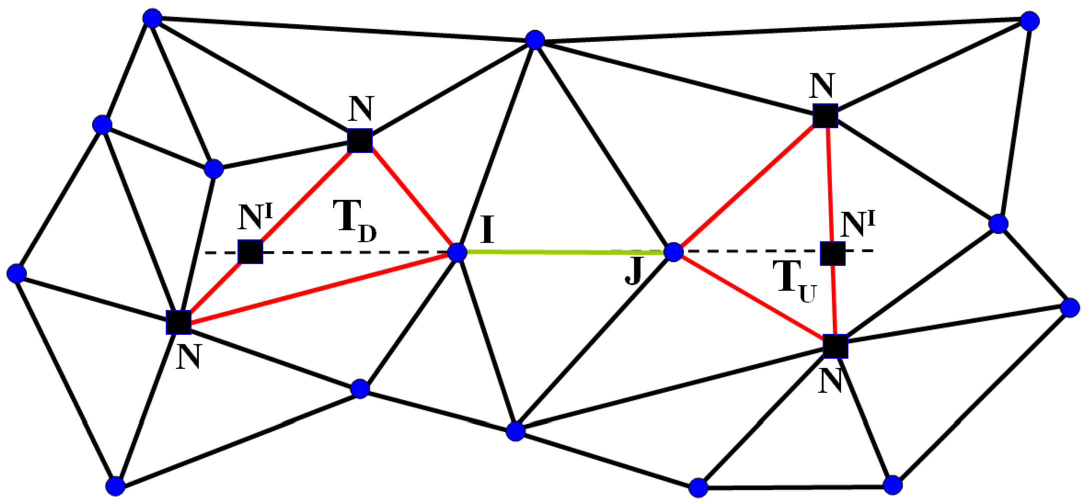
Numerical Algorithms

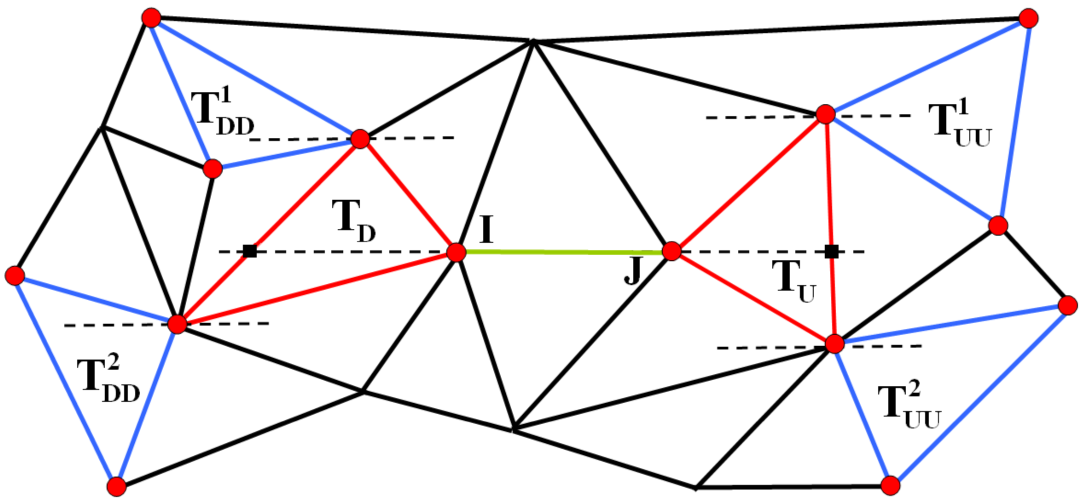
- Higher-accuracy multi-parameter vertex-centered edge-based schemes - upto the 6^{th} order (for structured meshes)
- Explicit 4^{th} order Runge-Kutta method and Its linear version of arbitrary high order
- Implicit second-order time integration based on Newton linearization of equations system with block linear solvers: SOR including Gauss-Seidel, SSOR, Krylov-type (GMRES, BCG,…)
- Non-reflecting boundary conditions (flux splitting-based BC, characteristic BC, Tam radiation and outflow BC)

Parallel Implementation

- SMP Systems (OpenMP), MPP Systems (MPI), Hybrid Systems (MPI + OpenMP)
- Efficient computations using thousands of CPU

Edge-Based Higher-Accuracy Scheme for Unstructured Grids





Central gradient

$$\left(\nabla \mathbf{F}\right)^{C} = \mathbf{F}_j - \mathbf{F}_i$$

in green

Nodal gradients in nodes **N**

$$\left(\nabla \mathbf{F}\right)_i = \frac{1}{|C_i|}\sum_{j\in\Omega_i}\frac{|T_j|}{3}\sum_{k\in T_j}\mathbf{F}_k\nabla\varphi_k$$

in blue – participating nodes

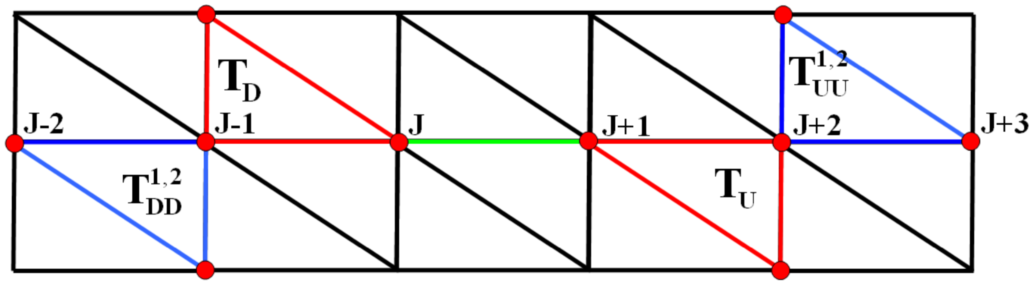
Gradient on a triangle

$$\nabla \mathbf{F}\Big|_T = \sum_{k\in T}\mathbf{F}_k\nabla\varphi_k$$

in red - first level upwinding triangles,

in blue - second level upwinding triangles

Edge-Based Higher-Accuracy Scheme for Unstructured Grids



2D	1D
$(\nabla \mathbf{F})_{ij}^{c} \cdot \mathbf{ij} = \mathbf{F}_j - \mathbf{F}_i$	$\Delta \mathbf{F}_{j+1/2} = \mathbf{F}_{j+1} - \mathbf{F}_j$
$\nabla \mathbf{F}\big\|_{T^D} \cdot \mathbf{ij}$	$\Delta \mathbf{F}_{j-1/2} = \mathbf{F}_j - \mathbf{F}_{j-1}$
$\nabla \mathbf{F}\big\|_{T^U} \cdot \mathbf{ij}$	$\Delta \mathbf{F}_{j+3/2} = \mathbf{F}_{j+2} - \mathbf{F}_{j+1}$
$\nabla \mathbf{F}\big\|_{T^{DD}} \cdot \mathbf{ij}$	$\Delta \mathbf{F}_{j-1/2} = \mathbf{F}_j - \mathbf{F}_{j-1}$
$\nabla \mathbf{F}\big\|_{T^{UU}} \cdot \mathbf{ij}$	$\Delta \mathbf{F}_{j+5/2} = \mathbf{F}_{j+3} - \mathbf{F}_{j+2}$
$(\nabla \mathbf{F})_i^N \cdot \mathbf{ij}$	$\Delta_0 \mathbf{F}_j = \mathbf{F}_{j+1} - \mathbf{F}_{j-1}$
$(\nabla \mathbf{F})_j^N \cdot \mathbf{ij}$	$\Delta_0 \mathbf{F}_{j+1} = \mathbf{F}_{j+2} - \mathbf{F}_j$

Central gradient

$$(\nabla \mathbf{F})^C = \mathbf{F}_j - \mathbf{F}_i$$

in green

Nodal gradients in nodes **N**

$$(\nabla \mathbf{F})_i = \frac{1}{|C_i|} \sum_{j \in \Omega_i} \frac{|T_j|}{3} \sum_{k \in T_j} \mathbf{F}_k \nabla \varphi_k$$

in blue – supporting nodes

Gradient on a triangle

$$\nabla \mathbf{F}\big|_T = \sum_{k \in T} \mathbf{F}_k \nabla \varphi_k$$

in red - first level upwinding triangles,

in blue - second level upwinding triangles

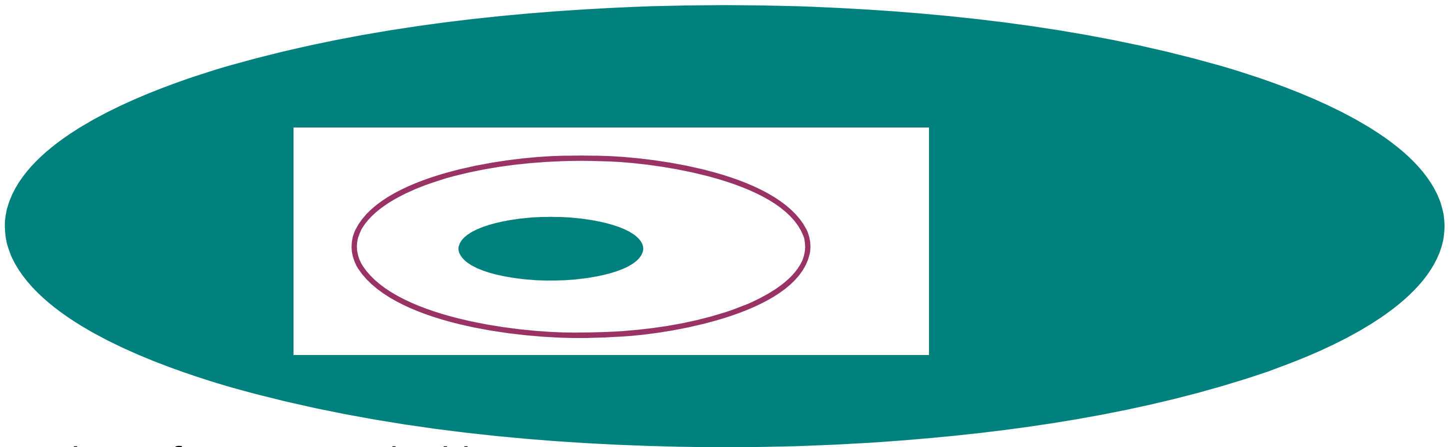
Edge-Based Higher-Accuracy Scheme for Unstructured Grids

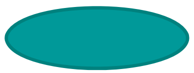
Idea – to gain the advantages of structured and unstructured grids within one algorithm

Requirements to the grid:

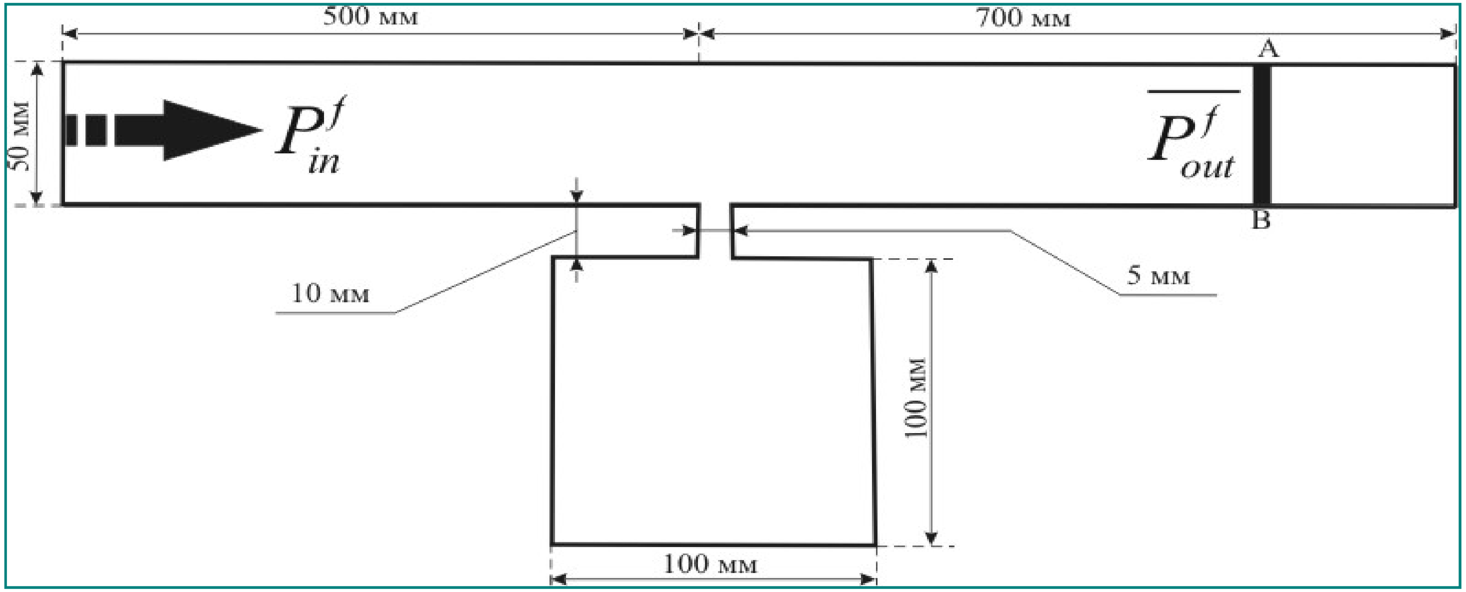
– to have as large “structured” subdomains as possible
– to have rather smooth interfaces between “structured” and “unstructured” regions (not well developed yet)

