Application of a shadowgraphy method
to measure the shock front
of spark-generated $N$-waves in air

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Outline

• Introduction
  - sonic boom problem and laboratory scale experiment

• Measurement of spark-generated $N$-waves
  - microphone measurements
  - numerical simulation of $N$-wave propagation
  - rise time overestimation in the measurement

  Shadowgraphy method to resolve the fine structure of shocks
  - experimental setup
  - interpretation of shadowgrams
  - geometrical optics versus diffraction model

• Comparison of shadowgraphy and numerical results

• Conclusions
Sonic boom problem: the application involving shock wave measurement

Atmospheric turbulence

- U-wave
- Rounded waveform
- Multi-peaked waveform

100-200 Pa amplitude

Distortions of the N-wave

Classic N-wave

Measured waveforms, Lee and Downing, 1991

Practical importance of prediction and measurement of sonic boom noise: impact on people, buildings etc.
Laboratory-scaled experiment:

$N$-wave propagation through jet turbulence

- Jet
- Turbulence
- Spark source
- Distorted $N$-wave
- Microphones

Distorted $N$-wave - $U$-wave

M.V. Averianov et al., presentation tomorrow
Spark generated $N$-wave in homogeneous air: typical waveform measured with a microphone

Characteristic parameters:
- peak pressure: $P_{\text{max}}$ 1500 Pa
- duration: $T$ 30 – 50 $\mu$s
- rise time: $\tau_{sh}$ 3 $\mu$s

How to define the duration of the pulse? What are the major effects that result in distortion of the pulse waveform?
Theoretical model

Burgers equation extended to include **spherical divergence** and **relaxation**

\[
\frac{\partial p}{\partial r} + \frac{p}{r} = \frac{\varepsilon}{\rho_0 c_0^3} p \frac{\partial p}{\partial t} + \frac{b}{2\rho_0 c_0^3} \frac{\partial^2 p}{\partial t^2} + \sum_{\nu=1}^{2} d_\nu \frac{\partial}{\partial t} \int_{-\infty}^{t} \exp\left(-\frac{t-t'}{t_\nu}\right) \frac{\partial p}{\partial t'} \, dt'
\]

in addition to **acoustic nonlinearity** and **thermoviscous absorption**

\(p\) – acoustic pressure, \(r\) – radial propagation distance, \(\varepsilon = 1.21\) – nonlinear parameter in air, \(b = 5.2 \cdot 10^{-5}\) Pa·s – thermoviscous absorption parameter, 
\(\rho_0 = 1.29\) kg/m\(^3\) – density of the air, \(c_0 = 343.67\) m/s – ambient sound speed

Two relaxation processes of the oxygen and nitrogen molecules are included:

\(\Delta c_1 = 0.11\) m/s, \(\tau_1 = 6.0\) \(\mu\)s (oxygen \(O_2\)),

and \(\Delta c_2 = 0.023\) m/s, \(\tau_2 = 531\) \(\mu\)s (nitrogen \(N_2\))

\[d_\nu = \frac{c_\infty - c_0}{c_0^2} = \frac{\Delta c}{c_0^2}\]
Characteristic time and frequency scale

\[ N\text{-pulse pressure waveform } p, \text{ Pa} \]

\[ \tau, \mu s \]

\[ T = 30 \, \mu s \]

\[ P = 1000 \, \text{Pa} \]

\[ N\text{-pulse spectrum } S \cdot 10^{-3}, \text{Pa} \cdot s \]

Change in sound speed and absorption with frequency due to relaxation

\[ C_0 \]

\[ \alpha \]

\[ f, \text{ Hz} \]
Results of simulations:
effects of nonlinearity, thermoviscous absorption, and relaxation on \( N \)-wave propagation

Thermoviscous absorption:
- reduces the pulse amplitude
- broadens the shock front
- no change in symmetry
- no change in duration of the pulse

Relaxation:
- asymmetric waveform distortion
- reduces the pulse amplitude
- displaces both front and distal shocks towards the direction of propagation

Nonlinearity:
- lengthening of the \( N \)-pulse
- decrease of the shock amplitude
- no change in symmetry
Distorted $N$-wave in modeling and experiment: definition of pulse duration

Pulse duration is defined in the frequency domain by matching the positions of the minima in the measured/modeled pulse spectrum with those in the spectrum of an ideal $N$-wave with an infinitely thin front.

