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Application of a shadowgraphy method to measure the shock front of spark-generated *N*-waves in air

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Outline

- Introduction
    - *sonic boom problem and laboratory scale experiment*
- Measurement of spark-generated *N*-waves
    - *microphone measurements*
    - *numerical simulation of N-wave propagation*
    - *rise time overestimation in the measurement*

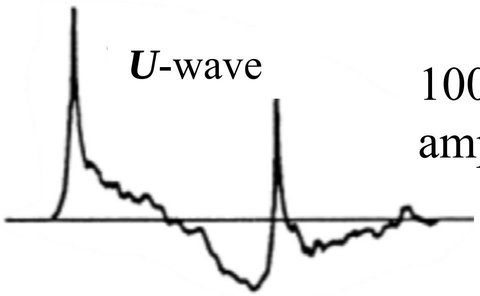
Shadowgraphy method to resolve the fine structure of shocks

- *experimental setup*
- *interpretation of shadowgrams*
- *geometrical optics versus diffraction model*

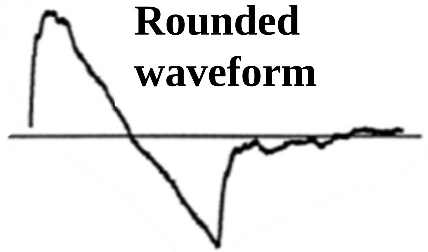
- Comparison of shadowgraphy and numerical results
- Conclusions

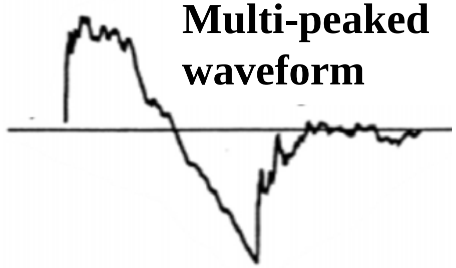
Sonic boom problem:

the application involving shock wave measurement



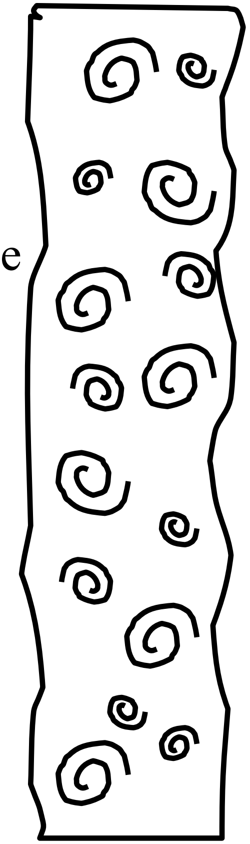
100-200 Pa
amplitude

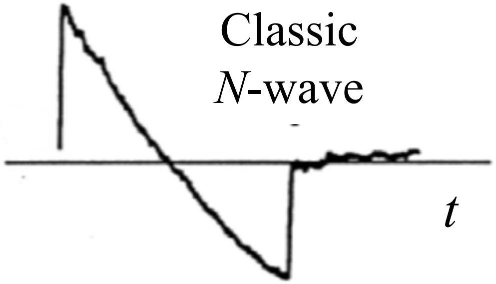




Distortions of the
N-wave

Atmospheric turbulence



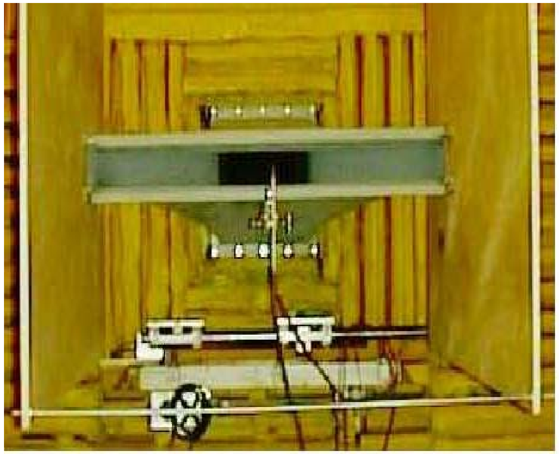


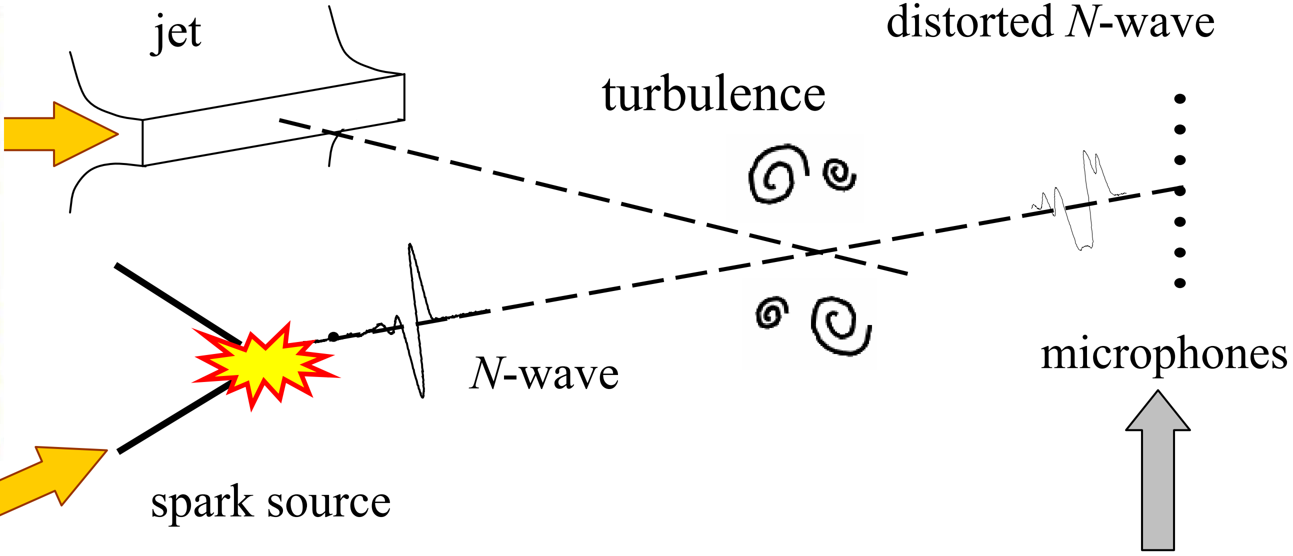
Measured waveforms,
Lee and Downing, 1991

Practical importance of prediction and measurement of sonic boom noise:
impact on people, buildings *etc.*

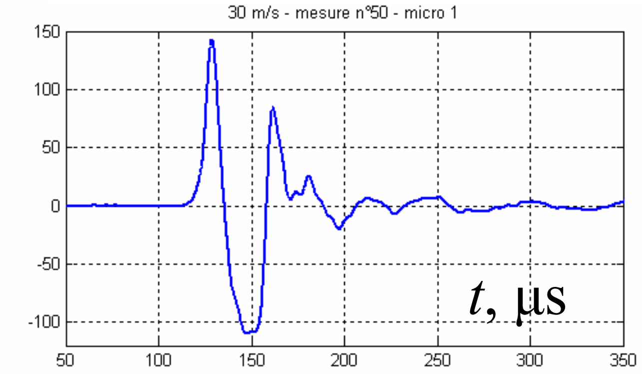
Laboratory-scaled experiment:

N-wave propagation through jet turbulence





distorted *N*-wave - *U*-wave

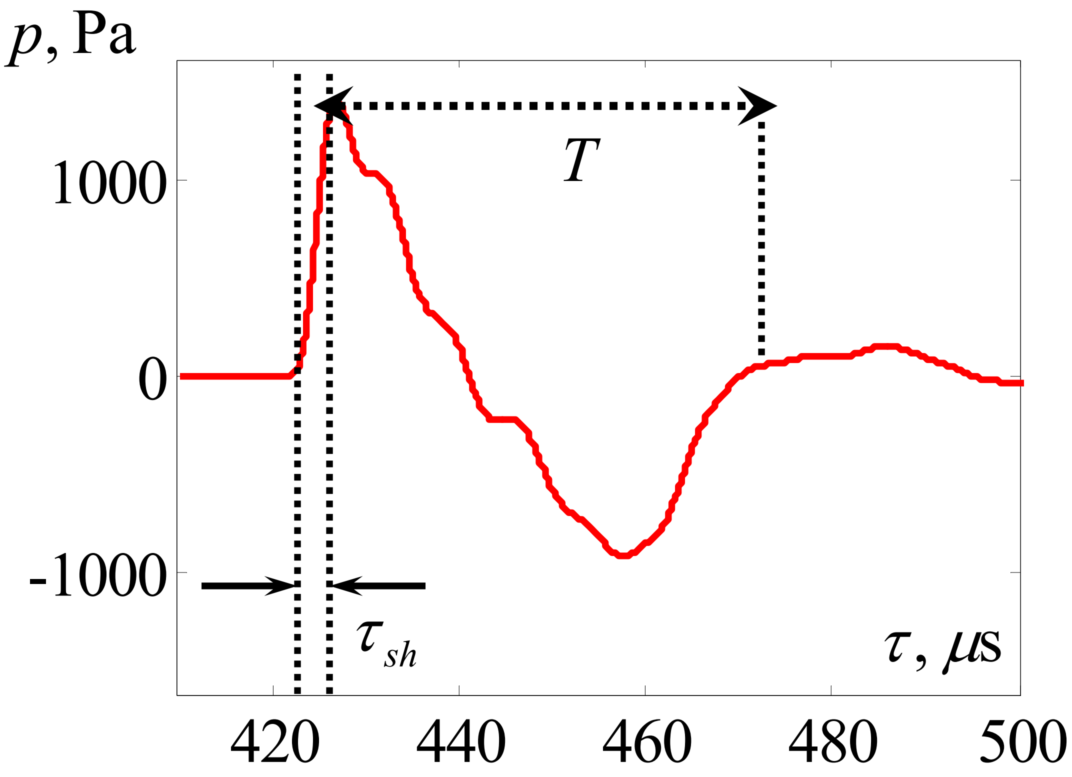


Brüel&Kjær 1/8’’

M.V Averiyanov *et al*., presentation tomorrow

Spark generated *N*-wave in homogeneous air:

typical waveform measured with a microphone



distance:
15 cm from the spark source

Characteristic parameters:

peak pressure: $p_{\max}$ 1500 Pa

duration: T 30 – 50 µs

rise time: τ_{sh} 3 µs

How to define the duration of the pulse? What are the major effects that result in distortion of the pulse waveform?