Desirable structure of the grid



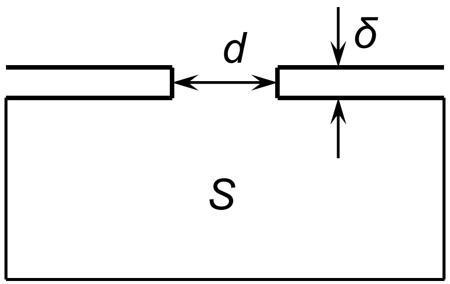
 - regions of unstructured grid

2D Numerical Results



Validation on Helmholtz resonator

Theoretical estimation of characteristic frequency



Oscillating mass in hole

$$m \cong \rho\, d \left[\delta + d \ln\left(\frac{H}{d} \right) \right]$$

Elasticity coefficient

$$\kappa = \frac{d^2}{\beta\, S}$$

$$\beta = \frac{1}{\rho\, c^2}$$

$$\omega_0^2 \cong \frac{\kappa}{m} \cong \frac{c^2}{\left(\frac{\delta}{d} + \alpha \ln\left(\frac{H}{d} \right) \right) S}$$

$$f_0 \approx 261.7\ Hz$$

$$\omega_0 = 2\pi f_0$$

Computational Details

In-house code NOISEtte:

- Edge-Based Scheme of the 5th order
- 4th order Runge-Kutta scheme
- boundary conditions based on Steger-Worming flux splitting

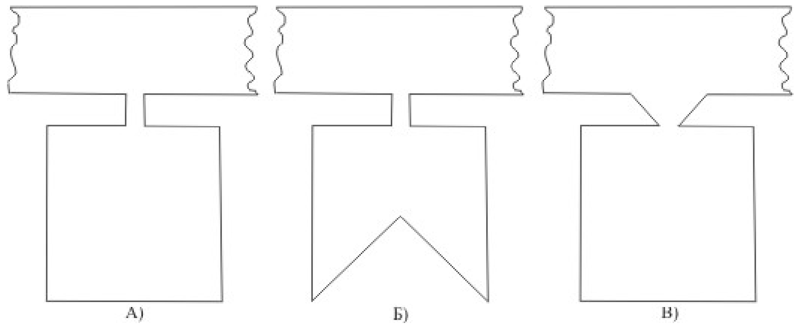
Analysis
Fast Fourier transform

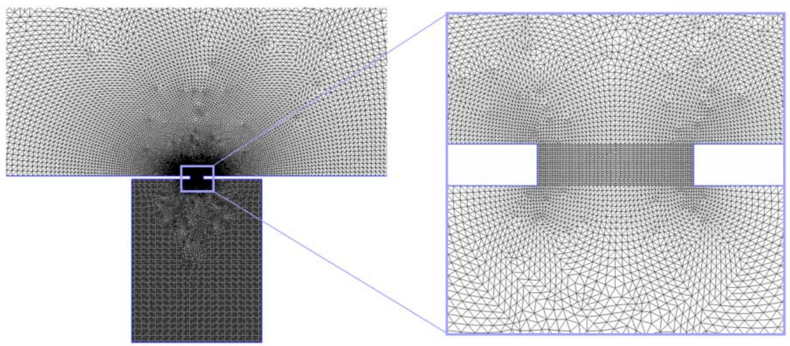
Grids
Structured in channel, unstructured in the vicinity of hole, 20K -150K nodes

Estimation of boundary layer thickness for the upstream flow

$$\delta_{\mathrm{BL}} \sim \frac{1}{\sqrt{\mathrm{Re}\left(D_{channel}\right)}}$$

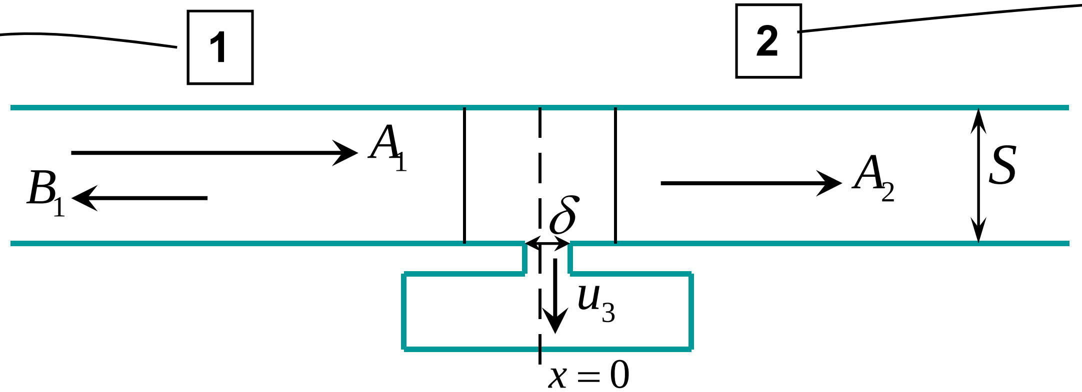
3 configuration of resonators under study





Incoming acoustic radiation
Uniformly distributed broadband noise

How to Calculate Impedance



$$p_2 = A_2 e^{ikx}$$

$$u_2 = \frac{1}{\rho c} A_2 e^{ikx}$$

$$p_1 = A_1 e^{ikx} + B_1 e^{-ikx}$$

$$u_1 = \frac{1}{\rho c}\left(A_1 e^{ikx} - B_1 e^{-ikx}\right)$$

at $x = 0$

$$p_1(0) = p_2(0)$$

$$Su_1 - Su_2 = -\delta z^{-1} p \qquad z \equiv \frac{p}{u_3}$$

Mass conservation law

Reflection and transmission coefficients for harmonics k

$$V = \frac{B_1}{A_1} \qquad W = \frac{A_2}{A_1}$$

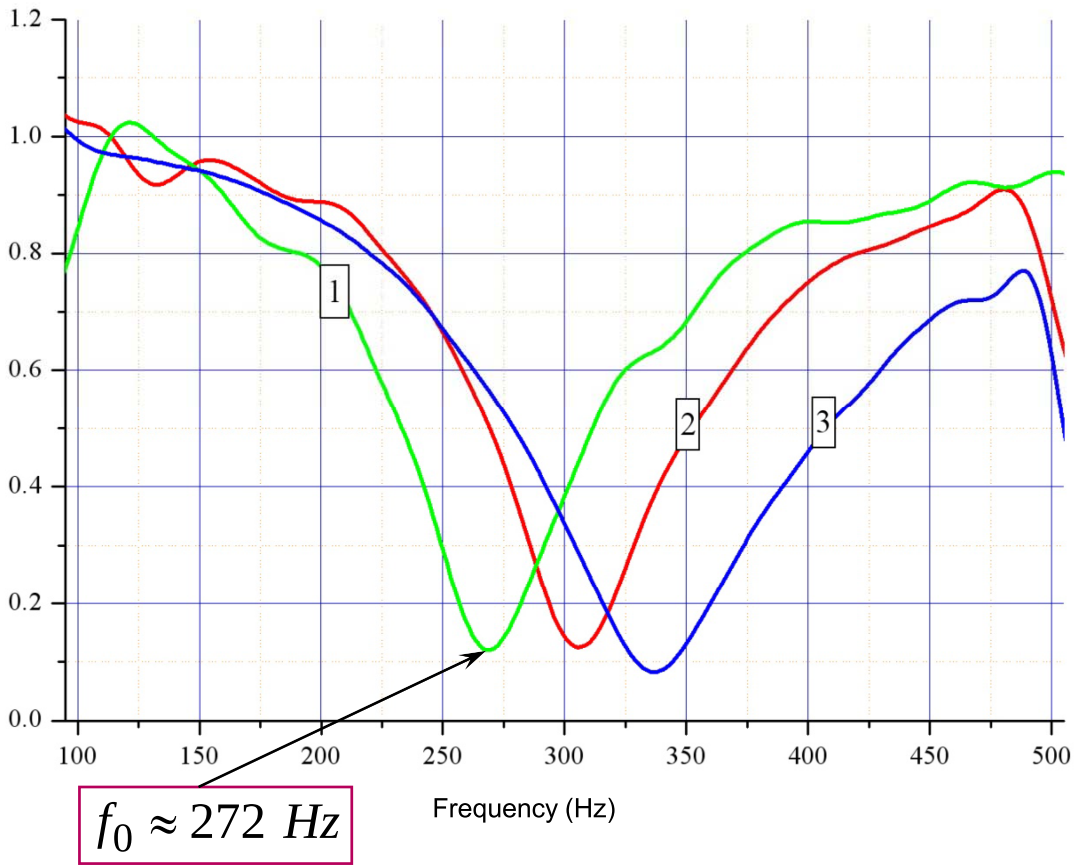
$$W = V + 1$$

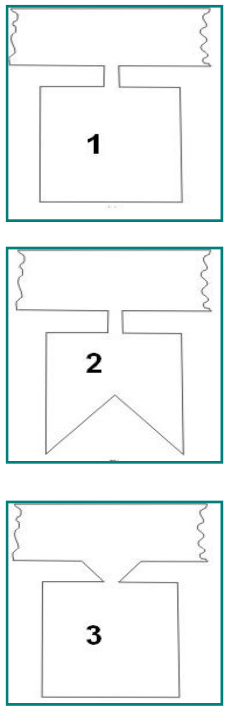
Acoustic impedance

$$\frac{z}{\rho c} = \frac{\delta}{S}\frac{W}{2(1-W)}$$

Validation on Helmholtz Resonator

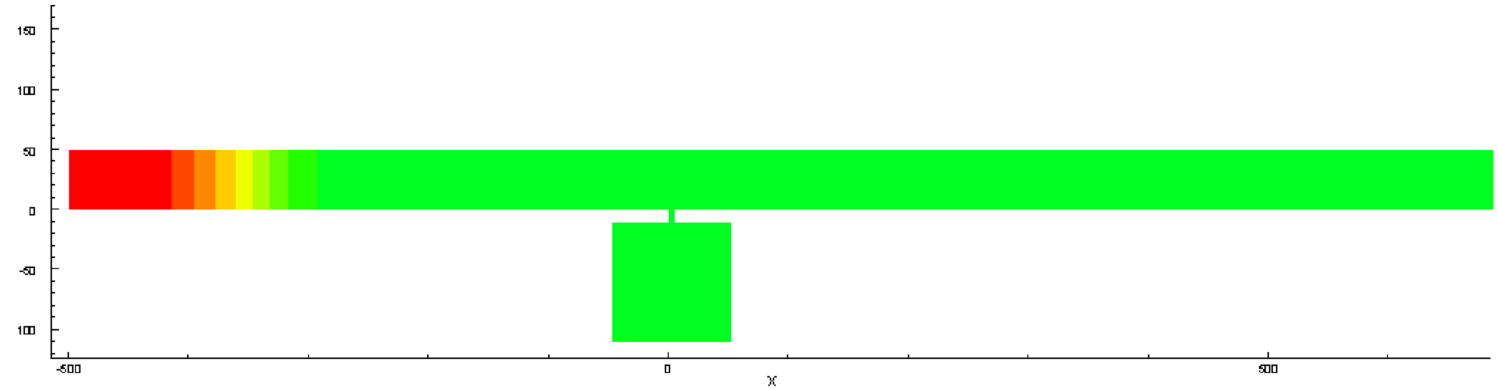
Linear resonance: Transmission coefficient