$r = 15 \text{ cm}$

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peak positive pressure, $p_+ r/r_0$, Pa
half pulse duration, $T$, μs

Pulse duration increases mainly due to nonlinear propagation effect:
can be used to set boundary condition in simulation of the experiment
Simulation of the experiment:
calibration of initial pulse amplitude and duration

The change in pulse duration is mainly due to AMPLITUDE dependent NONLINEAR EFFECTS.

Weak shock theory can be applied:

\[ T^2 = T_0^2 \left[ 1 + \frac{\varepsilon}{\rho_0 c_0^3} \frac{p_0}{T_0} r_0 \ln \left( \frac{r}{r_0} \right) \right] \tag{#} \]

Two unknown parameters:
- initial half duration \( T_0 \) and
- peak pressure \( p_0 \)
in the linear dependence of \( T^2 \) over propagation variable \( \ln \left( \frac{r}{r_0} \right) \) in the solution (\#) were obtained by fitting the experimental values over distances 15 cm – 3 m using the method of least squares.

Experimental and modeled waveforms: comparison over propagation distance

Measured and modeled waveforms are in a good agreement

Calibration of the measurement track: the ratio of the experimental and modeled spectrum = total system response
**N-wave parameters: simulations and experiment**

- **pr / r₀, Pa**
  - **Pressure waveform**
  - **theory** (black line)
  - **experiment** (red line)

- **distance:** 42 cm from the spark source

- **pulse duration**
  - **T / 2, μs**
  - **pulse duration**

- **peak pressure**
  - **p_{max} r / r₀, Pa**
  - **peak pressure**

- **r, cm**
  - **r, cm**
N-wave parameters: simulations and experiment

Measured risetime is overestimated up to 10 times

Limited bandwith of the microphone: cut off at ~200 kHz
Shadowgraphy method to measure rise time of the shock with higher accuracy

How to resolve fine structure of the shock front in experiment?

Light intensity distribution in the observation plane is a shadow
Shadowgram of the moving shock front: very short flash of white light

Nanolite KL-L flashlamp – very short 20 ns white light pulses

propagating shock (340 m/s) is a “frozen” object

Shorter light flash duration – better image of the moving object
Experimental setup

- Spark source
- N-wave
- Focusing lens
- Flash lamp
- Camera objective
- CCD camera, 1600×1186 pixels
- 4138 B&K 1/8” microphone in a baffle
How to interpret shadowgrams and what causes inversion of light intensity?
Images processing: from 2D to 1D

Spherical curvature compensation + vertical averaging significantly suppress noise. As an output – 1D intensity distribution along $x$ axis.
Inversion of intensity distribution: explanation

Refraction of rays on the shock

real light intensity in space

imaginary light intensity in space

“Virtual” shadows are antisymmetric to real shadows relatively to the plane $y = 0$
Modeling of light propagation through shock front inhomogeneity

Parabolic equation

\[ \frac{\partial E}{\partial y} = \frac{i}{2k_0} \frac{\partial^2 E}{\partial x^2} + \frac{i \Delta n(x, y)}{n_0} k_0 E \]

From pressure to refraction index spatial variations

diffraction

Geometrical optics

\[ \frac{\partial \vec{s}}{\partial \theta} = \vec{k} \]
\[ \frac{\partial \vec{k}}{\partial \theta} = \frac{1}{2} \nabla n^2 \]

refraction

“frozen” approximation of propagation media

\[ \Delta n(x, y) = \frac{kp(x, y)}{c_0^2} \]

Two models were used to propagate the light through the shock
Shadowgram intensity patterns: geometrical optics and parabolic equation

Shadows, obtained by the two models are different
Definition of a shadow width

Shadow width $\Delta x$ is the distance between minimum and maximum of intensity distribution in the observation plane.
Shadow width: geometrical optics, parabolic equation, and experiment

How to determine shock thickness from images: use diffraction model for the light, use shadowgrams measured close to grazing point $y = 0$.
Shock rise time: comparison of simulations, microphone measurements, and shadowgraphy

\( \tau_{sh}, \mu s \)

- Microphone
- Shadowgraphy
- Theory

Good agreement between the results of modeling and shadowgraphy experiment
Conclusions

Simulations and measurements of nonlinear $N$-wave propagation in homogeneous air

Simulations made it possible

- to evaluate the relative roles of absorption, relaxation, and nonlinearity under experimental conditions
- to set a boundary condition for the model using experimental data as nonlinear effects were shown to be dominant in the pulse lengthening
- to obtain frequency response of the broadband measuring system
- to interpret and quantify shadowgraphy images simulating the light propagation through inhomogeneities of the refraction index
- to demonstrate that shadowgraphy method provides accurate resolution of the shocks of 0.15 μs rise time
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Thank you for your attention!