Theoretical model

Burgers equation extended to include spherical divergence and relaxation

$$\frac{\partial p}{\partial r} + \boxed{\frac{p}{r}} = \frac{\varepsilon}{\rho_0 c_0^{3}} p \frac{\partial p}{\partial t} + \frac{b}{2\rho_0 c_0^{3}} \frac{\partial^2 p}{\partial t^2} + \boxed{\sum_{v=1}^{2} d_v \frac{\partial}{\partial t} \int_{-\infty}^{t} \exp\left(-\frac{t-t'}{t_v}\right) \frac{\partial p}{\partial t'} dt'}$$

in addition to acoustic nonlinearity and thermoviscous absorption

p – acoustic pressure, *r* – radial propagation distance, $\varepsilon = 1.21$ – nonlinear parameter in air, $b = 5.2 \cdot 10^{-5}$ Pa·s – thermoviscous absorption parameter, $\rho_0 = 1.29$ kg/m³ – density of the air, $c_0 = 343.67$ m/s – ambient sound speed

Two relaxation processes of the oxygen and nitrogen molecules are included:

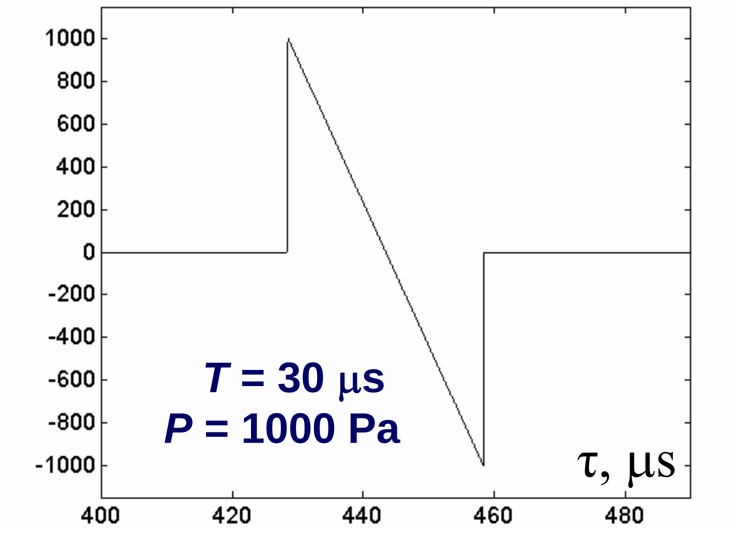
$\Delta c_1 = 0.11$ m/s, $\tau_1 = 6.0$ μs (oxygen O_2) ,

and $\Delta c_2 = 0.023$ m/s, $\tau_2 = 531$ μs (nitrogen N_2)

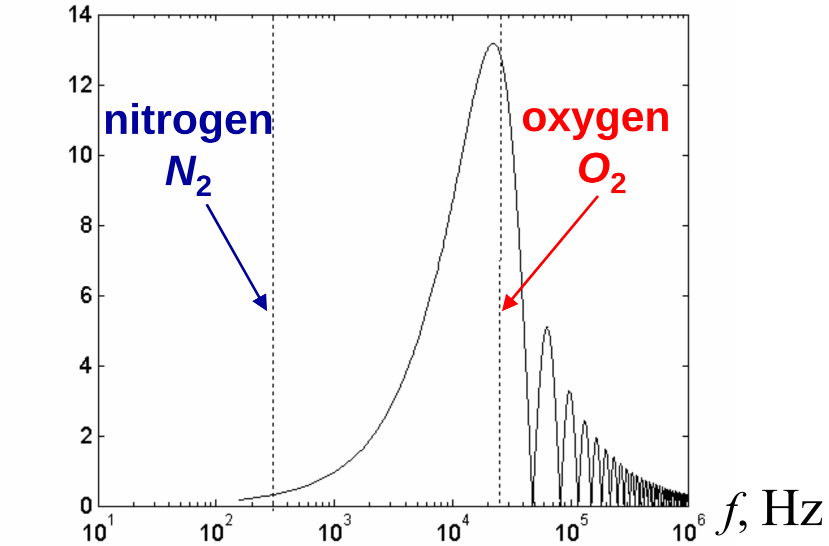
$$d_v = \frac{c_\infty - c_0}{c_0^{2}} = \frac{\Delta c}{c_0^{2}}$$

Characteristic time and frequency scale

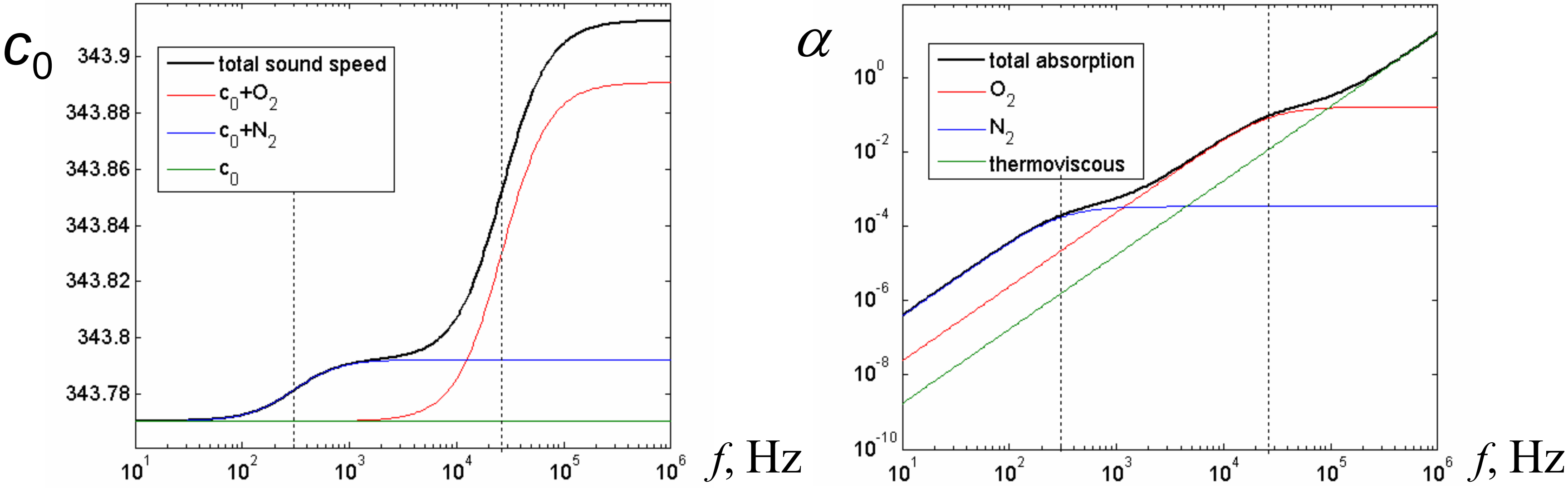
N-pulse pressure waveform $p, \mathrm{P}a$



N-pulse spectrum $S \cdot 10^{-3}, \mathrm{P}a \cdot s$

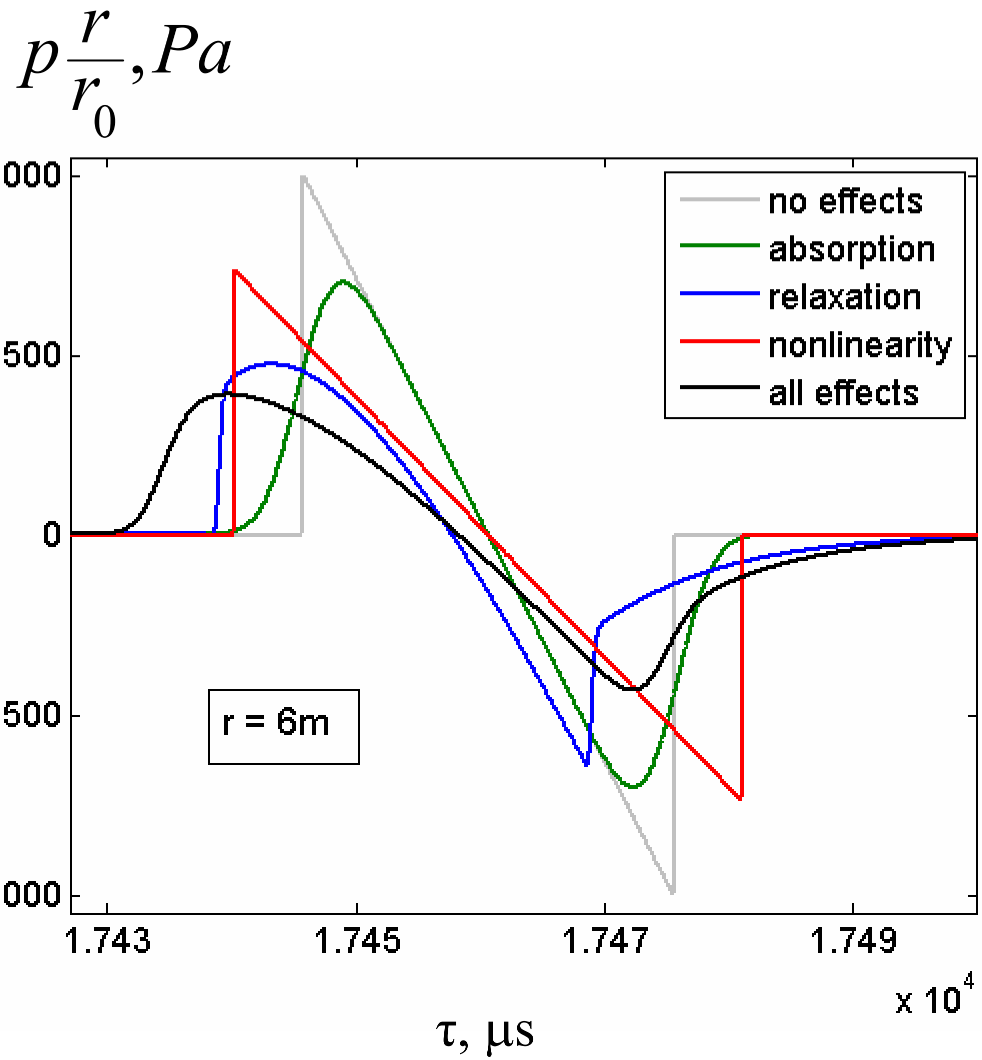


Change in sound speed and absorption with frequency due to relaxation



Results of simulations:

effects of nonlinearity, thermoviscous absorption, and relaxation on *N*-wave propagation



Thermoviscous absorption :

- *reduces the pulse amplitude*
- *broadens the shock front*
- *no change in symmetry*
- *no change in duration of the pulse*

Relaxation :

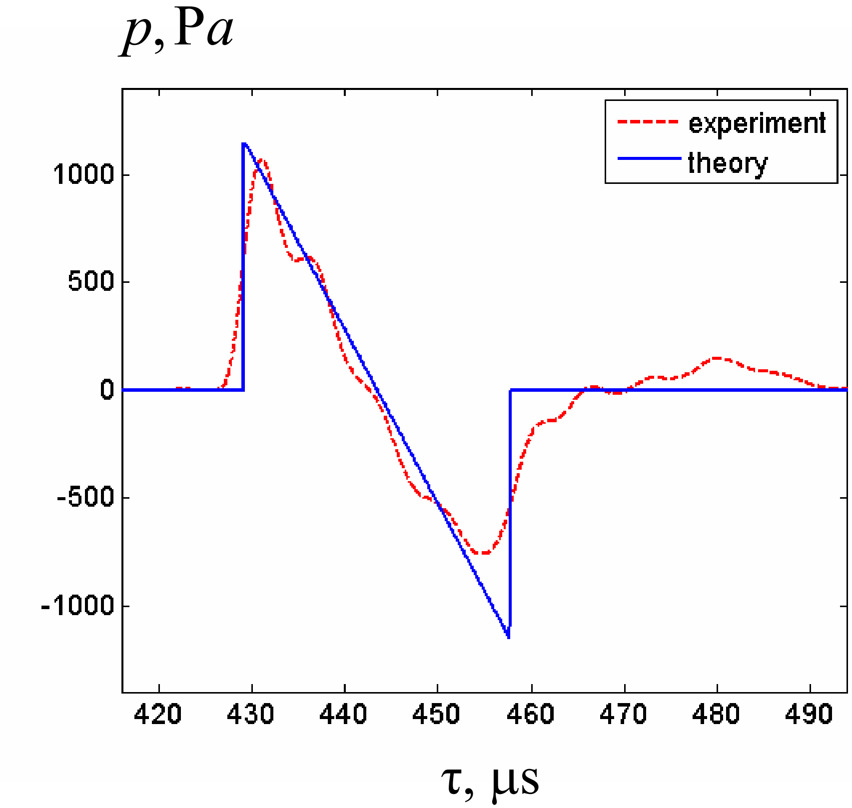
- *asymmetric waveform distortion*
- *reduces the pulse amplitude*
- *displaces both front and distal shocks towards the direction of propagation*