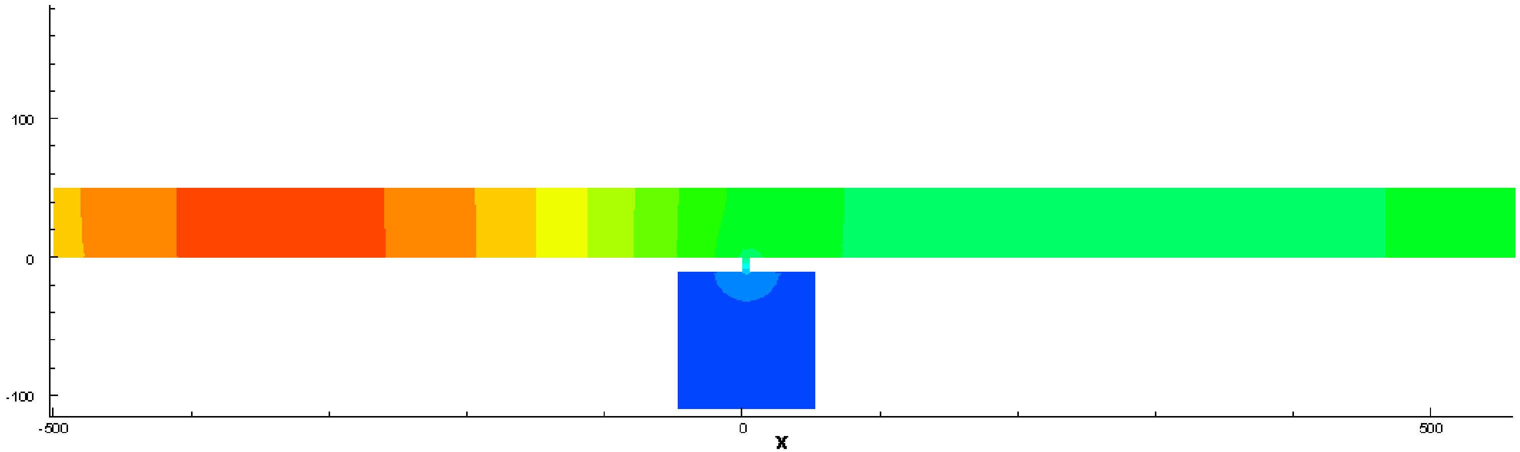




Validation on Helmholtz Resonator

Pressure waves at non-resonance (top) and resonance (bottom) frequencies

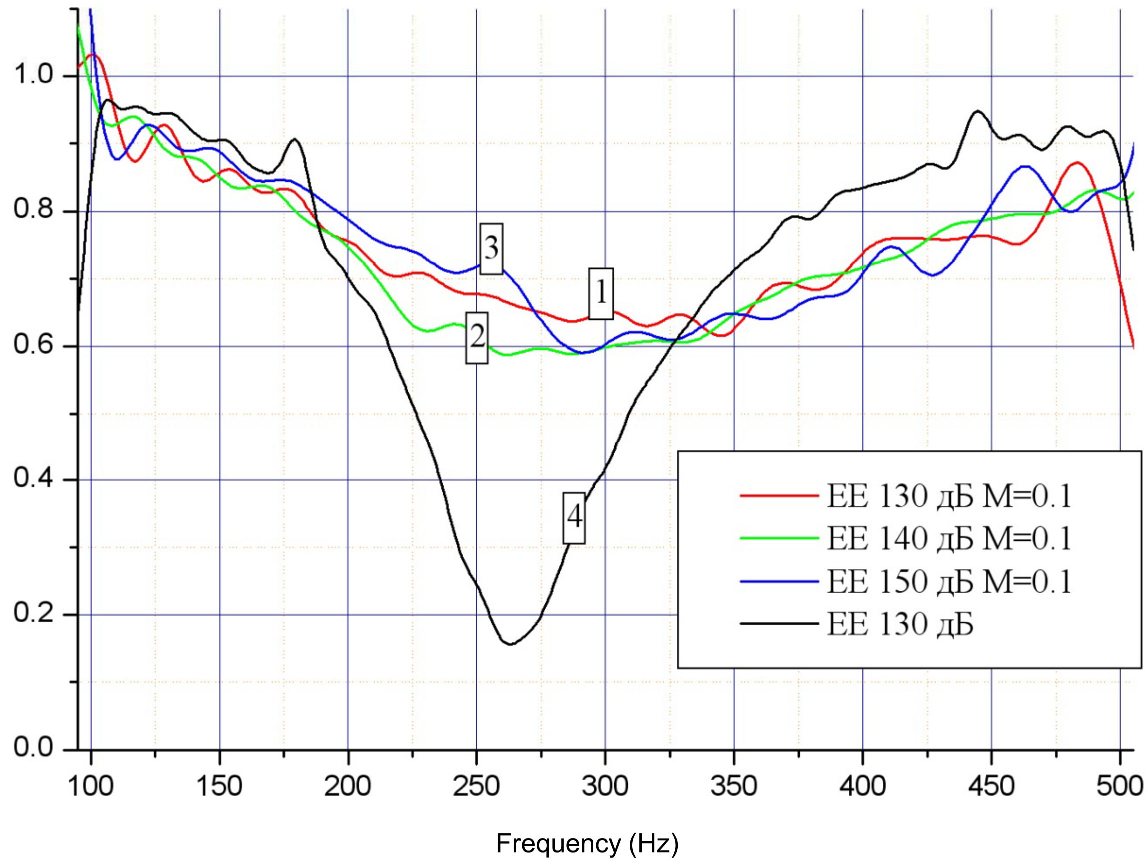




мультики

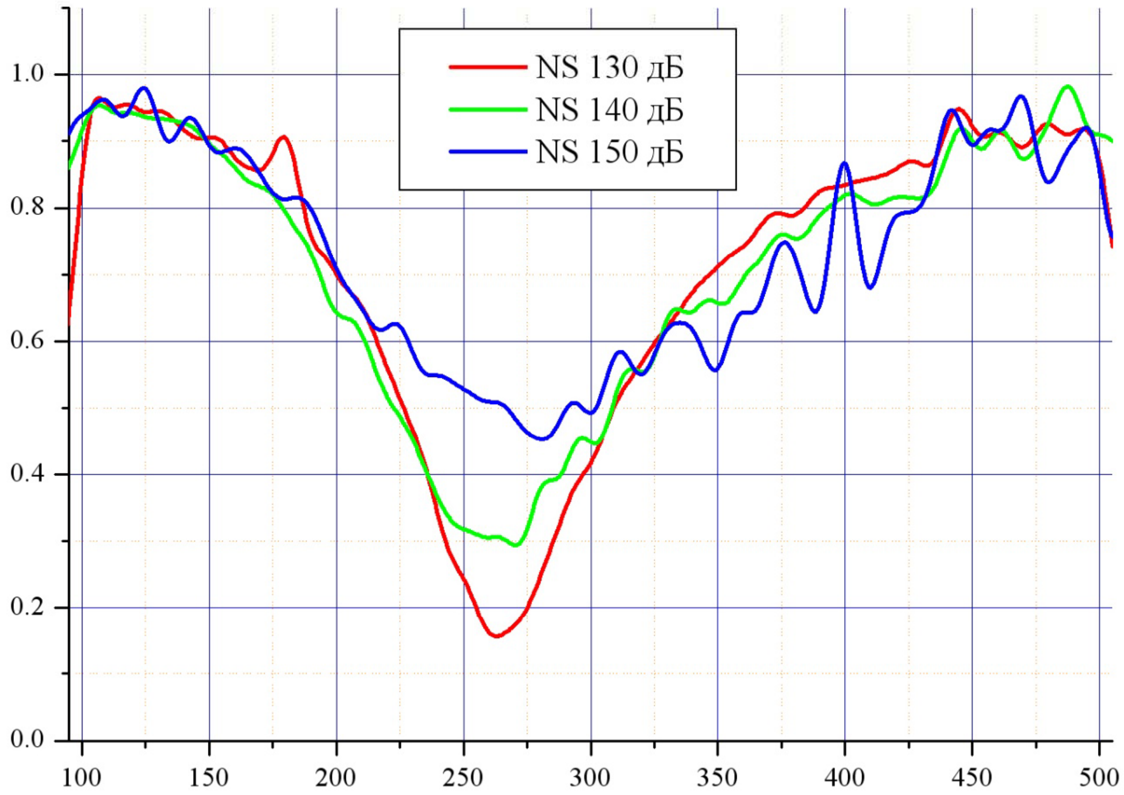
Euler equations: Transmission coefficient

(without and with upstream flow, M=0.5)



Navier-Stokes equations: Transmission coefficient

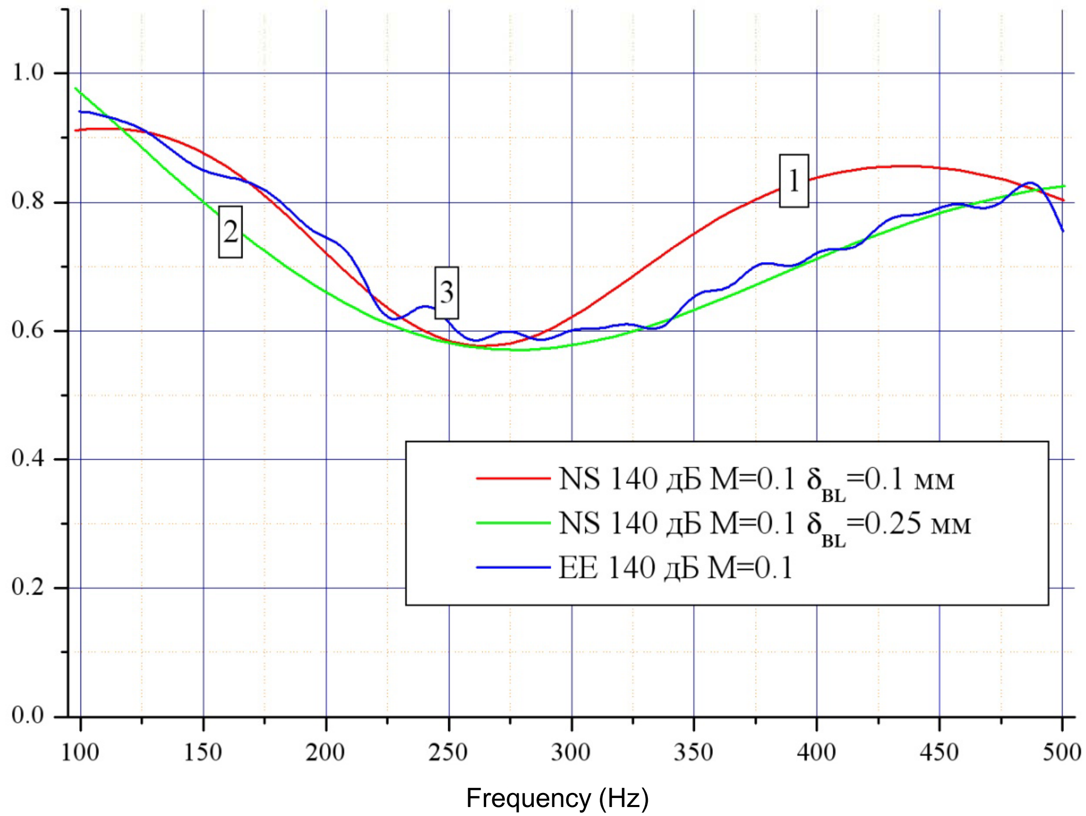
(no upstream flow)



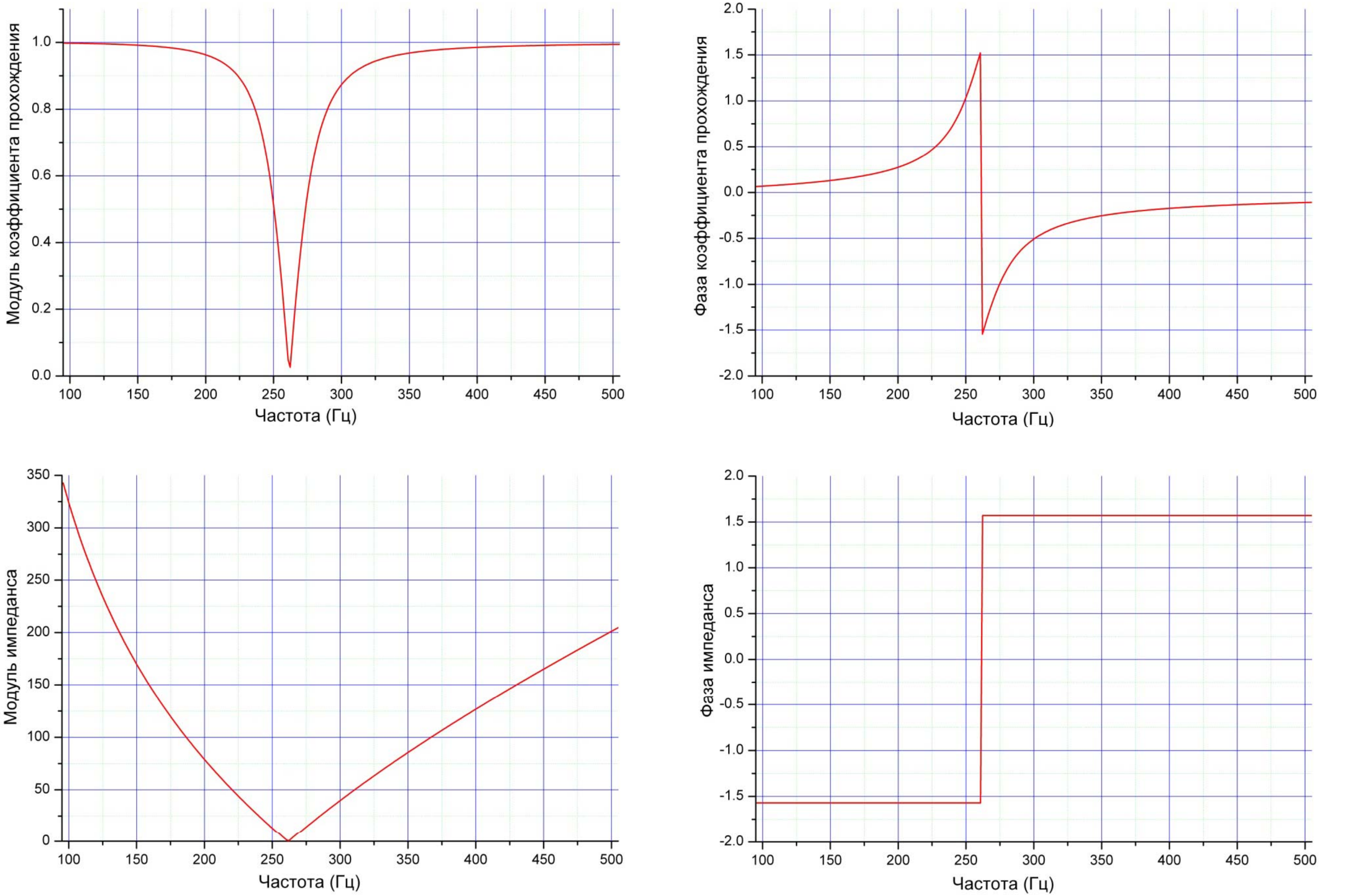
Frequency (Hz)