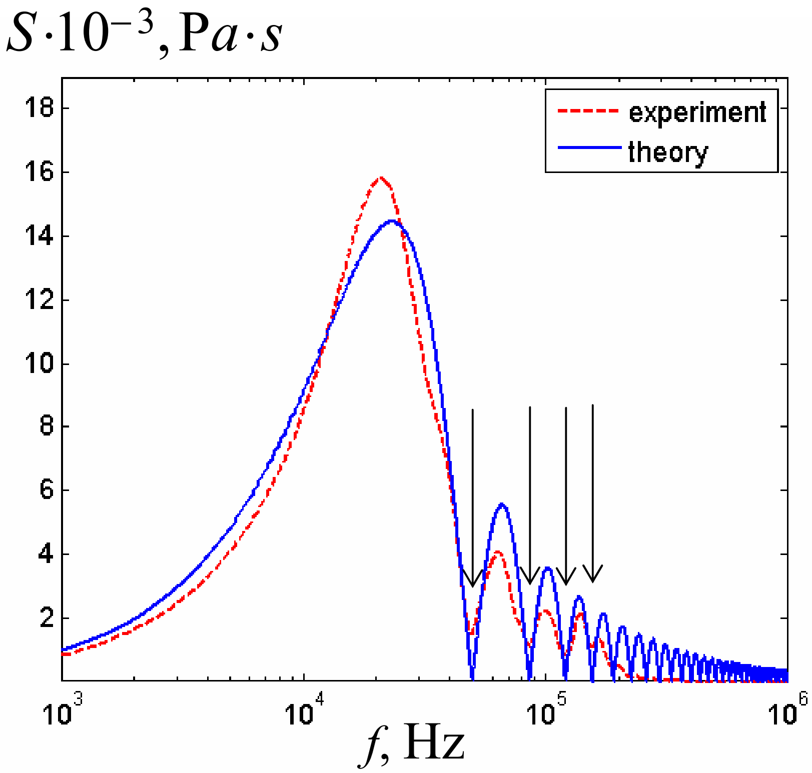
Nonlinearity:

- *lengthening of the N-pulse*
- *decrease of the shock amplitude*
- *no change in symmetry*

Distorted *N*-wave in modeling and experiment:

definition of pulse duration



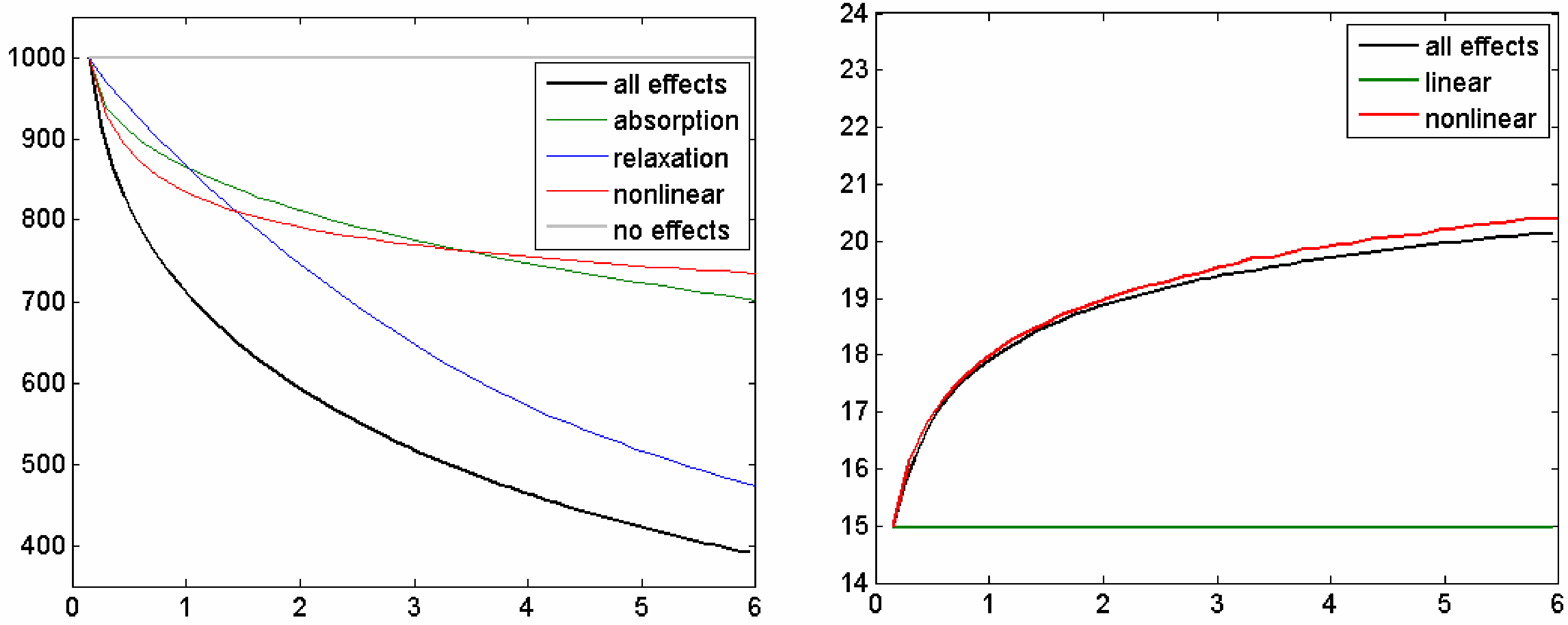


r = 15 cm

Pulse duration is defined in the frequency domain by matching the positions of the minima in the measured/modeled pulse spectrum with those in the spectrum of an ideal *N*-wave with an infinitely thin front

Results of simulations:

effects of nonlinearity, thermoviscous absorption, and relaxation on N-wave propagation



peak positive pressure, $p_+ \, r / r_0$, Pa

half pulse duration, T, μs

propagation distance r, m

Pulse duration increases mainly due to nonlinear propagation effect: can be used to set boundary condition in simulation of the experiment

Simulation of the experiment:

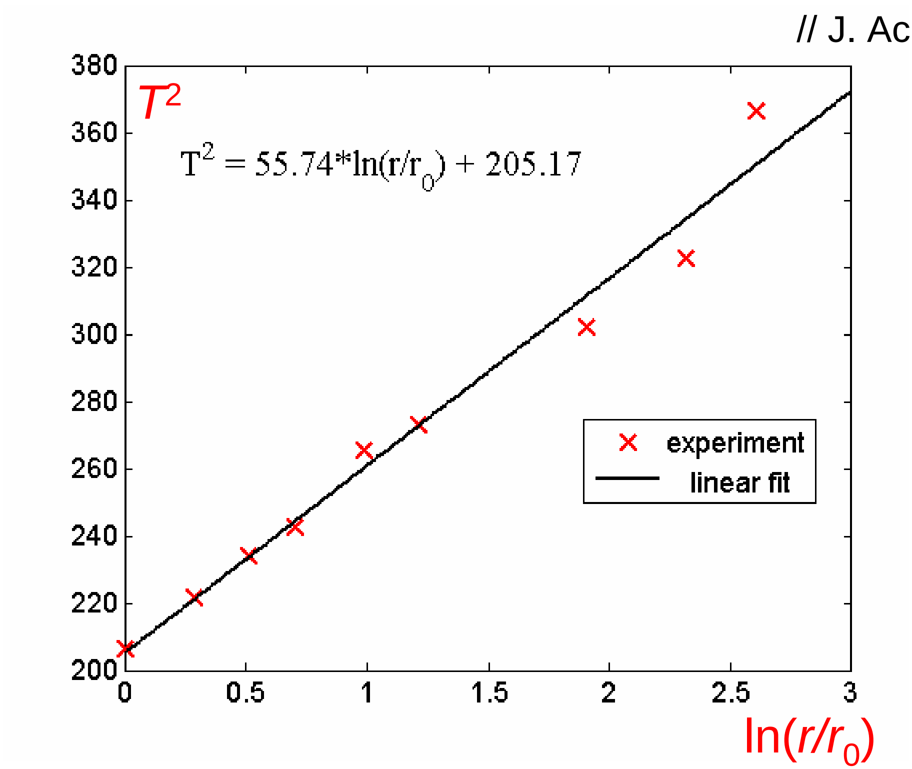
calibration of initial pulse amplitude and duration

The change in pulse duration is mainly due to AMPLITUDE dependent NONLINEAR EFFECTS

Weak shock theory can be applied

$$T^2 = T_0^2 \left[1 + \frac{\varepsilon}{\rho_0 c_0^{\ 3}} \frac{p_0}{T_0} r_0 \ln\left(r / r_0 \right) \right] \qquad (\#)$$

W.M. Wright. Propagation in air of *N*-waves produced by sparks. // J. Acoust. Soc. Am. 1983. V. 73(6). P.1948-1955



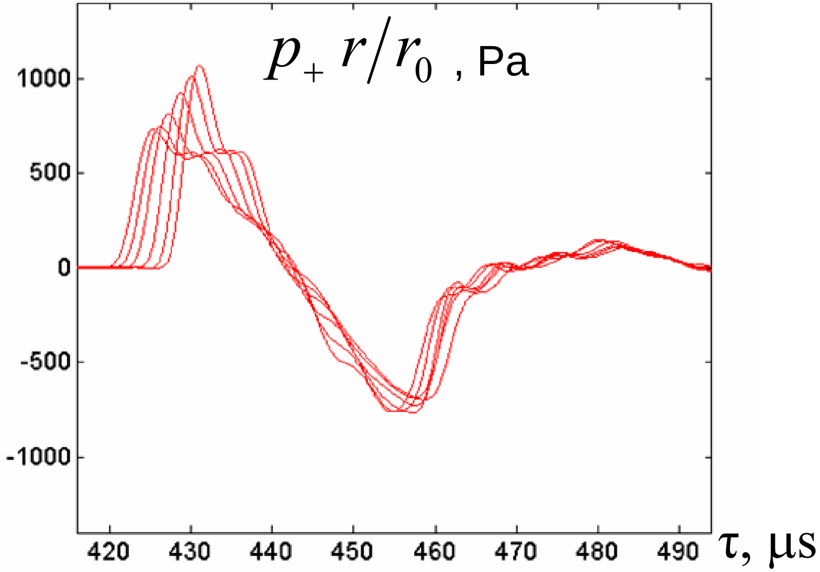
Two unknown parameters

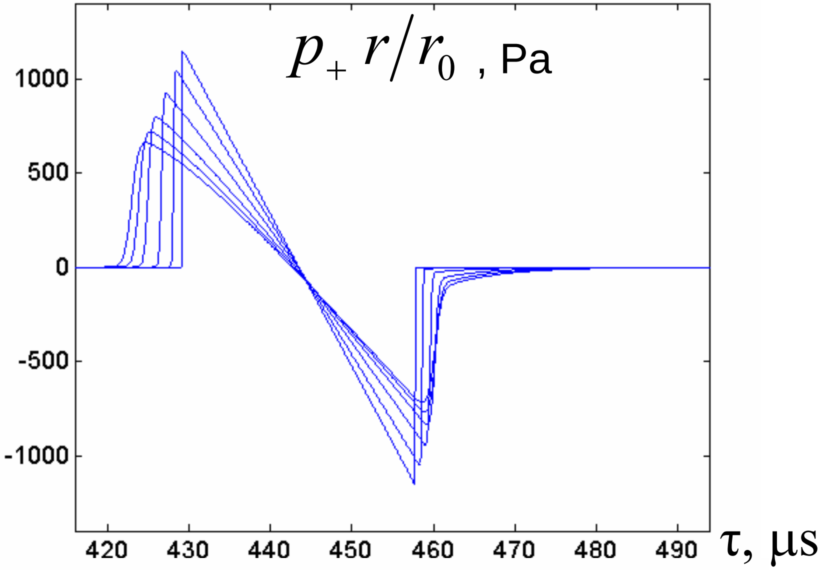
- initial half duration T_0 and
- peak pressure p_0

in the linear dependence of T^2 over propagation variable $\ln(r/r_0)$ in the solution (#) were obtained by fitting the experimental values over distances 15 cm – 3 m using the method of least squares

Experimental and modeled waveforms:

comparison over propagation distance

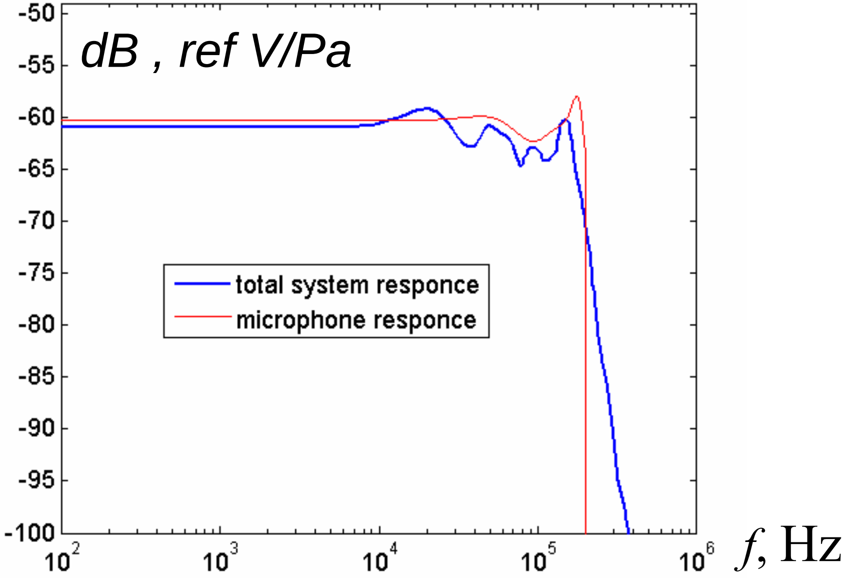




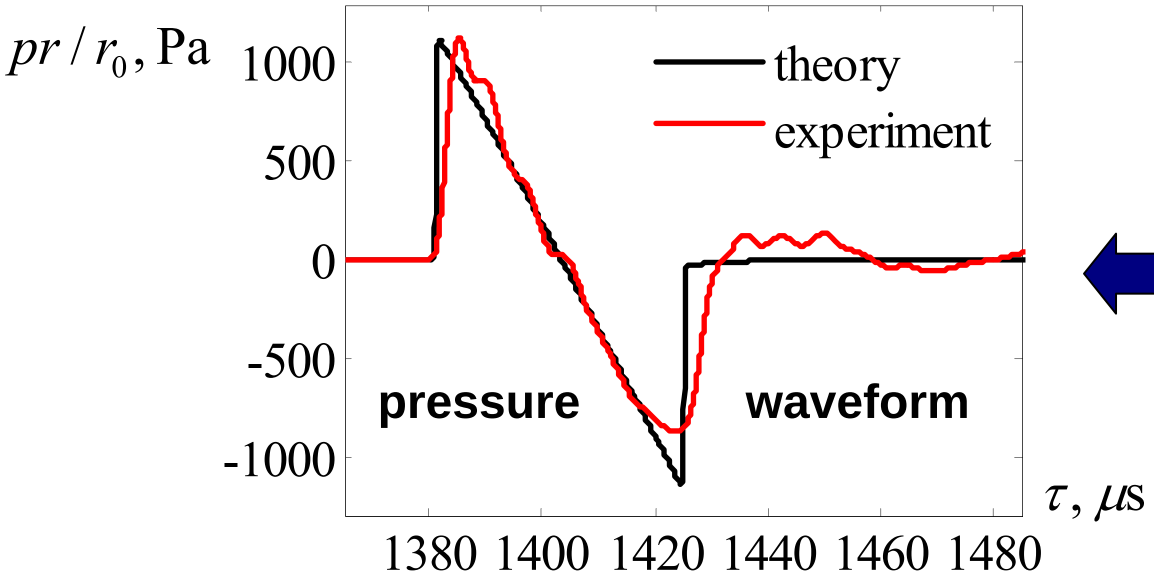
Measured and modeled waveforms are in a good agreement

****

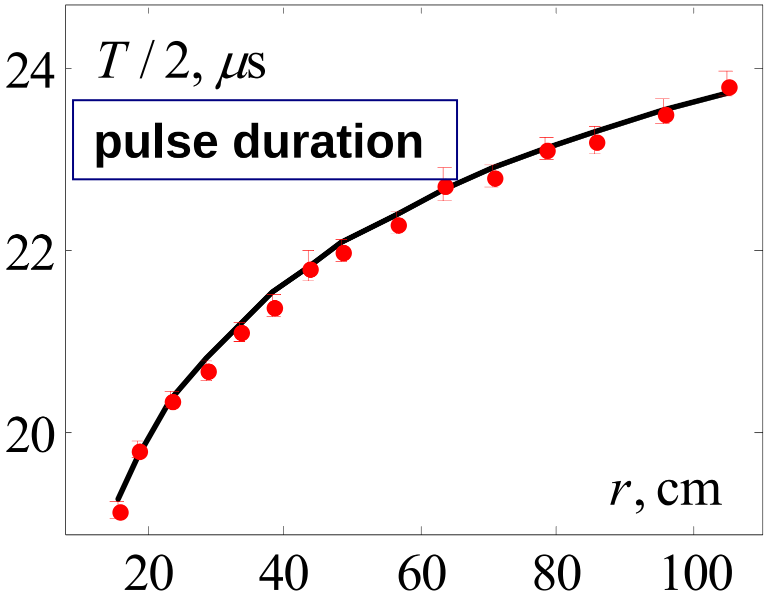
Calibration of the measurement track: the ratio of the experimental and modeled spectrum = total system response