Navier-Stokes equations: Transmission coefficient

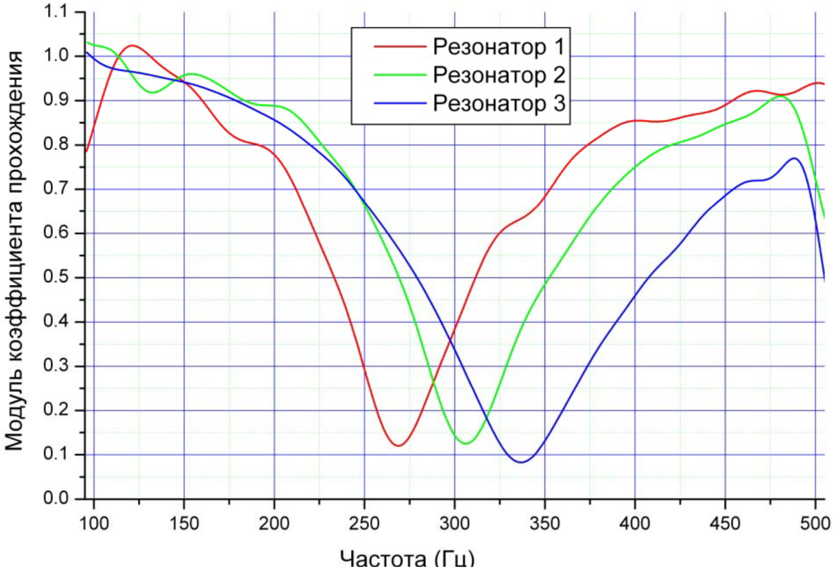
(upstream flow with boundary layers, M=0.1)