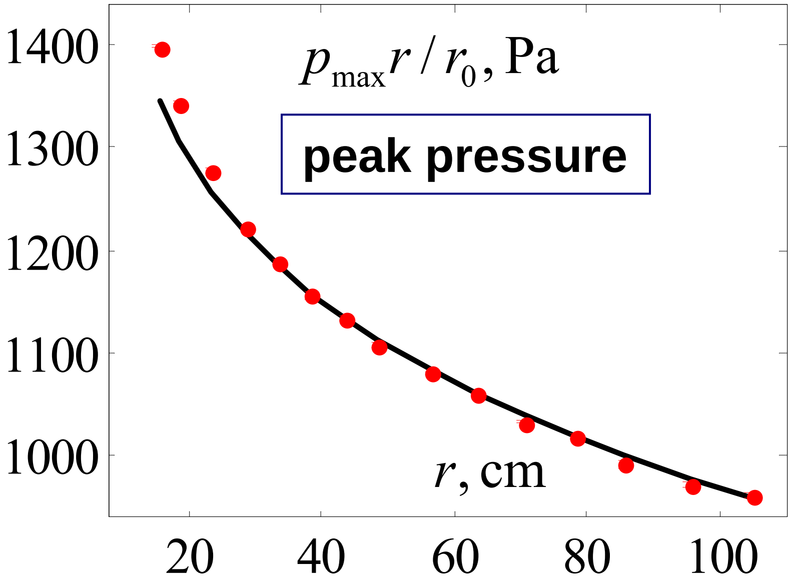


N-wave parameters: simulations and experiment

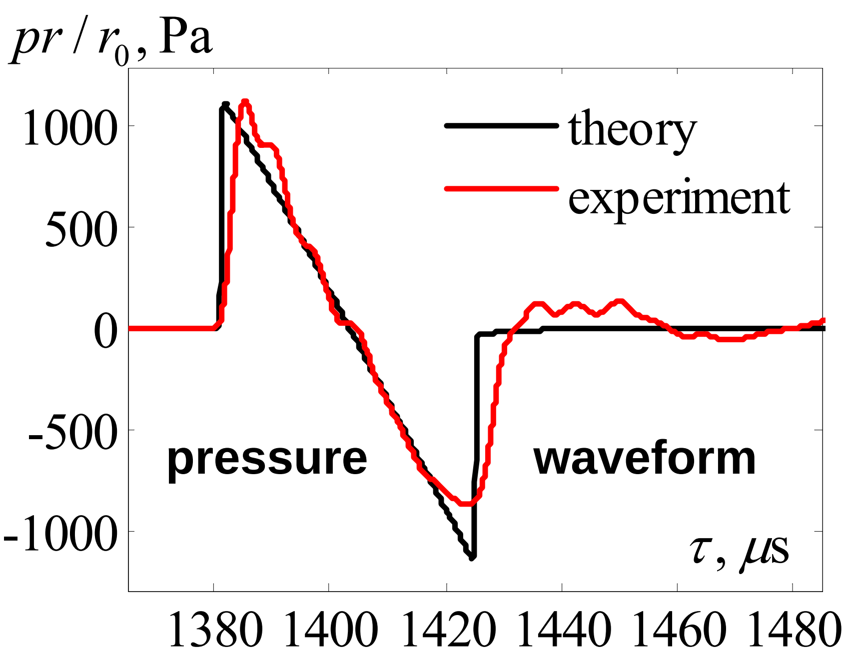


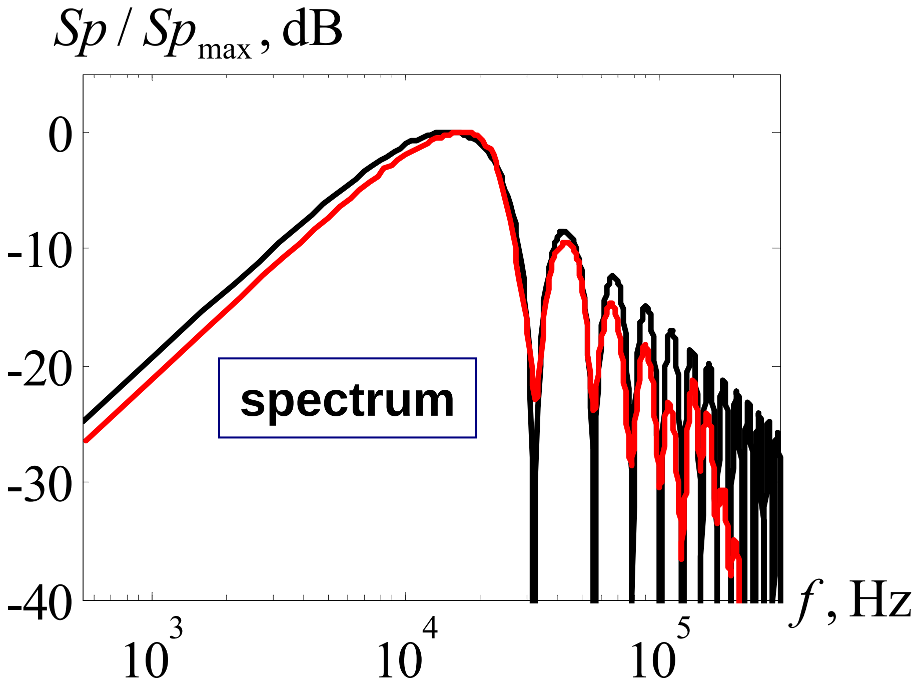
distance:
42 cm from
the spark
source

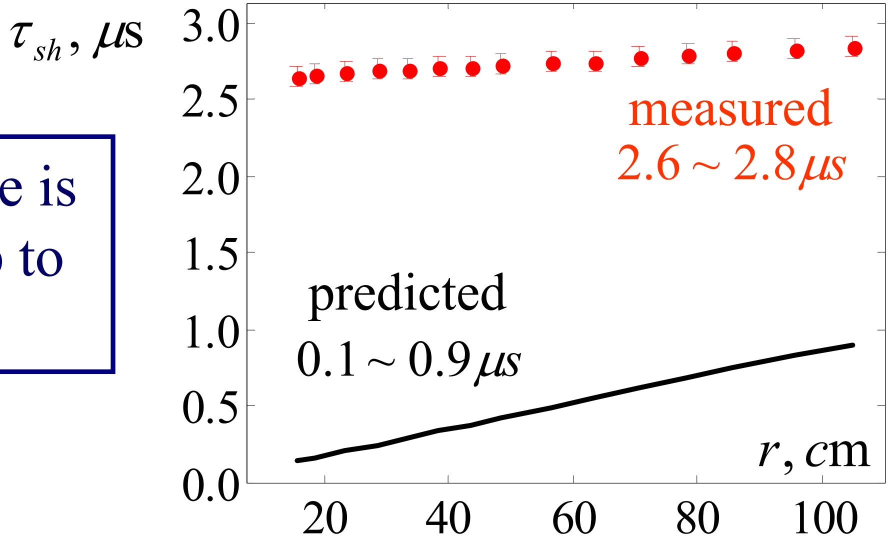




N-wave parameters: simulations and experiment





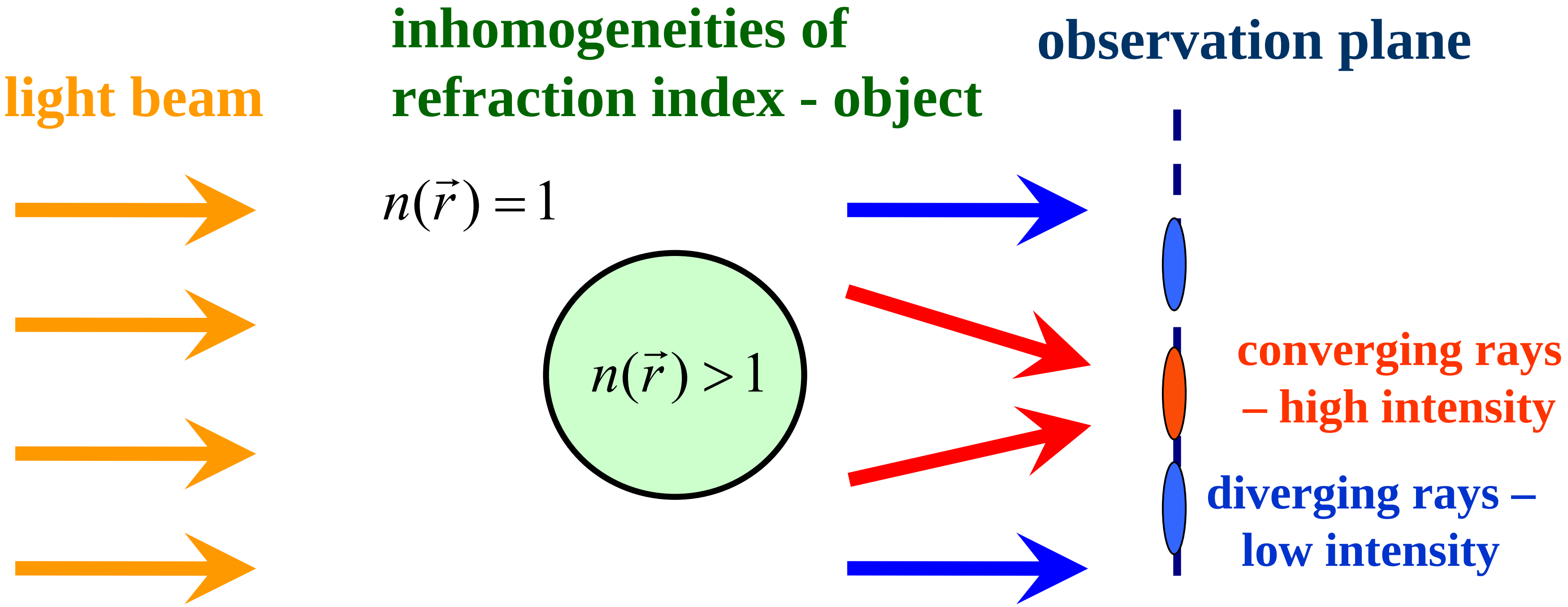


Measured risetime is overestimated up to 10 times

Limited bandwith of the microphone:
cut off at ~200 kHz

How to resolve fine structure of the shock front in experiment?

Shadowgraphy method to measure rise time of the shock with higher accuracy



light beam

inhomogeneities of refraction index - object

observation plane

$n(\vec{r}) = 1$

$n(\vec{r}) > 1$

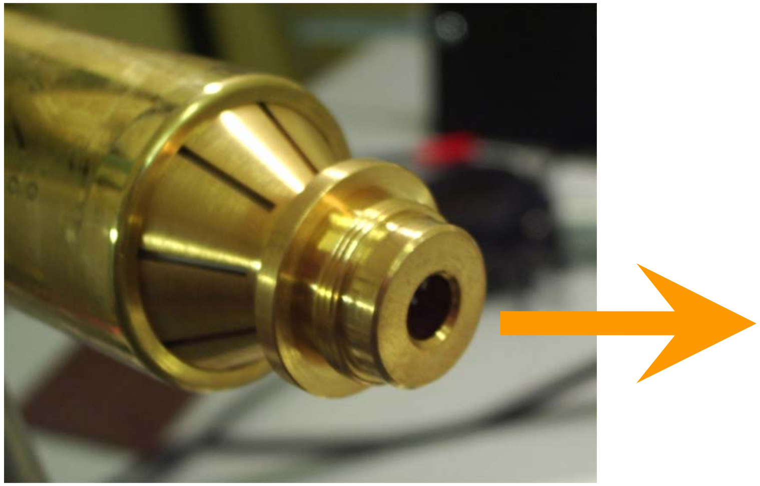
converging rays – high intensity

diverging rays – low intensity

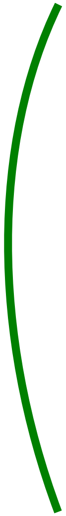
Light intensity distribution in the observation plane is a shadow

Shadowgram of the moving shock front: very short flash of white light

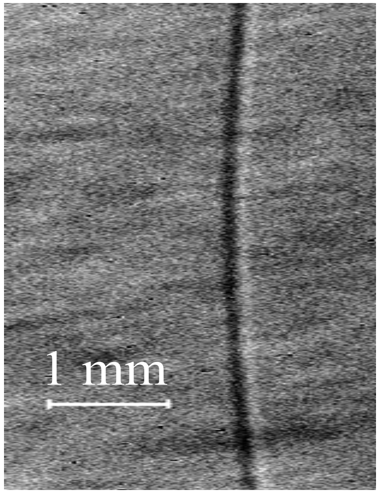
Nanolite KL-L flashlamp – very short 20 ns white light pulses



propagating shock (340 m/s) is a “frozen” object

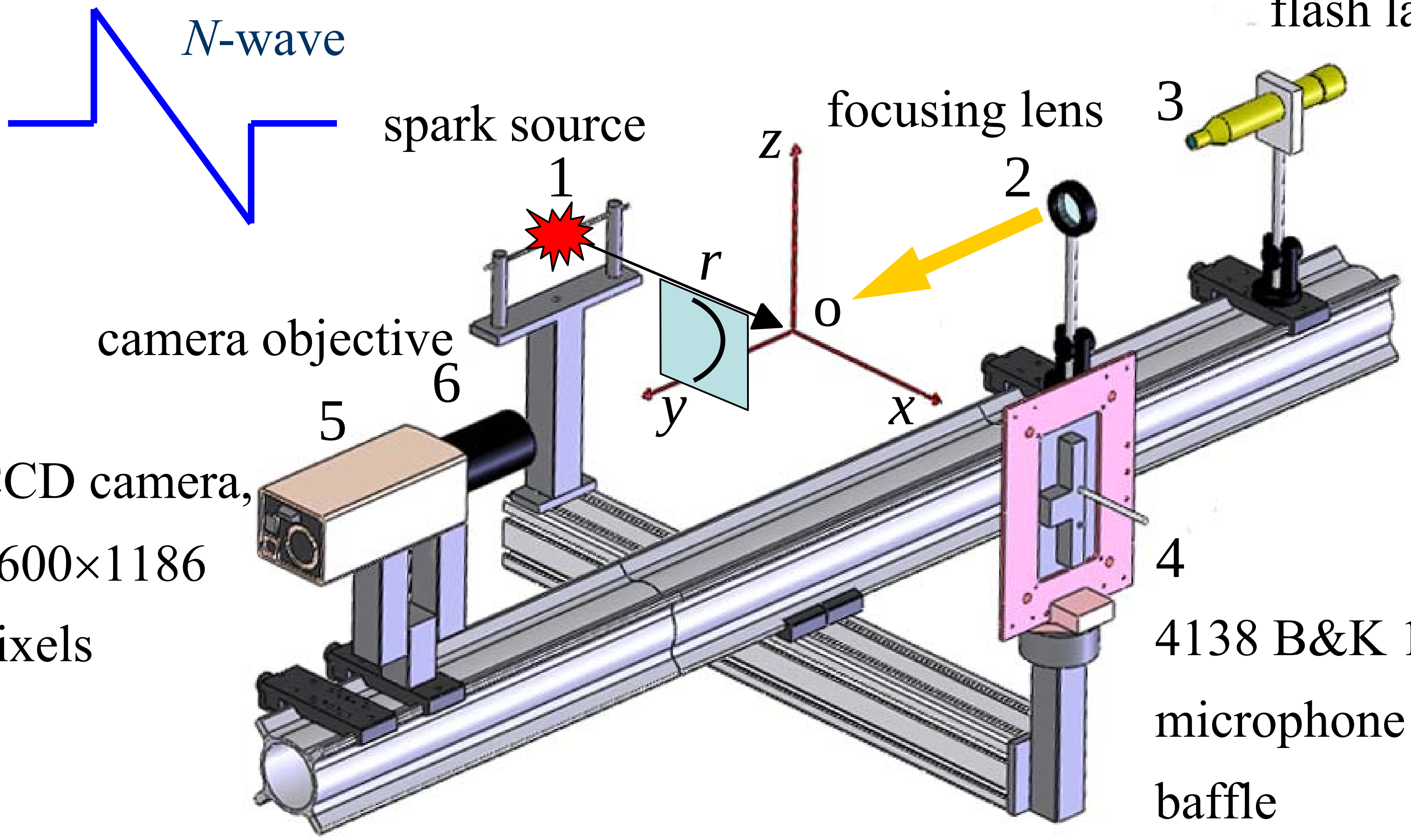


the shadow



Shorter light flash duration – better image of the moving object

Experimental setup



N-wave

spark source

focusing lens

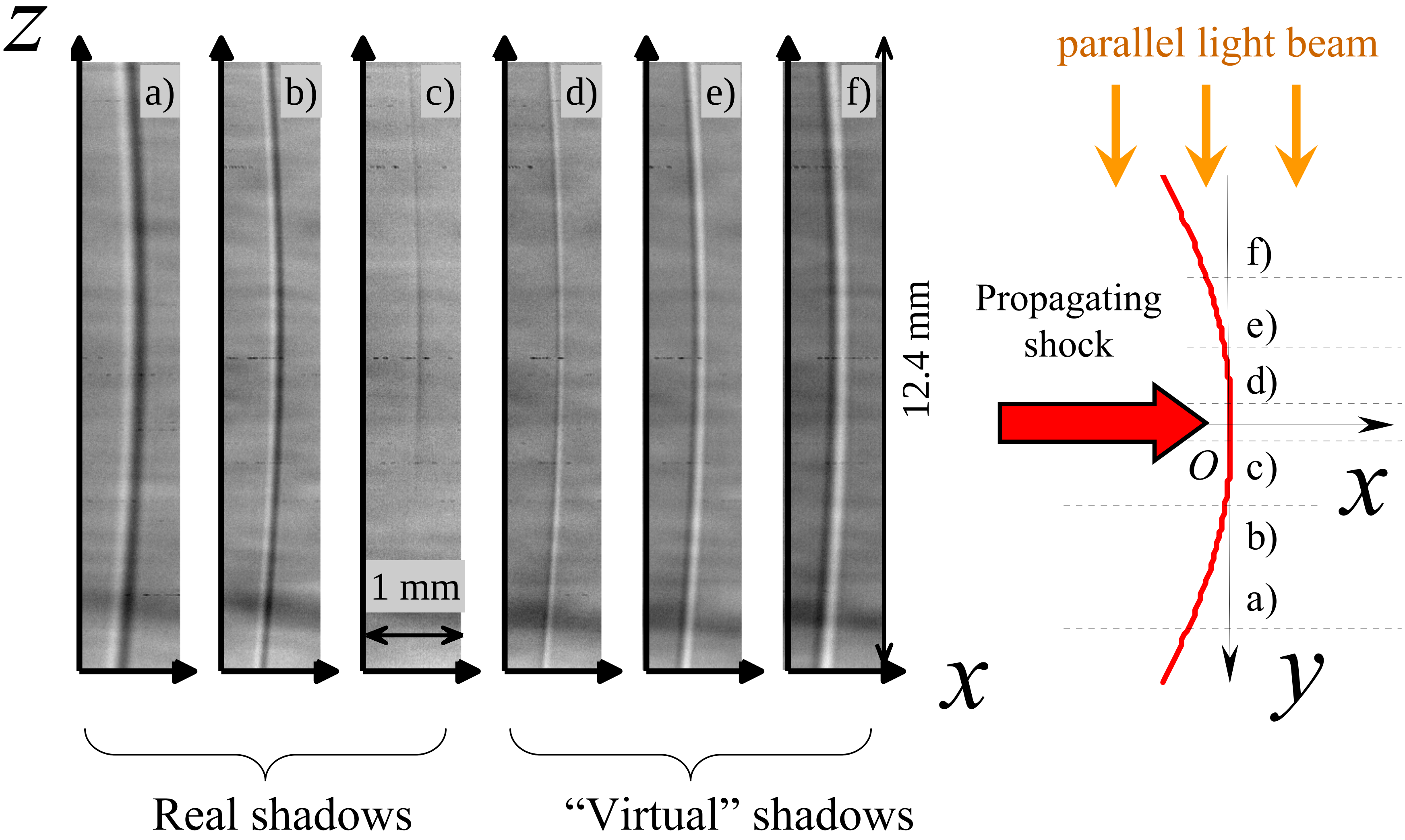
flash lamp

camera objective

CCD camera,
1600×1186
pixels

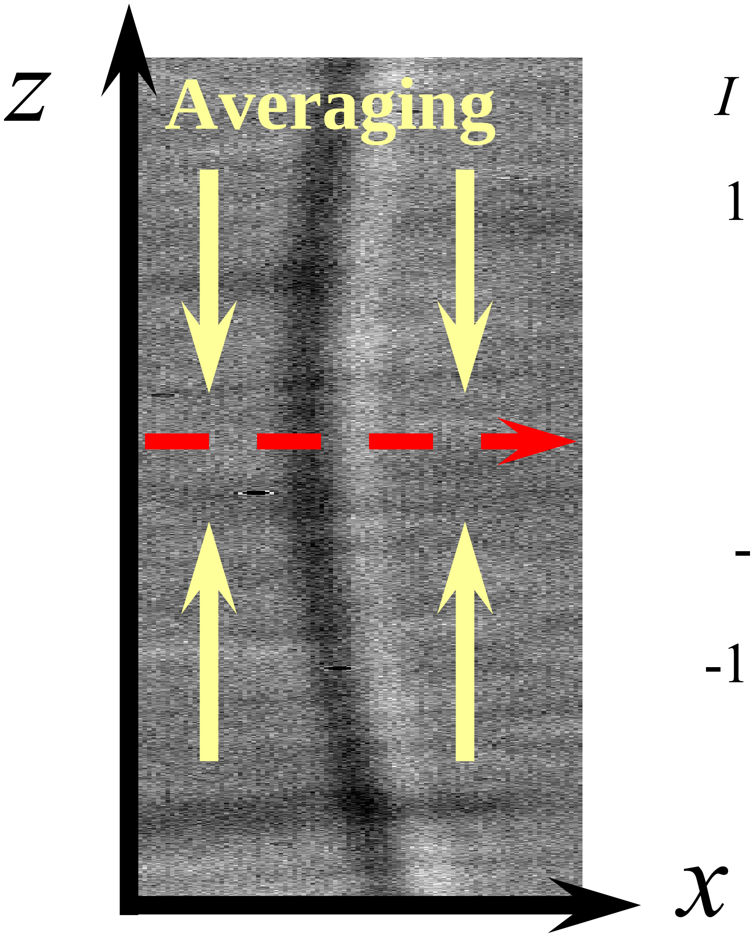
4138 B&K 1/8’’
microphone in a
baffle

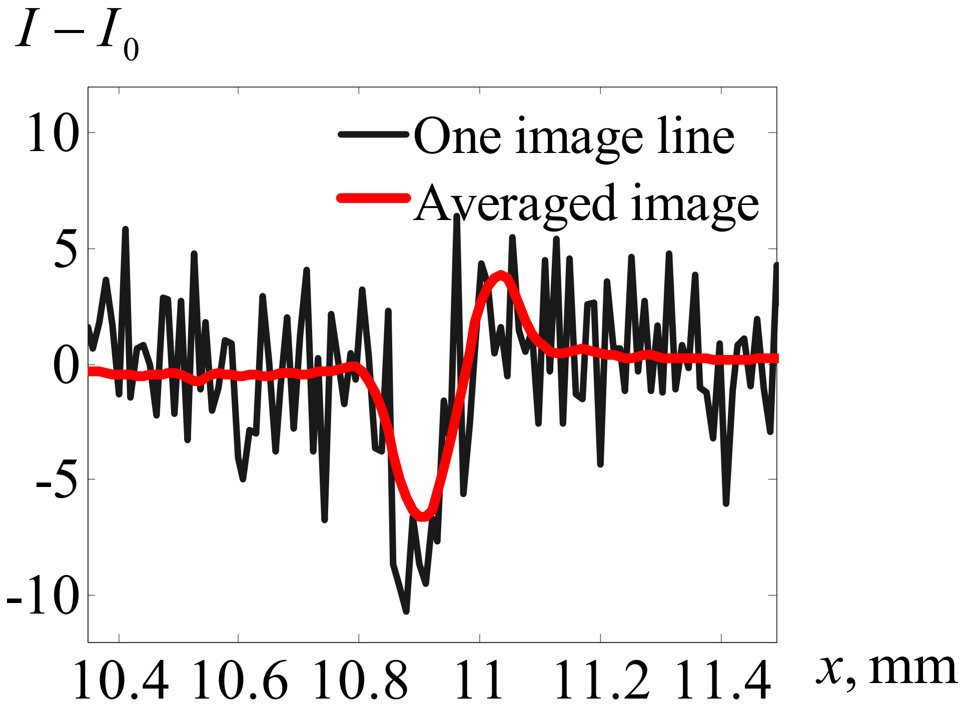
Measured shadowgrams



How to interpret shadowgrams and what causes inversion of light intensity?