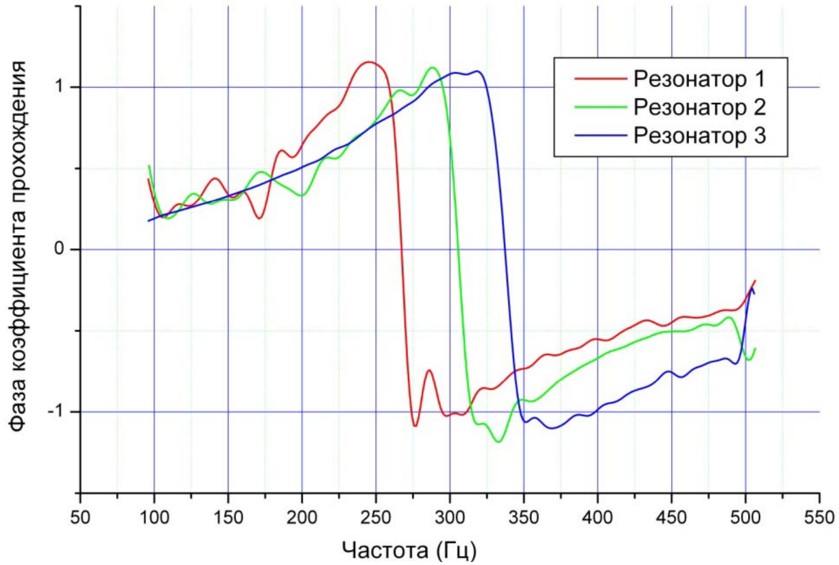


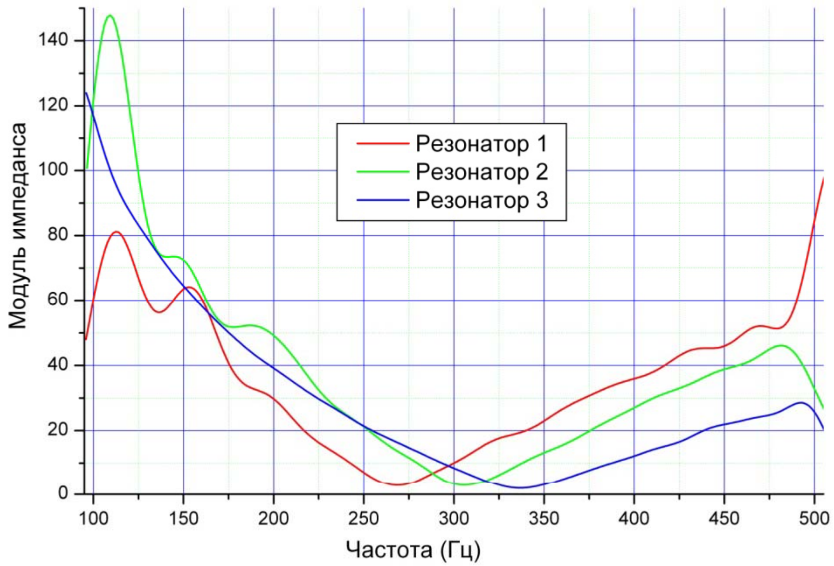
Analytical behavior in linear case

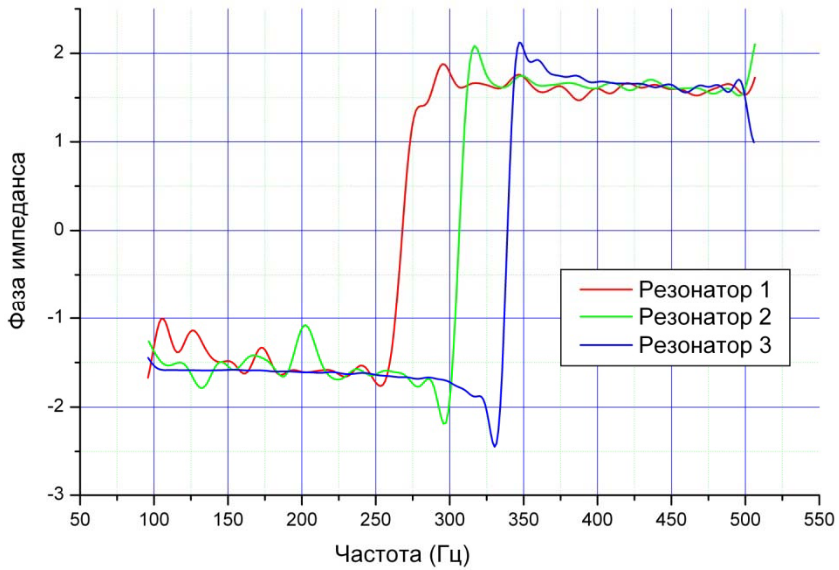


Numerical results in linear case

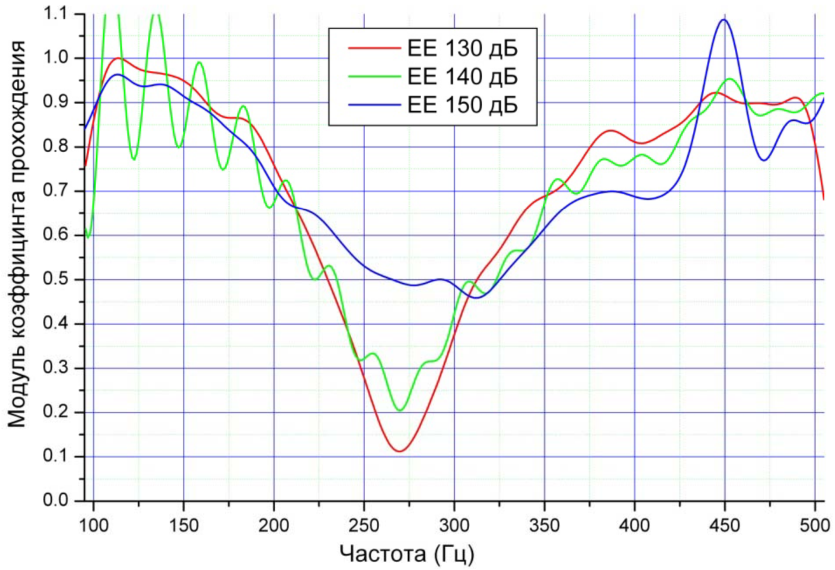


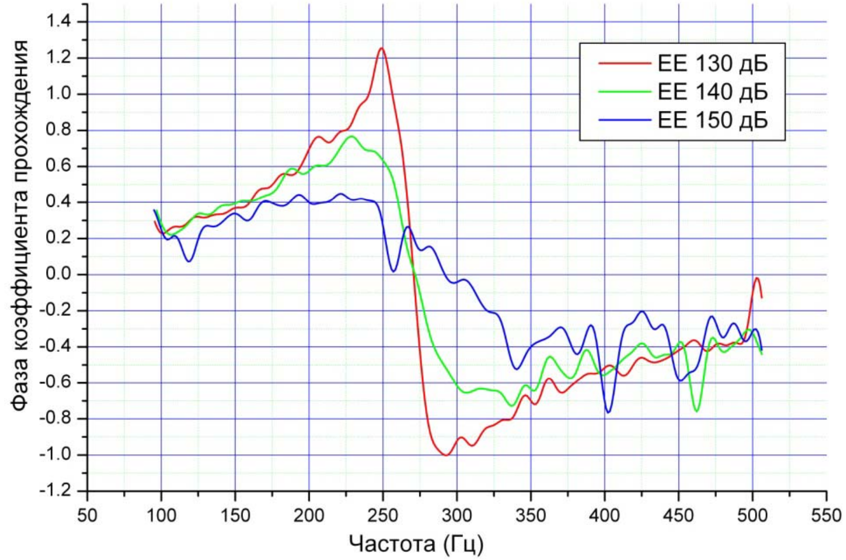


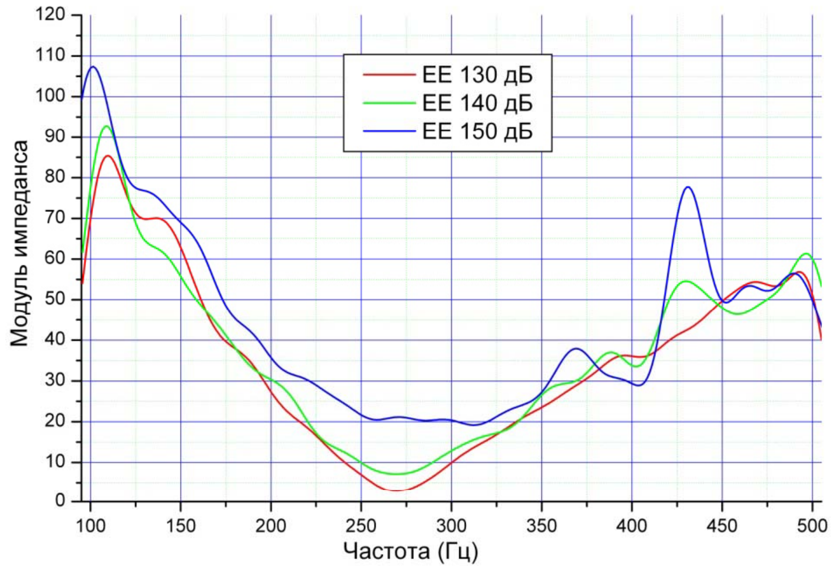


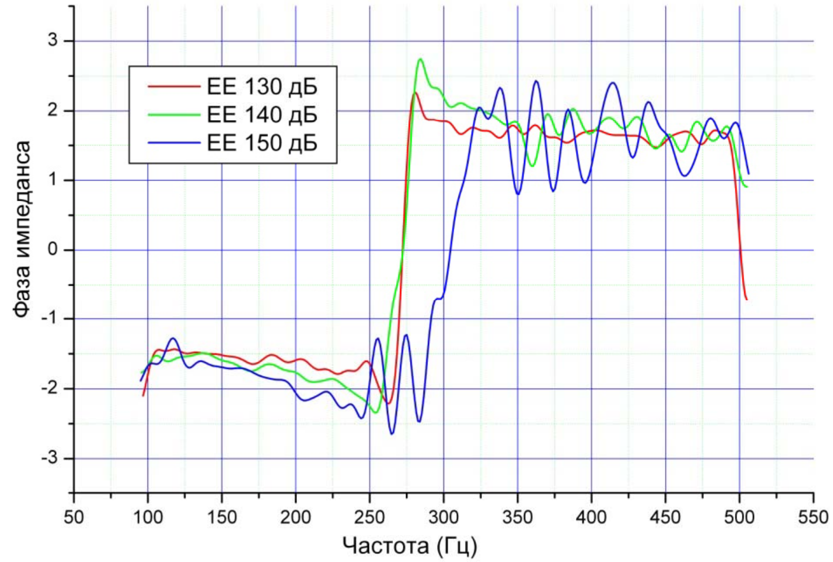


Numerical results in nonlinear case

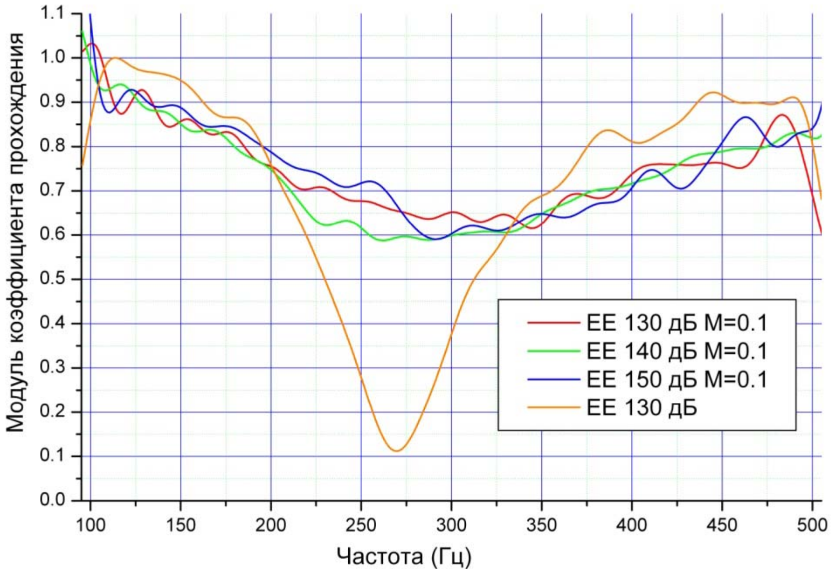


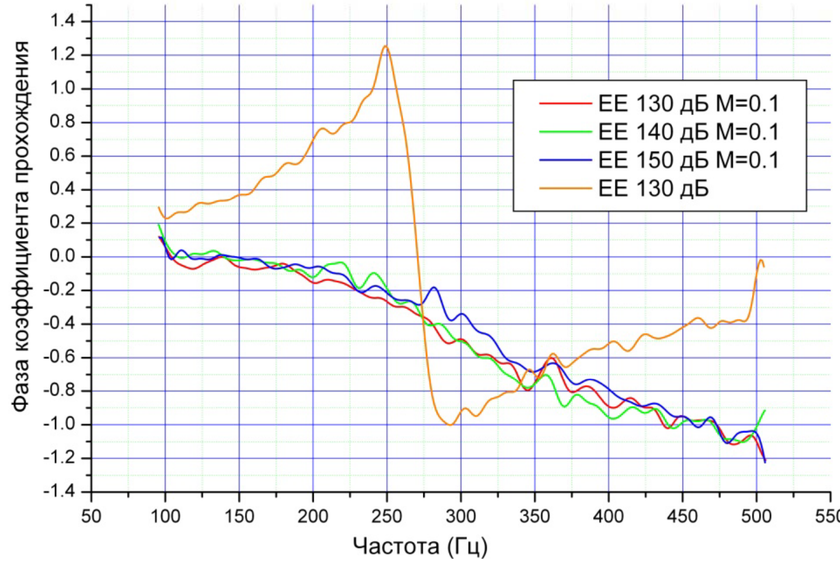


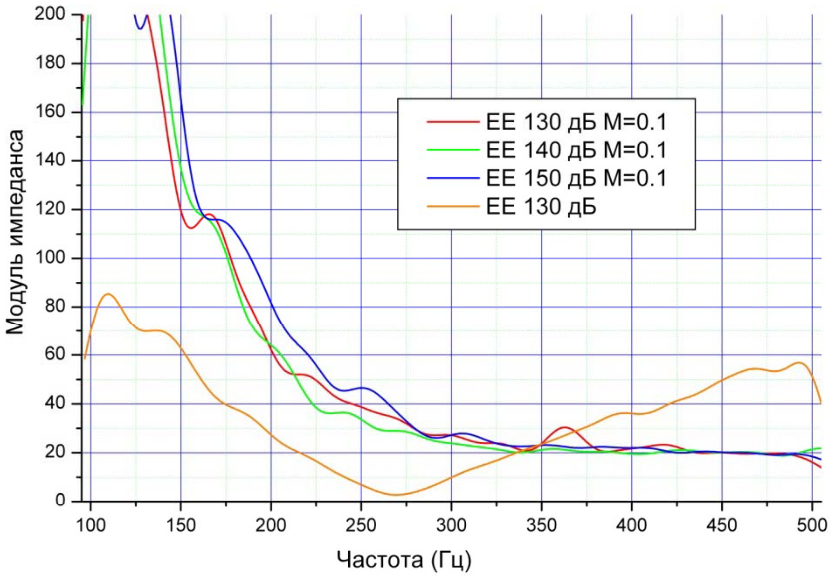


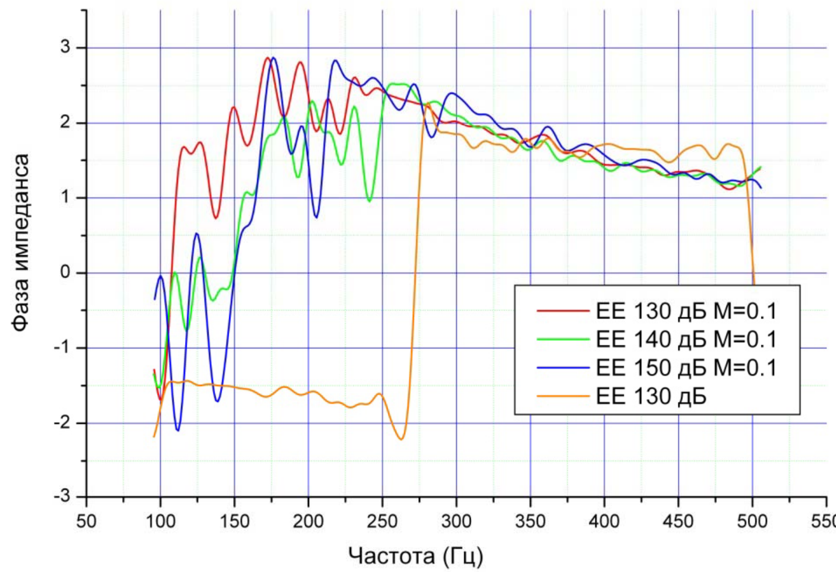


Numerical results in nonlinear case + upstream flow

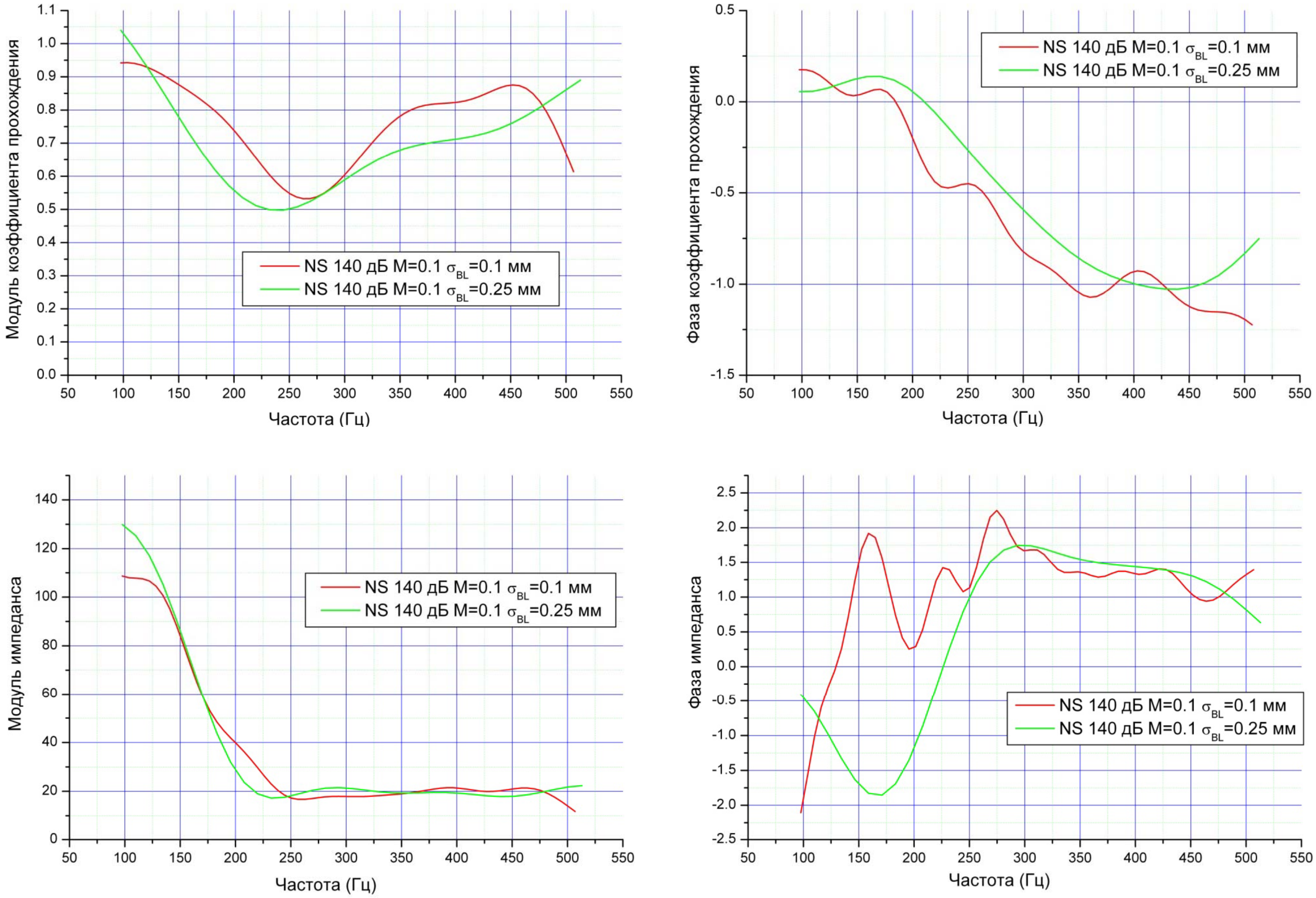




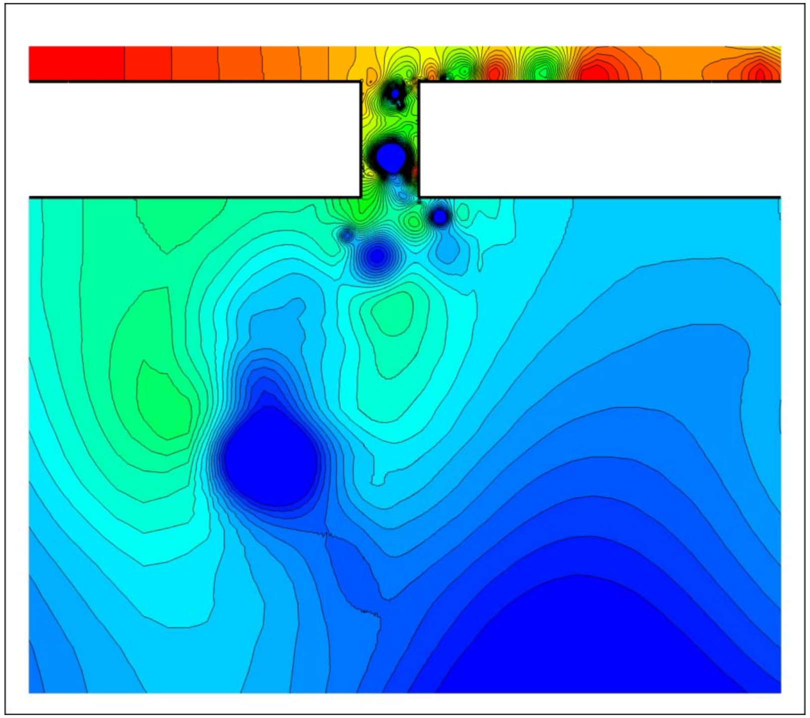




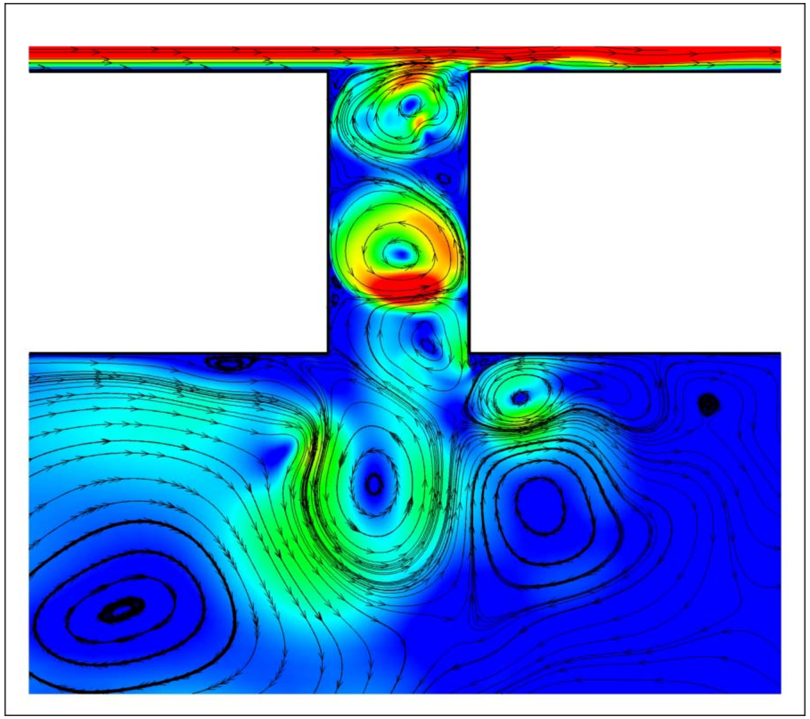
Numerical results in nonlinear case + upstream flow + boundary layer



2D Numerical Results

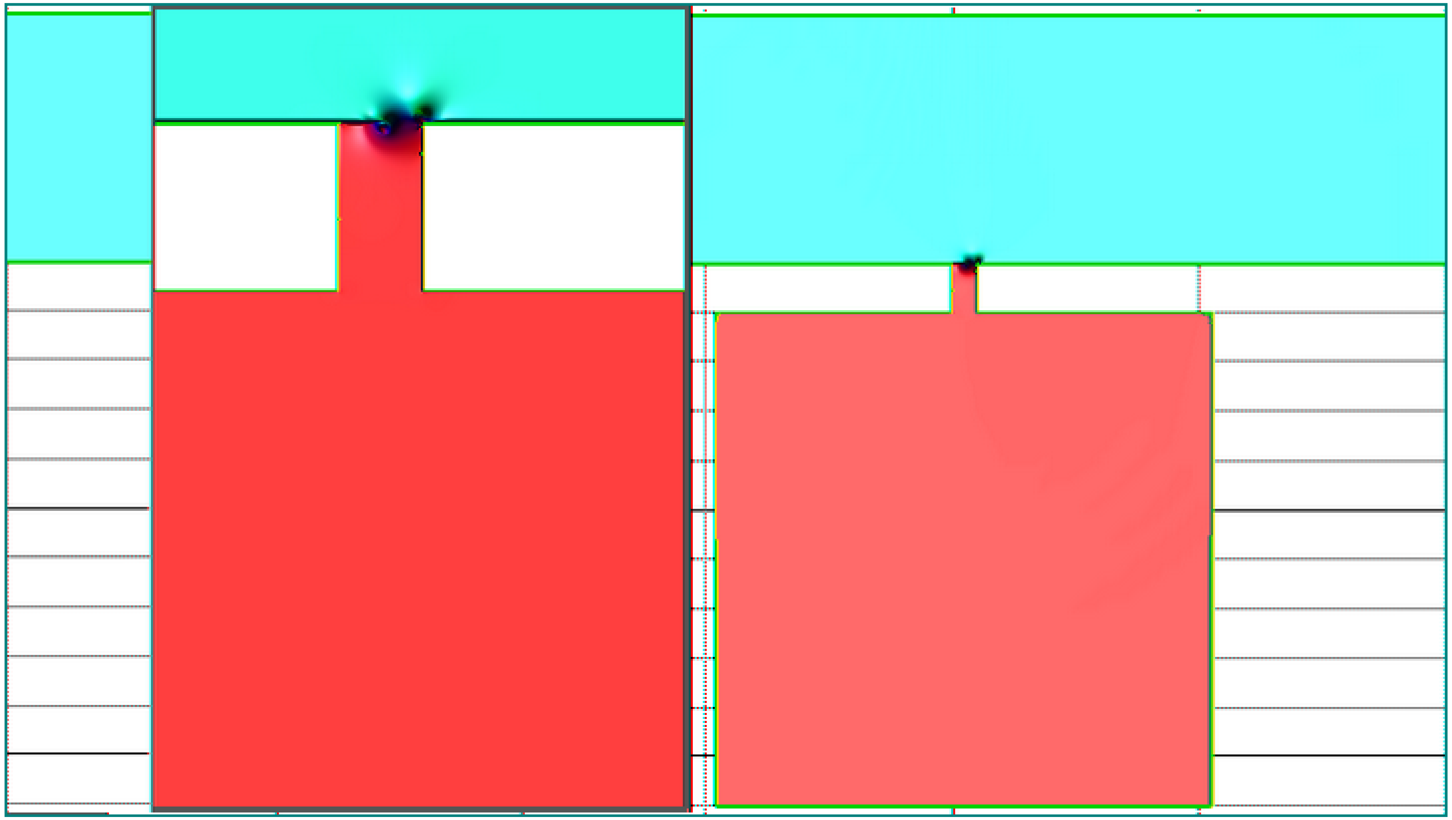


Isolines of pressure



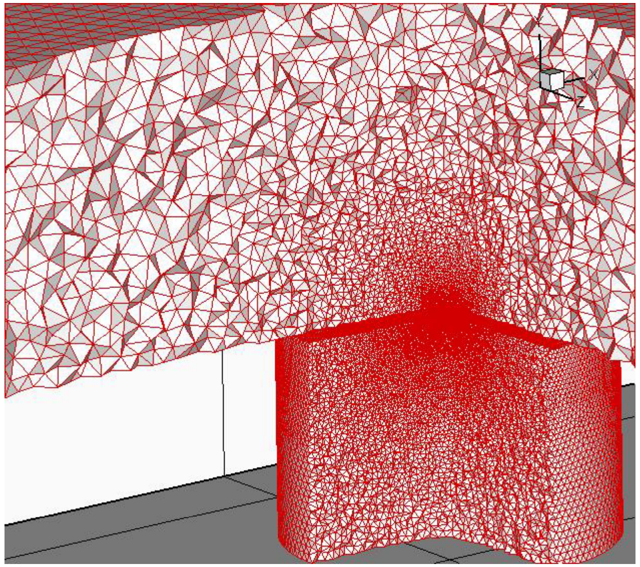
Absolute velocity field and streamtraces

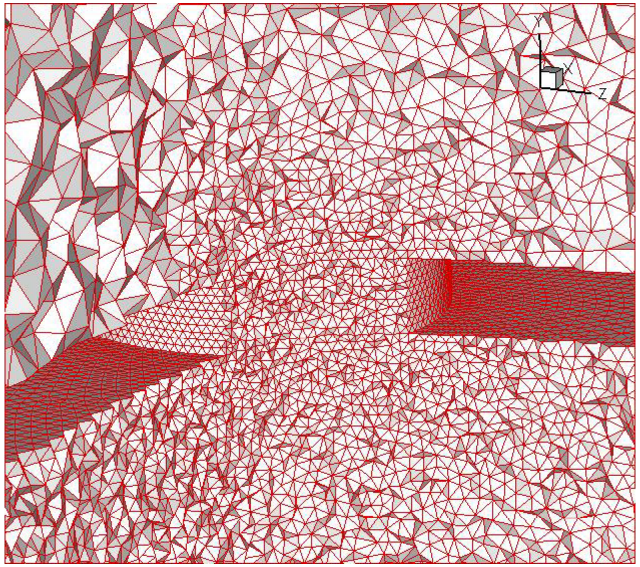
2D Numerical Results



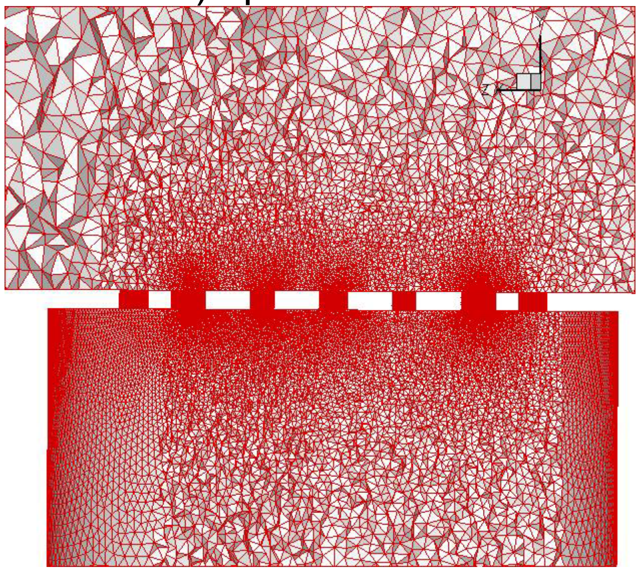
3D Numerical Results

Cases: 1) liner cell a single hole





2) perforated liner cell



Grids:

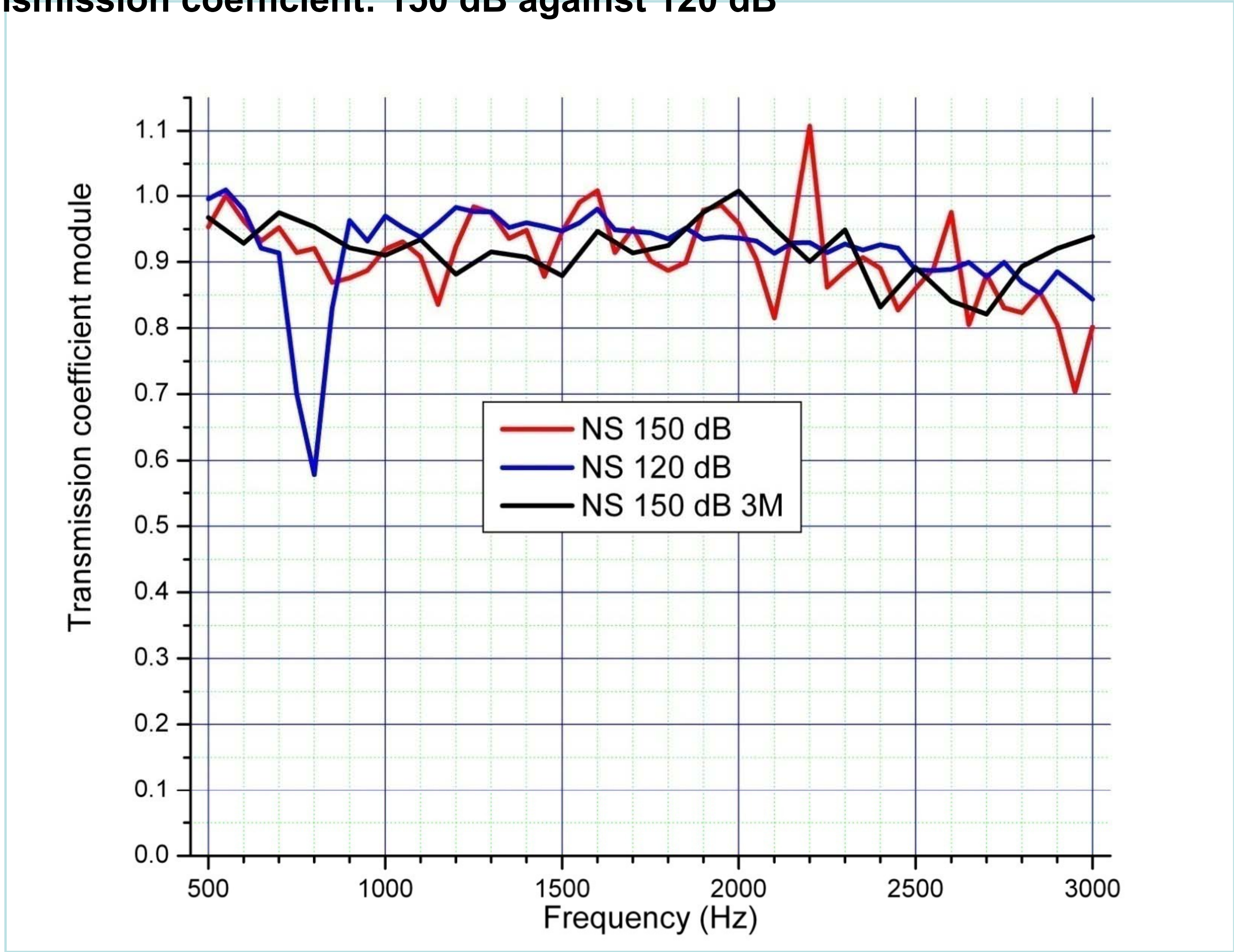
1) 613K nodes / 3498K tetras (coarse)
   3420K nodes / 20124K tetras (fine)