Images processing: from 2D to 1D

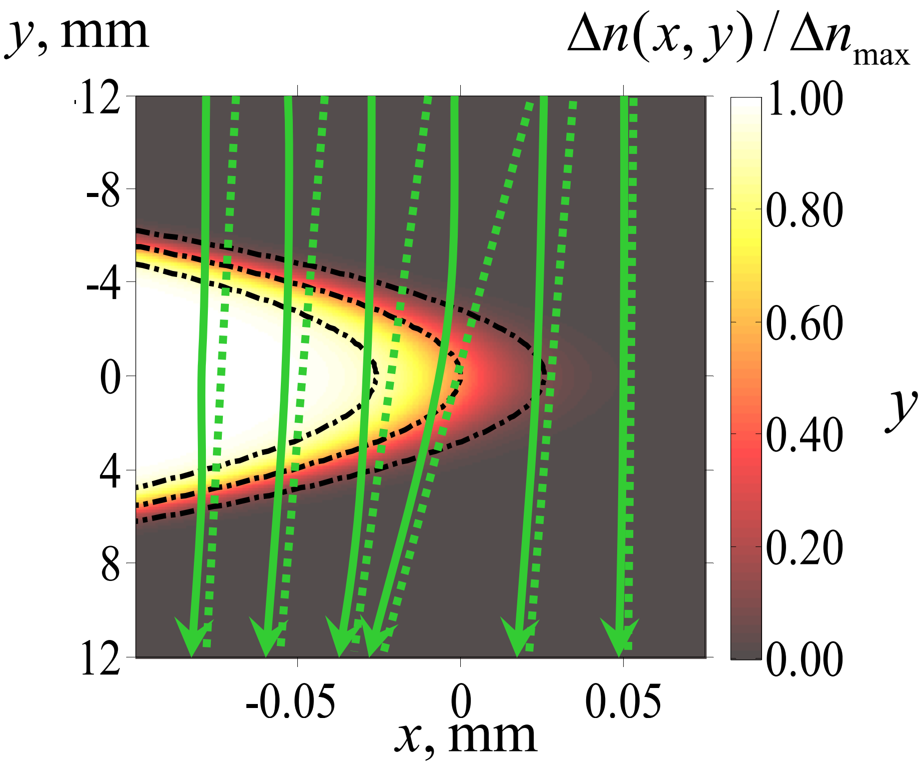




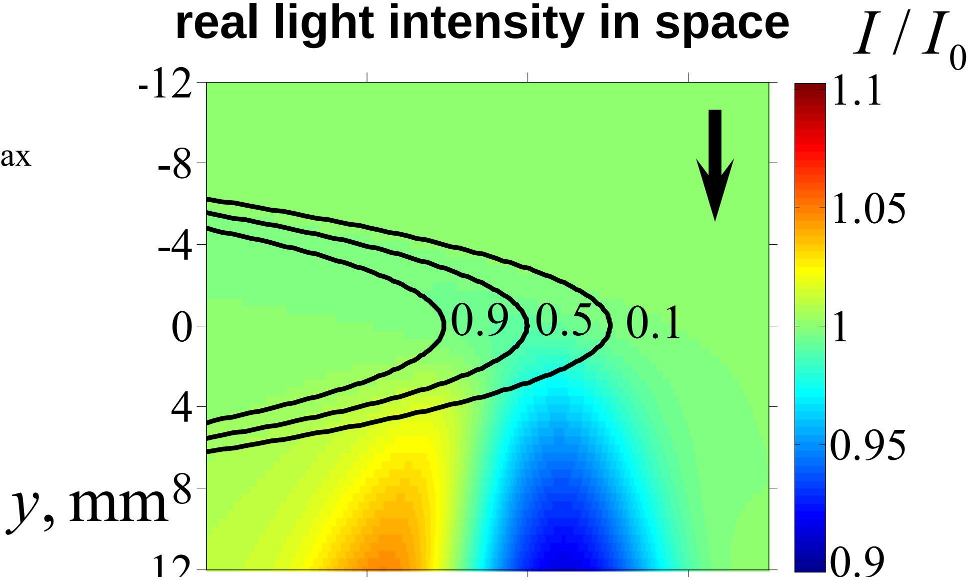
Spherical curvature compensation + vertical averaging significantly suppress noise. As an output – 1D intensity distribution along *x* axis

Inversion of intensity distribution: explanation

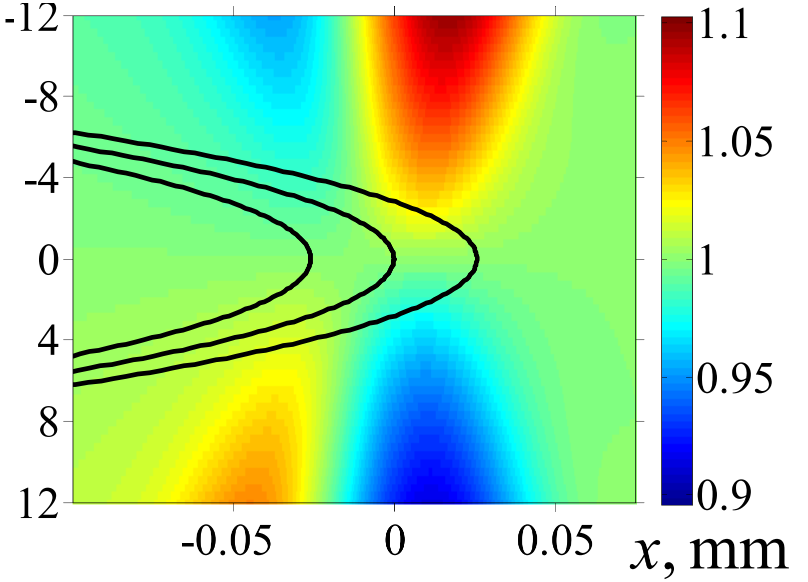
Refraction of rays on the shock



real light intensity in space

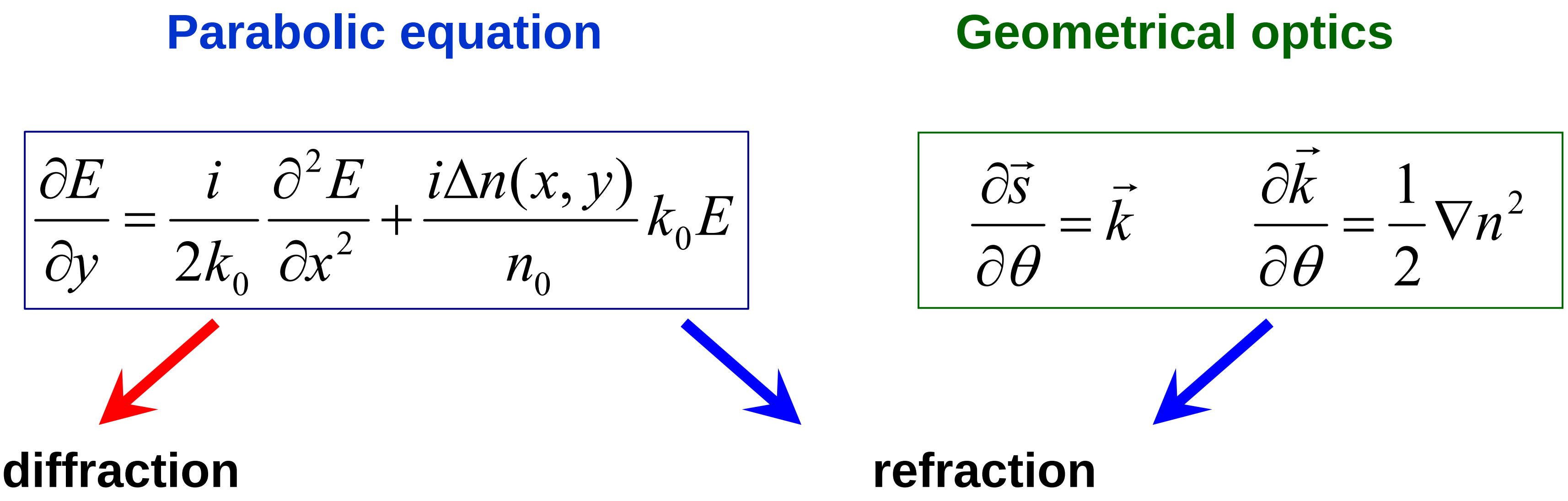


imaginary light intensity in space



“Virtual” shadows are antisymmetric to real shadows relatively to the plane $y = 0$

Modeling of light propagation through shock front inhomogeneity



Parabolic equation

$$\boxed{\frac{\partial E}{\partial y} = \frac{i}{2k_0}\frac{\partial^2 E}{\partial x^2} + \frac{i\Delta n(x,y)}{n_0}k_0 E}$$

Geometrical optics

$$\boxed{\frac{\partial \vec{s}}{\partial \theta} = \vec{k} \qquad \frac{\partial \vec{k}}{\partial \theta} = \frac{1}{2}\nabla n^2}$$

diffraction

refraction

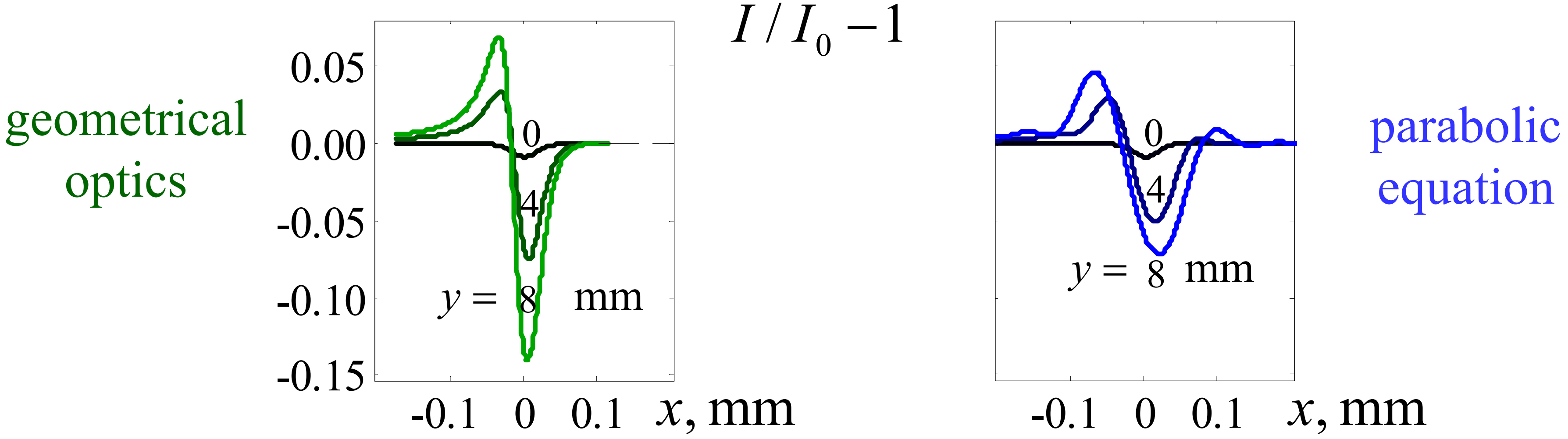
From pressure to refraction index spatial variations

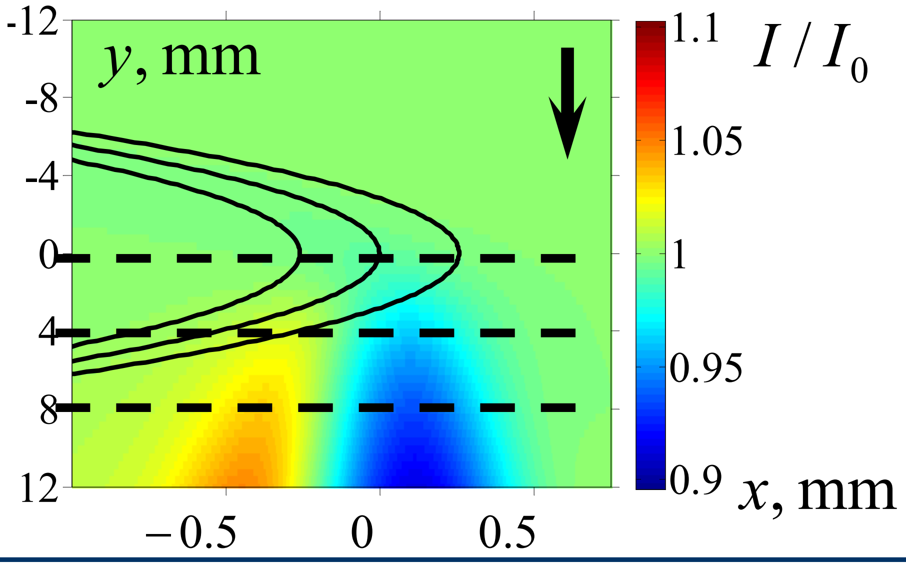
$$\boxed{\Delta n(x,y) = \frac{kp(x,y)}{c_0^2}}$$