2) 1295K nodes / 7463K tetras

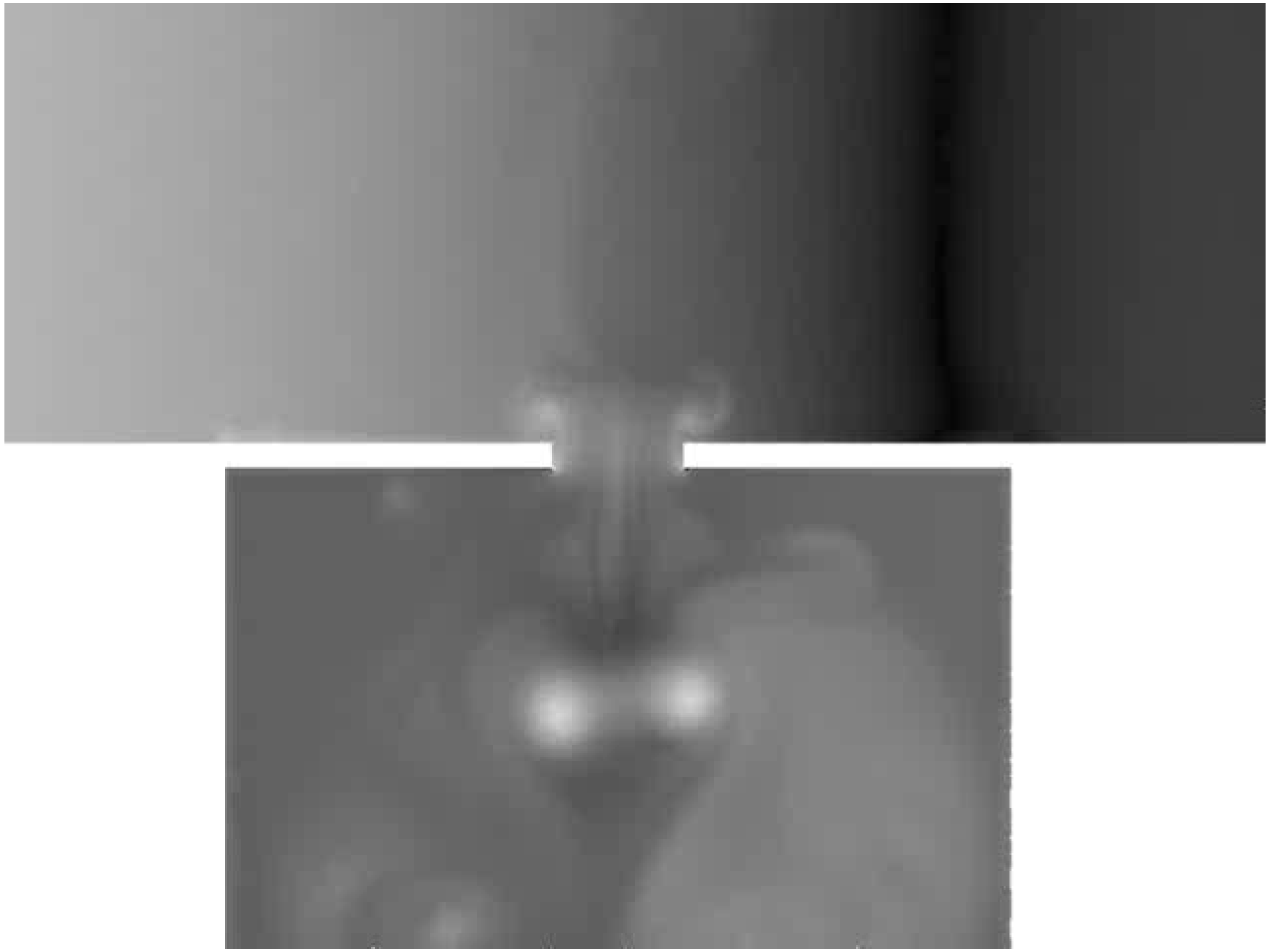
DNS

Liner Cell with Single Hole

Transmission coefficient: 150 dB against 120 dB

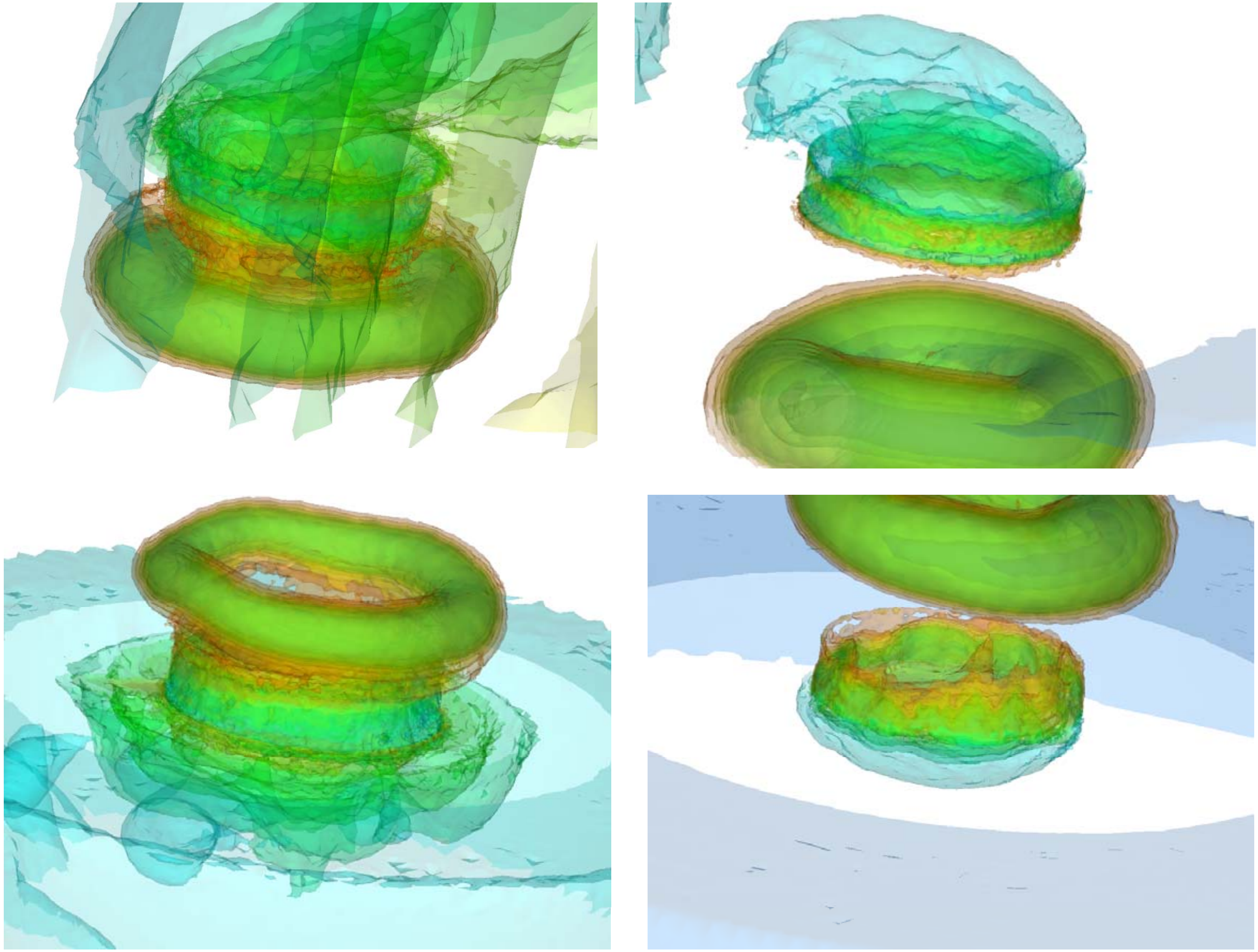


Liner Cell with Single Hole



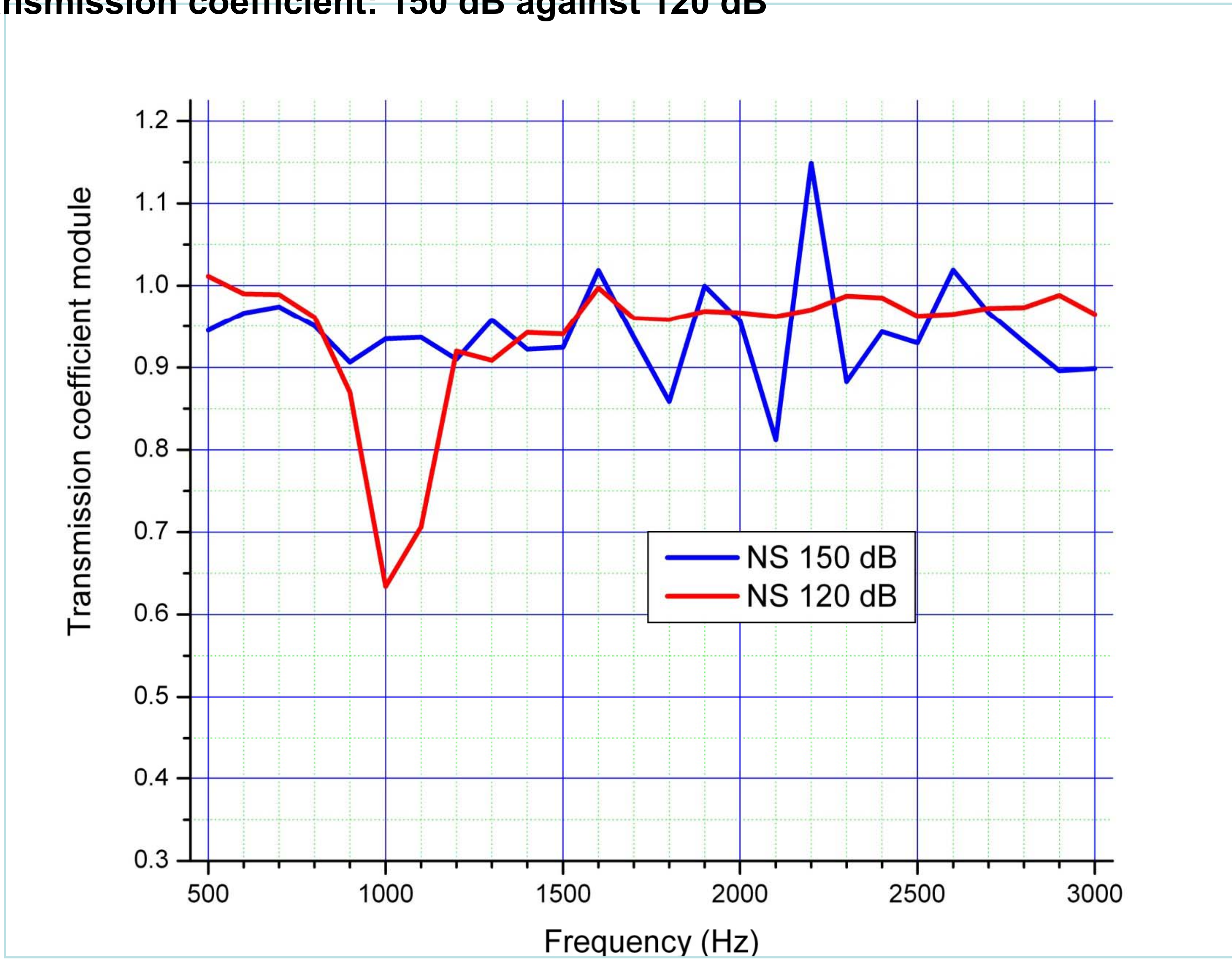
Density

Liner Cell with Single Hole

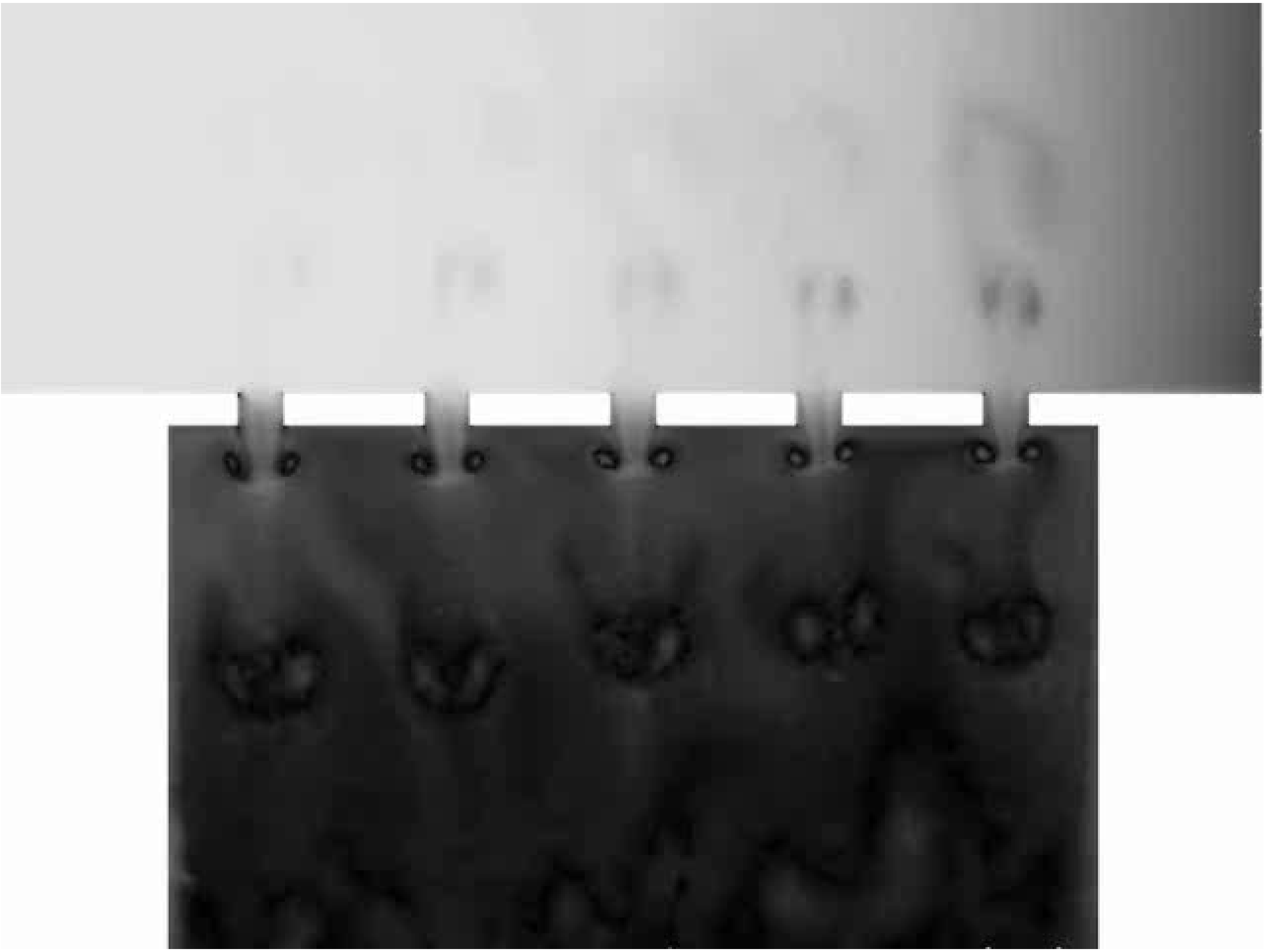


Perforated Liner Cell with 11 Holes

Transmission coefficient: 150 dB against 120 dB



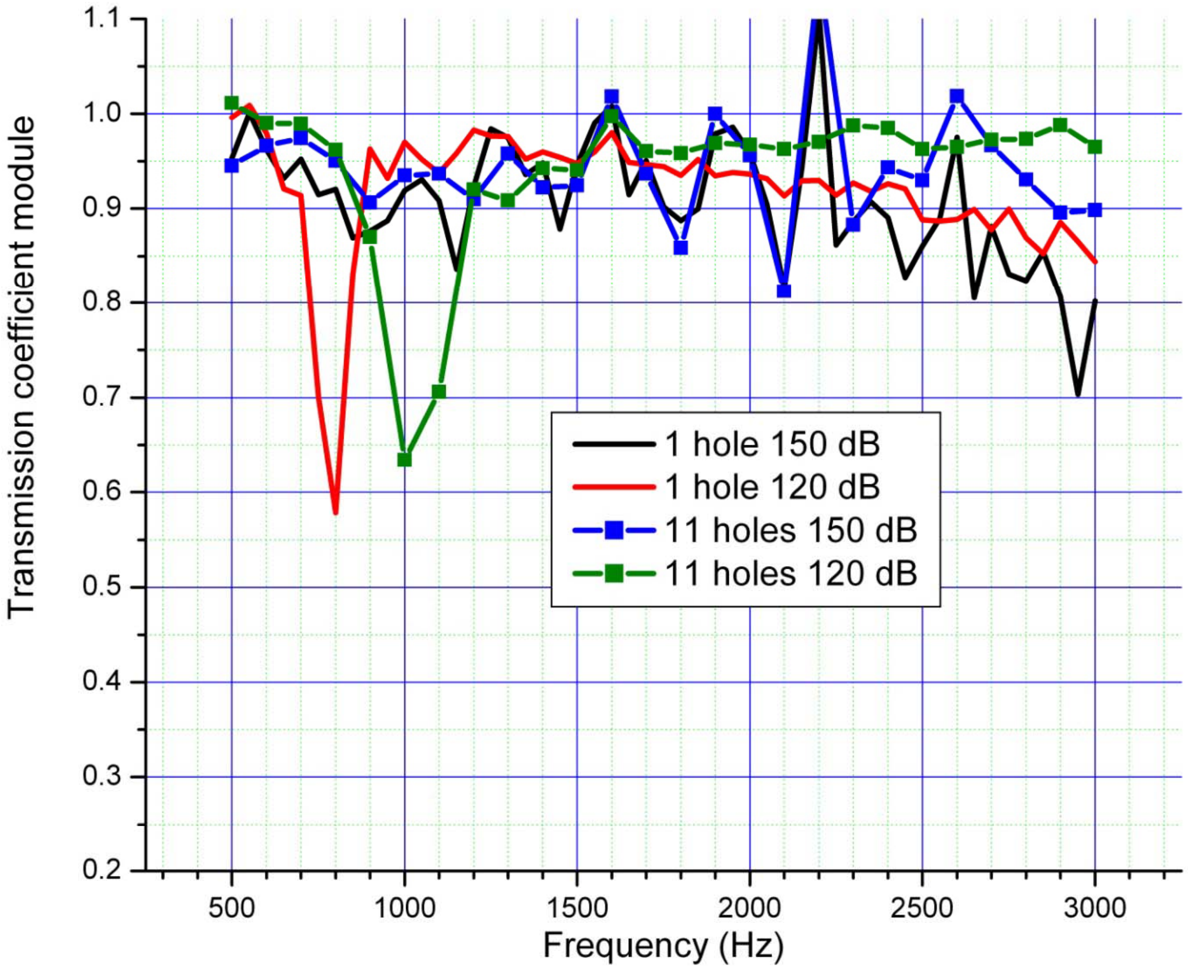
Perforated Liner Cell with 11 Holes



Density

3D Numerical Results

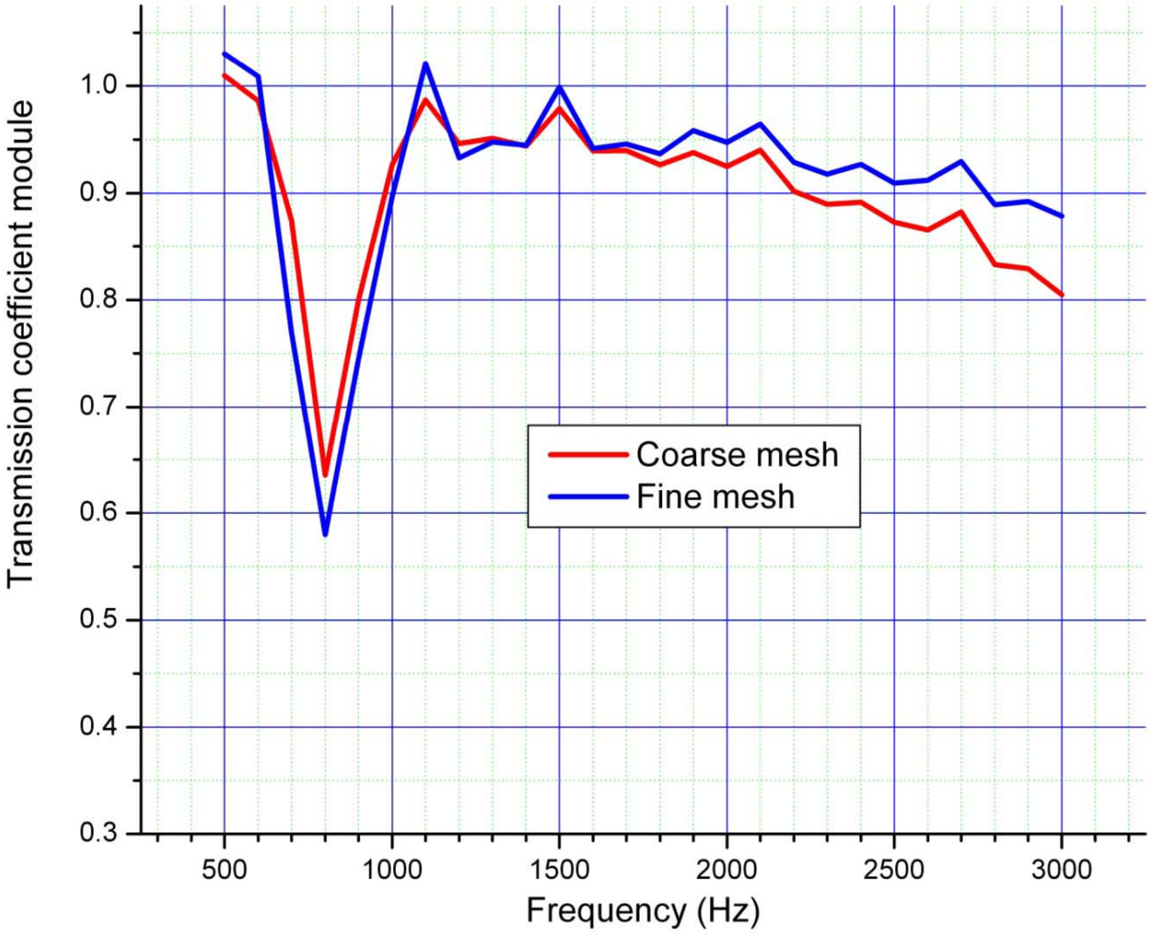
Transmission coefficient: 150 dB against 120 dB, 11 holes against 1 hole



Perforation coefficient is equal

3D Numerical Results

Transmission coefficient: fine grid against coarse grid



Concluding Remarks

1. Computational “testing facilities”

   aimed at studying and optimizing the liner cells

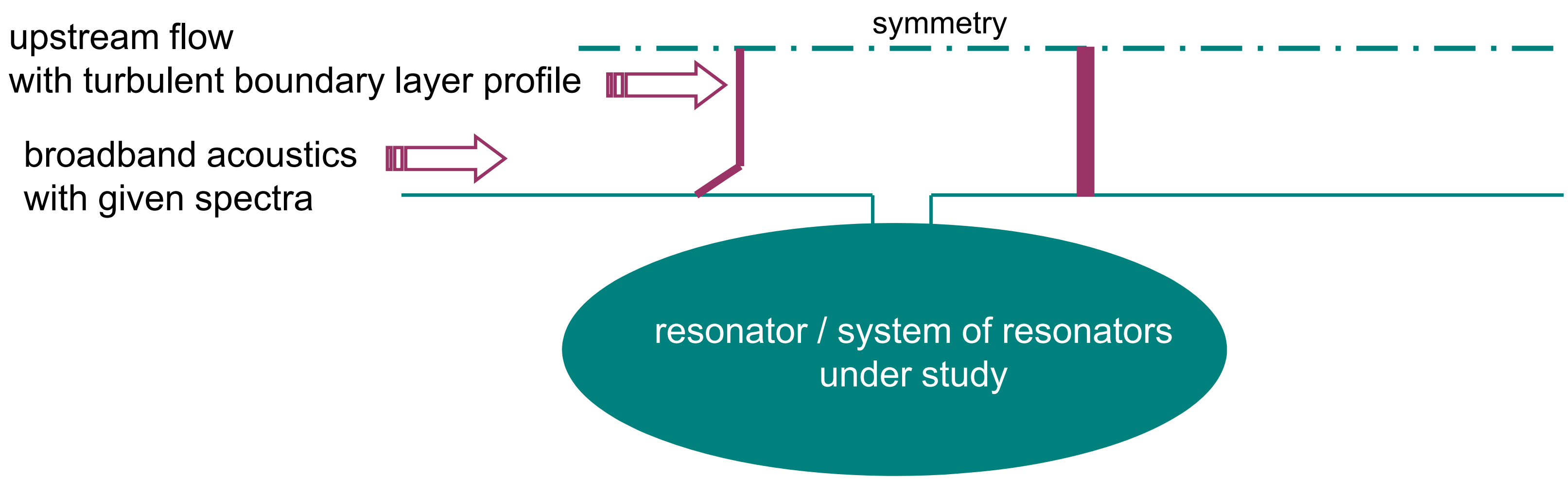
2. Extendable to

   - systems of a few resonators
   - cells of different configurations
   - cells of different properties

On-going and Future Work

1. **Extension to turbulent formulations (DES-family)**

⇨ **Careful elaboration of computational experiment formulation**



2. **Optimization of computational efforts, maximal automatization,…**

Thank you!