“frozen” approximation of propagation media

Two models were used to propagate the light through the shock

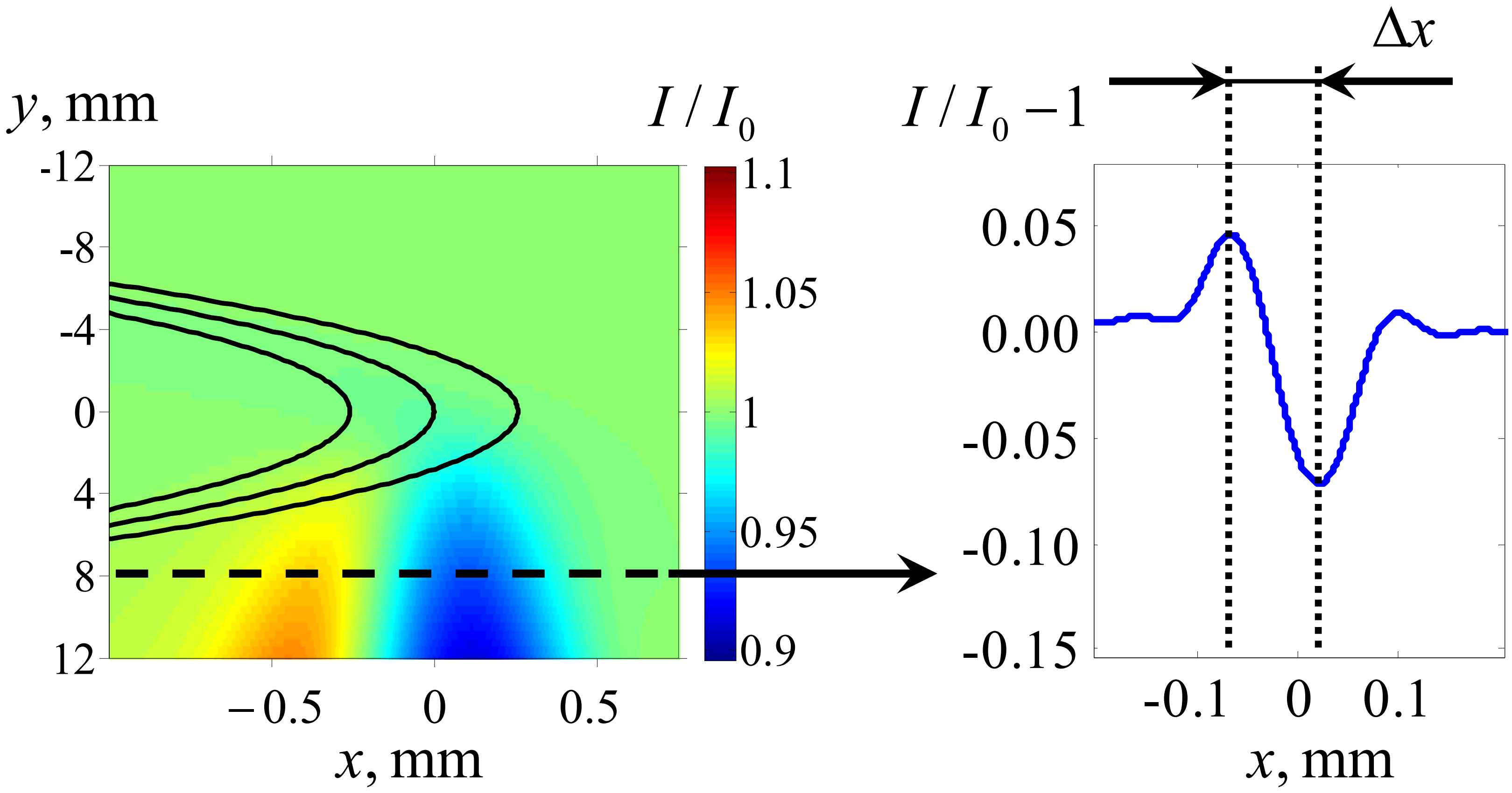
Shadowgram intensity patterns: geometrical optics and parabolic equation





Shadows, obtained by the two models are different

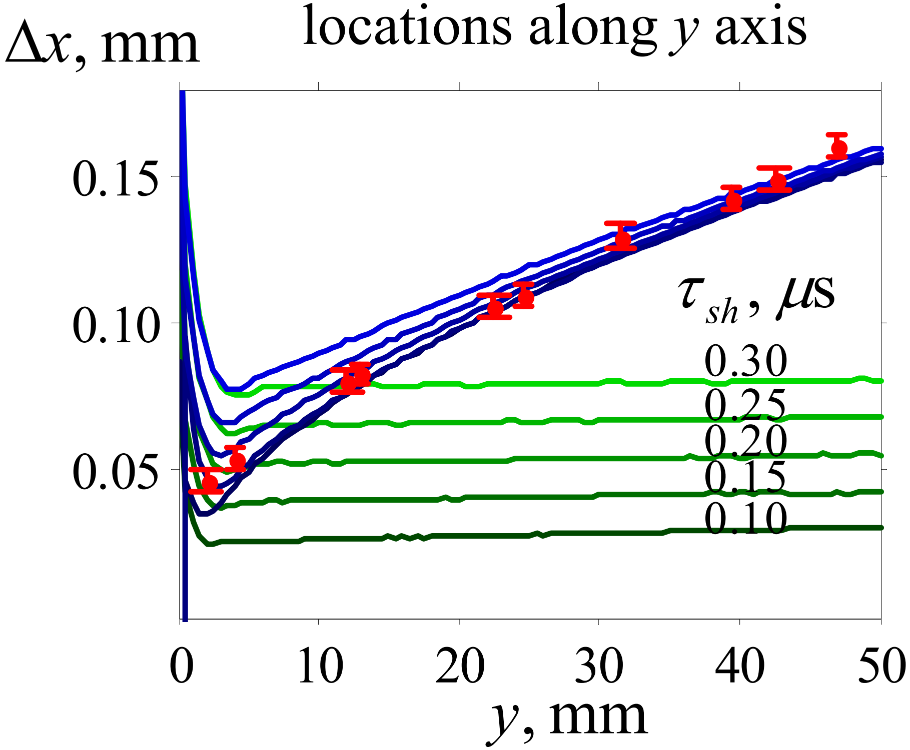
Definition of a shadow width



Shadow width Δx is the distance between minimum and maximum of intensity distribution in the observation plane

Shadow width: geometrical optics, parabolic equation, and experiment

shadow width in different locations along *y* axis



parabolic equation

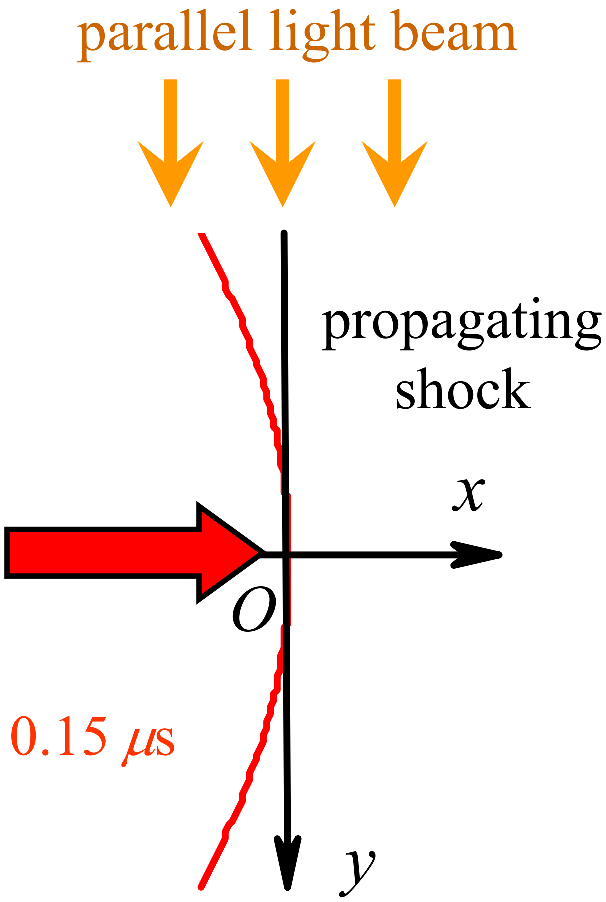
🟦

geometrical optics

🟩

shadowgraphy measurements

🟥 $\tau_{sh} = 0.15\ \mu$s

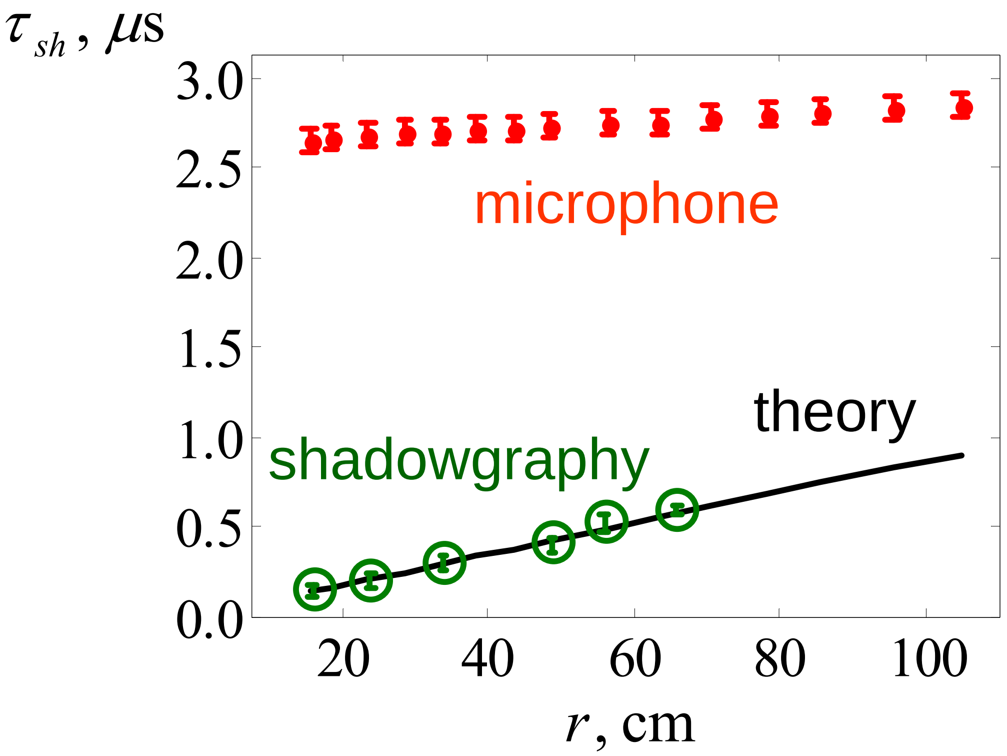


parallel light beam

propagating shock

How to determine shock thickness from images: use diffraction model for the light, use shadowgrams measured close to grazing point $y = 0$

Shock rise time: comparison of simulations, microphone measurements, and shadowgraphy



Good agreement between the results of modeling and shadowgraphy experiment

Conclusions

Simulations and measurements of nonlinear *N*-wave propagation in homogeneous air

Simulations made it possible

- to evaluate the relative roles of absorption, relaxation, and nonlinearity under experimental conditions
- to set a boundary condition for the model using experimental data as nonlinear effects were shown to be dominant in the pulse lengthening
- to obtain frequency response of the broadband measuring system
- to interpret and quantify shadowgraphy images simulating the light propagation through inhomogeneities of the refraction index
- to demonstrate that shadowgraphy method provides accurate resolution of the shocks of 0.15 μs rise tme

Recent publications

P.V. Yuldashev, M.V. Averiyanov, V.A. Khokhlova, S. Ollivier, Ph. Blanc-Benon. **Nonlinear spherically divergent shock waves propagating in a relaxing medium.** Acoust. Phys., 2008, 54(1), pp. 32–41.

P.V. Yuldashev, S. Ollivier, M. Averiyanov, O. Sapozhnikov, V. Khokhlova, Ph. Blanc-Benon. **Nonlinear propagation of spark-generated *N*-waves in air: modeling and measurements using acoustical and optical methods.** J. Acoust. Soc. Am., 2010 (accepted)

Acknowledgments

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and a stipend from the French Government for cotutelle PhD program

Thank you for your attention!